(3+1)-D self-focusing dynamics using split-step quasi-discrete Hankel transform

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We present a method for the numerical calculation of the split-step quasi-discrete Hankel transform (SSQDHT) that has a high computational efficiency and an accuracy. The (3+1)-D nonlinear self-focusing dynamics can be investigated by SSQDHT. The results show that the (3+1)-D self-focusing characterization is simulated successfully for different initial noise, chirp and coupled beams.

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1. Introduction

Self-focusing dynamics of optical beams has attracted much more interest. In particular, the spatiotemporal self-focusing of ultrashort optical pulses has been the subject of intense investigations. The remarkable richness of spatial and temporal nonlinear phenomena are observed, such as self-steepening, pulse splitting, multiphoton ionization, multiple filamentation, the universal self-similar spatial collapse profile known as the *Townes* profile, and supercontinuum generation [1-7]. The evolution of self-focusing dynamics could be described by (3+1)-D nonlinear Schrödinger equation (NLSE). It is necessary to solve the NLSE to understand various dynamics occurring during the self-focusing beam transmission. However, except in some special cases, it is not possible to solve the NLSE analytically when the nonlinear, diffraction and dispersion effect are considered. A large number of methods can be used for this purpose, such as the split-step Fourier transform (SSFT) and Crank-Nicholson scheme [8-15]. However, most of the approach known takes long time and does not have enough accuracy because the spatiotemporal self-focusing problems are considered in a lot of computations and CPU time is required. In this paper, we demonstrate the split-step quasi-discrete Hankel transform (SSQDHT) for speeding up the computations and high precision for its simple matrix-vector multiplication.

Based on Dini series expansion, the quasi-discrete Hankel transform (QDHT) algorithm can not only directly fast simulate the values on symmetry axis, but also keep very high precision after numerous transformations [16-19]. The results show our method runs fast and has high accuracy. Thus, the QDHT is really appropriate to study the spatiotemporal self-focusing dynamics.

In this paper, firstly we propose an approach called SSQDHT which is fast and has high precision. Then the (3+1)-D self-focusing dynamics is theoretically analyzed for different initial noise, different initial chirp and different initial separation of the coupled beam. The final section is the conclusion.

2. Quasi-discrete Hankel transform (QDHT) algorithm

In propagation of optical beams through systems with a spatiotemporal symmetry, the half order Hankel transform pairs are [16-19]

$$g(\rho) = 2\pi\int_0^\infty f(r) J_{\frac{1}{2}}(2\pi r\rho)\, r\, dr$$
$$f(r) = 2\pi\int_0^\infty g(\rho) J_{\frac{1}{2}}(2\pi r\rho)\, \rho\, d\rho \quad (1)$$

where $r^2 = x^2 + y^2 + t^2$ is the spatiotemporal coordinate; ρ is the spatiotemporal frequency; $J_{\frac{1}{2}}$ is the half-order Bessel function of the first kind; f(r) and g(ρ) can be either real or complex functions and are of axial symmetry mathematically, representing the field distributions in a spatial domain and spatial frequency domain, respectively.

Based on the half order Bessel series, we evaluate the $r = a_n / 2\pi\beta$ and $\rho = a_m / 2\pi b$, then the f(r) and g(ρ) of the equation(1) can be expanded

$$g(\frac{a_m}{2\pi b}) = \frac{1}{\pi\beta^2}\sum_{n=1}^{N} f(\frac{a_n}{2\pi\beta})J_{\frac{3}{2}}^{-2}(a_n)J_{\frac{1}{2}}(\frac{a_n a_m}{S})$$
$$f(\frac{a_n}{2\pi\beta}) = \frac{1}{\pi b^2}\sum_{m=1}^{N} g(\frac{a_m}{2\pi b})J_{\frac{3}{2}}^{-2}(a_m)J_{\frac{1}{2}}(\frac{a_n a_m}{S}) \qquad (2)$$

where a_n (n = 1, 2, 3 . . .) are the roots of the half-order Bessel function , $S \equiv 2\pi\beta\mathrm{b}$.

