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A noise robust method of distance measurement based on synchronization of chaotic oscillator

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In this paper, a noise robust method of measuring target distance through chaotic synchronization is proposed. In the method, a reference sinusoidal signal is superimposed to the radar transmitted signal generated by the master chaotic system, and the radar received signal forces the slave chaotic system to generate the chaotic signal embedded in it, which can recover the sinusoidal signal by chaotic synchronization. The phase error of the reference sinusoidal signal and the recovered sinusoidal signal allows computation of the flight time of the transmitted signal. Thus, the distance of the radar from the target can be obtained. Unlike the existing methods, the proposed method is more robust to noise. The noise robust feature make the proposed method has more application potential than that of the existing methods. Finally, the proposed method is illustrated by numerical simulations to show its effectiveness on target distance measurement.

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Keywords: Chaos synchronization, Non-linear oscillators, Distance measurement

1. Introduction

Chaos synchronization which was first introduced by Pecora and Carroll [1], has attracted much attention in recent years, especially its applications to electronics [2], optical communication [3-6], and radar [7-16].

The target distance measurement is an important content of chaotic radar. It has been investigated by lots of researchers and a variety of methods have been proposed for target distance measurement. In[8], the transmitted signals are reconstructed through chaotic initial condition estimation methods. The target distance is obtained by computing the correlation function of the reconstructed transmitted signal and the received signal. This method is simple for operation. However, in practice, especially when added noise, initial condition estimation is not always accurate, and a small initial condition estimation error can cause large reconstruction error of transmitted signal. This limits the application of the method in practical engineering. In [9], a novel method of target distance measurement based on synchronization of Chua's chaotic system is proposed. The distance measurement is realized by comparing the phase of the transmitted signal with the received signal after synchronization. This method is easy to implement and the distance measurement is accurate in ideal condition. However, like the method in [8], the method in [9] is also sensitive to noise. Since in practical engineering, noise can not be avoided, thus the noise robust method is necessary needed.

In this paper, a noise robust method for measuring target distance is proposed. In the proposed method, firstly, a reference sinusoidal signal is superimposed to the radar transmitted signal. Then, let the radar received signal force the slave chaotic system to generate the same chaotic signal. In this way, the sinusoidal signal can be recovered by chaotic synchronization. The difference of the phases of the two sinusoidal signals allows computation of the flight time of the signal from radar to the target, and thus the target distance can be obtained.

Unlike the method in [8,9], the proposed method in this paper is more robust to noise by exploring the characters of a two-frequency system[14]. We find there are three advantages when use the two-frequency system in chaotic radar. The main one is: the slow part of two-frequency system is designed as a noise filter which makes the whole system robust to noise. This can overcome the noise sensitivity of the traditional chaotic system (such as Chua's) when doing synchronization. Even the signal-to-noise ratios (SNR) is low, the performance of chaotic synchronization is also well. The reason for this is illustrated in detail in section 3. The second one is: the high frequency signal make the radar transmitted signal have wide bandwidth and high distance resolution, which is suitable for high precision distance measurement. The third is: the low frequency signal can be extracted after synchronization in the receiving part. The narrowband signal is suitable for obtaining the Doppler shift of moving target and the Doppler shift error is always smaller than that when using wideband signal.

This paper is organized as follows. In section 2, the two-frequency chaotic system is introduced. In section 3, the analysis of noise robust feature of the two-frequency system is performed. In section 4, the noise robust target distance measurement method is proposed based on synchronization of the two-frequency chaotic system. In section 5, numerical simulation is done to verify effectiveness of the theory. Brief conclusion of this paper is drawn in section 6.

2. Two-frequency chaotic system

The two-frequency chaotic system is proposed by Carroll [14]. It is described as

$$\begin{cases} \dot{x}_1 = -(c_1x_1 + 0.5x_2 + x_3) \\ \dot{x}_2 = -(-x_1 - c_2x_2 + x_4) \\ \dot{x}_3 = -(-g(x_1) + x_3) \\ \dot{x}_4 = -c_3(c_4x_4 + 0.5x_5 + x_6 + c_5|x_1|) \\ \dot{x}_5 = -c_3(-x_4 + c_6x_5) \\ \dot{x}_6 = -c_3(-g(x_4) + x_6) \end{cases} \qquad (1)$$

$$g(x) = \begin{cases} 15(x-3), x > 3 \\ 0, x \le 3 \end{cases}$$

where $c_1 = 0.02, c_2 = 0.13, c_3 = 0.01, c_4 = 0.1, c_5 = 0.5$, $c_6 = 0.1$. Eq.(1) contains a high frequency part and a low frequency part. Its fast attractor and low attractor part is shown in Fig. 1 and Fig.2 respectively. The $x_1 \sim x_3$ equation describes a fast chaotic system which is the high frequency part. The $x_4 \sim x_6$ is a damped system coupled to the high frequency part. The waveform of x_1, x_2 and x_4, x_5 is shown in Fig. 3 and Fig.4. The frequency band of the damped system is determined by the time constant c_3, which is between 0 and 1. For example, $c_3 = 0.01$ means the frequency of the $x_4 \sim x_6$ system is one-hundredth of that of the $x_1 \sim x_3$ system. The synchronization of the two-frequency chaotic system is robust to added noise, which will be offered in section 3.

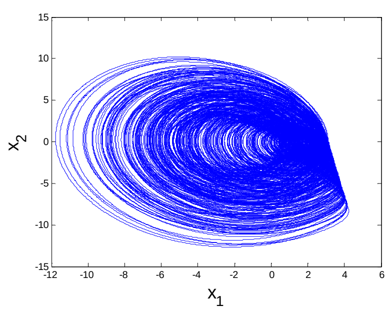


Fig. 1. x_1, x_2 attractor of fast part For Eq.(1).

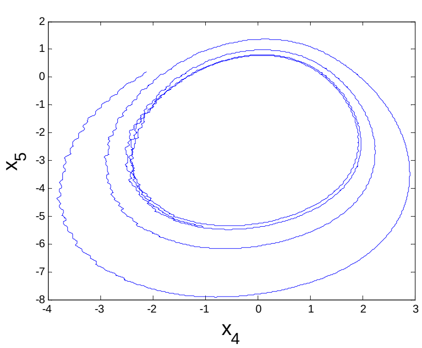


Fig. 2. x_4, x_5 attractor of slow part for Eq.(1)

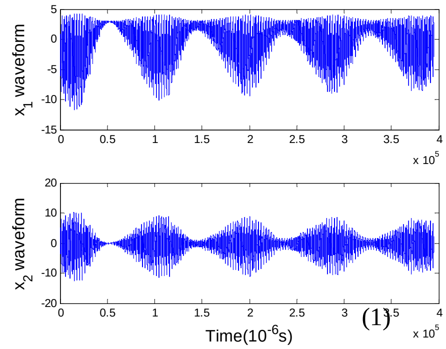


Fig. 3. x_1, x_2 waveform of fast part for Eq.(1).