Eq. (2) can be rewritten in a symmetric form by defining the vectors

$$G(m) = g(\frac{a_m}{2\pi b})\left|J_{\frac{3}{2}}^{-1}(a_m)\right|\beta$$
$$F(n) = f(\frac{a_n}{2\pi\beta})\left|J_{\frac{3}{2}}^{-1}(a_n)\right|b \qquad (3)$$

Therefore Eq. (3) can becomes

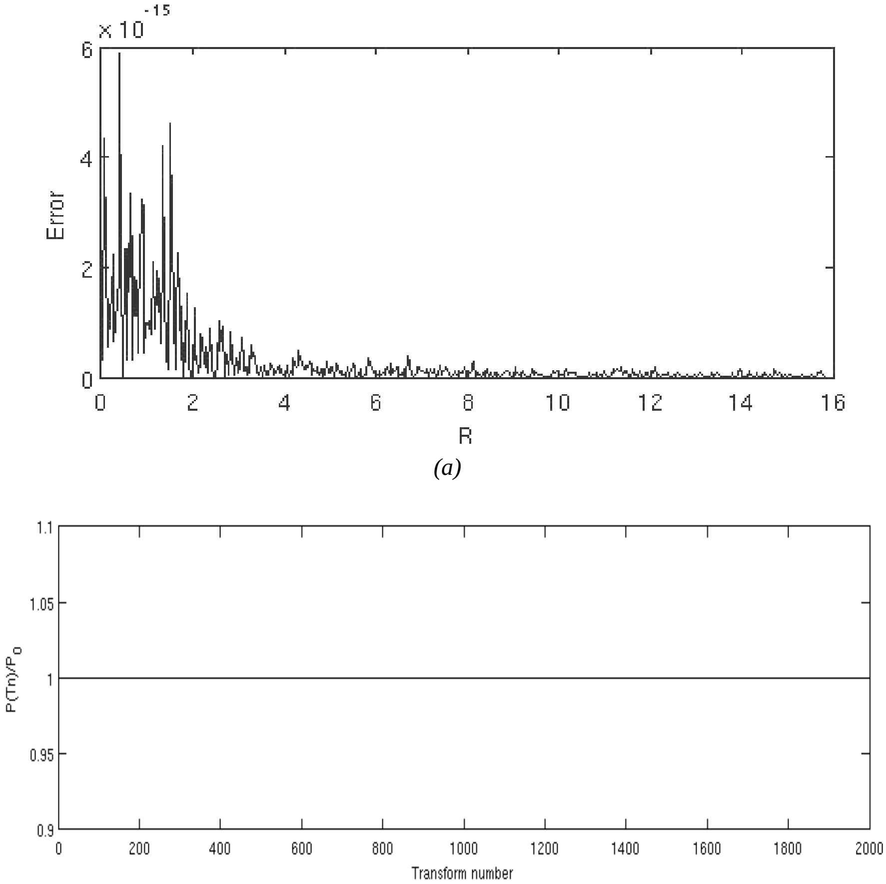
$$G(m) = \sum_{n=1}^{N} C_{nm}F(n) \quad ,$$
$$F(n) = \sum_{m=1}^{N} C_{nm}G(m) \qquad (4)$$

where C_{nm} is the elements of an (N+1)-dimensional transformation matrix C and given by

$$C_{nm} = \frac{2}{S}\left|J_{\frac{3}{2}}^{-1}(a_n)\right|\left|J_{\frac{3}{2}}^{-1}(a_m)\right|J_{\frac{1}{2}}(\frac{a_n a_m}{S}) \qquad (5)$$

When the sample number is N=500, the error is about 10^{-15} as Fig. 1 (a) shown. And the running time is T= $1.5625000\mathrm{E}\times10^{-2}+\mathrm{N_c}\times1.0742188\times10^{-2}$, where N_c is the cycle number. We can see from Fig. 1 (b) that the values of energy remain the same after 2000 transforming by QDHT, which indicates the program executes successfully.

The results illustrate the QDHT performs well, which not only has higher accuracy, but also is fast enough. Therefore, we can solve the (3+1)-D self-focusing dynamics based on QDHT.



(a)

(b)

Fig. 1. (a) The error versus radius for twice transformation; (b) the power with transform number by QDHT

3. (3+1) D Nonlinear self-focusing dynamics by SSQDHT

The time-dependent normalization paraxial wave equation in the presence of group-velocity dispersion (GVD) is

$$i\frac{\partial A}{\partial Z}+\frac{\beta_2}{2}\left(\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}-s_d\frac{\partial^2 A}{\partial t^2}\right)+\gamma A|A|^2=0 \qquad (6)$$

where A is the slowly varying envelope, and Z is the direction of propagation; β_2 is the dispersion coefficient and γ is nonlinear parameter; $s_d=\mathrm{sgn}(1)$ and $s_d=\mathrm{sgn}(-1)$ corresponding to normal dispersion and anomalous dispersion.

For anomalous dispersion, the Eq. (1) can be written in the form

$$i\frac{\partial A}{\partial Z}+\frac{\beta_2}{2}\Delta_\perp A+\gamma A|A|^2=0 \qquad (7)$$

where $\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial t^2}$ represents the transverse Laplacian, and vector $r\equiv(x,y,t)$ has space components *x* and *y* and time component *t*.

And the Eq. (7) can be written as

$$A_Z(r,Z)=(\hat{D}+\hat{N})\mathrm{A} \qquad (8)$$

where $\hat{D}$ is an operator that accounts for diffraction and dispersion in linear media and $\hat{N}$ is nonlinear operator that stands for the effect of the Kerr nonlinearities. These operators are given by

$$\hat{D}=\mathrm{i}\frac{\beta_2}{2}\Delta_\perp \qquad (9.a)$$

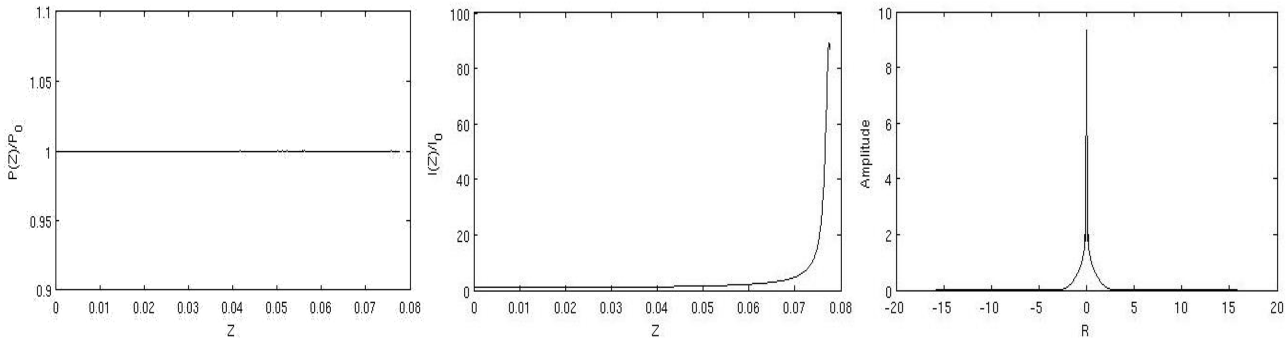
$$\hat{N}=\mathrm{i}\gamma|A|^2 \qquad (9.b)$$

$$A(r,Z+\mathrm{h}) \approx \exp\left(\frac{h}{2}\hat{D}\right)\exp\left(\int_Z^{Z+h}\hat{N}(Z')dZ'\right)\exp\left(\frac{h}{2}\hat{D}\right)\mathrm{A}(r,Z) \qquad (10)$$

Then the SSQDHT is as follows.

$$\begin{aligned}&A(r,Z+\mathrm{h})\\&=r^{(-\frac{1}{2})}QDHT(QDHT(r^{\frac{1}{2}}(r^{(-\frac{1}{2})}\\&QDHT(QDHT(r^{\frac{1}{2}}\mathrm{A}(r,Z)))\\&\exp(-i4\pi\rho^2\frac{h}{2}))i|A|^2h))\exp(-i4\pi\rho^2\frac{h}{2}))\end{aligned} \qquad (11)$$

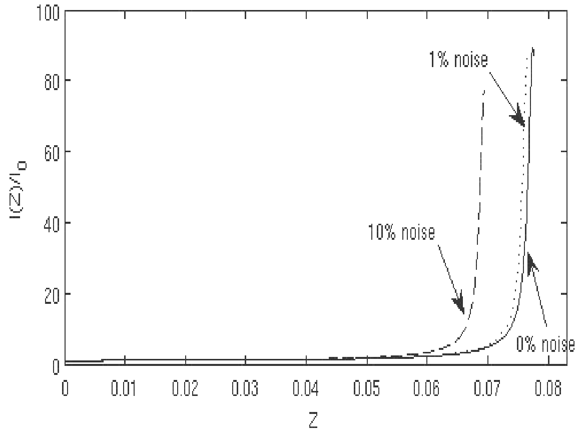
The following is the simulations by SSQDHT, and the input Gaussians beams is $A(0,r)=A_0e^{-r^2/2}=\sqrt{(P/4\pi)}e^{-r^2/2}$ with $P=50*11.7$.



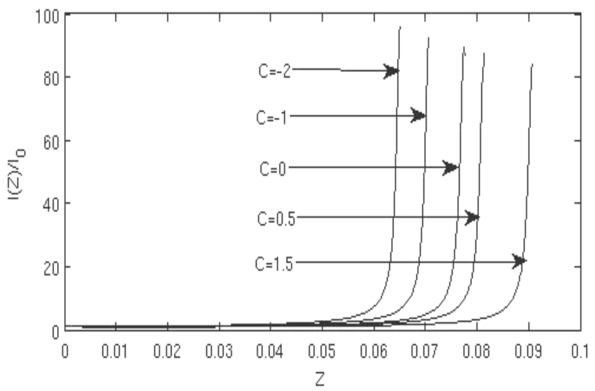
(a) (b) (c)

Fig. 2. (a) Power versus distance, (b) Axial intensity versus distance, (c) the profile of self-focusing beam by SSQDHT

Fig. 2 (a-c) indicates the self-focusing evolution of the beam. Fig. 2 (a) exhibits that the power remains unchanged during propagation. The beam collapses is about in 0.08 as Fig. 2 (b) shown. Fig. 2 (c) is the profile of the beam at self-focusing distance. From Fig. 2 (a-c) we can know the (3+1) D self-focusing dynamics can be well simulated by the SSQDHT algorithm. Therefore, the SSQDHT is appropriate to solve the problem of wave dynamics.



(a)



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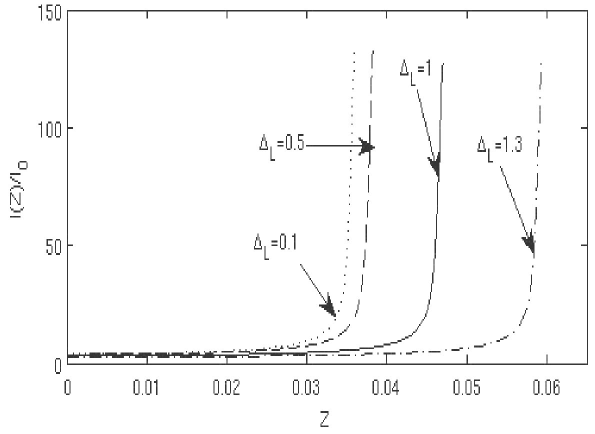

(c)

Fig. 3. Axial intensity versus propagation distance for (a) different initial noise, (b) chirp, and (c) separationΔL by SSQDHT

The (3+1) D collapse of the beam is highly sensitive because the self-focusing is characterized by a delicate balance between the diffraction, dispersion and Kerr nonlinearity. We apply SSQDHT model to demonstrate self-focusing dynamics for different initial noise, chirp and coupled beam separation as follows.

The beam always contains some noise because of various influences of quantum noise and fluctuations of various technical origins. It is necessary to study how the noise affects the beams self-focusing dynamics. Fig. 3 (a) illustrates the influence of initial noise. The beam collapses in a smaller distance and the on-axis intensity decreases for 10% random noise while they alter little for 1% random noise.

The spatiotemporal chirp is helpful for the suppression of longitudinal mode competition in the laser design. Fig. 3 (b) shows the self-focusing level varies with initial chirp and C is the spatiotemporal chirp. The collapse becomes faster and the on-axis intensity increases when the absolute value of the negative chirp increases. On the other hand, the beam collapses later for the positive chirp increasing. The accuracy of beam shaping relates to the degree of spatiotemporal chirp at the focal plane.

Fig. 3 (c) reveals the axial intensity of two coupled Gaussian beams changes with self-focusing distance for different initial Δ_L, and Δ_L is the initial spatiotemporal separation. The collapse varies with different Δ_L. The self-focusing distance enhances while the on-axis intensity reduces with the Δ_L increasing.

4. Conclusion

To sum up, we present a significant technique to numerically solve (3+1) D self-focusing problems. The SSQDHT model is simplicity, flexibility, good accuracy, and relatively low computing cost. We also demonstrate how to apply the SSQDHT to study the (3+1) D self-focusing dynamics for different initial noise, chirp and separation of the coupled beams.

With the help of SSQDHT, it is possible to accurately track a beam much closer to its (3+1) D physical collapse due to self-focusing than other existing methods. It is a significant method to simulate the light bullets which is very important for telecommunication system due to their self confined structure.

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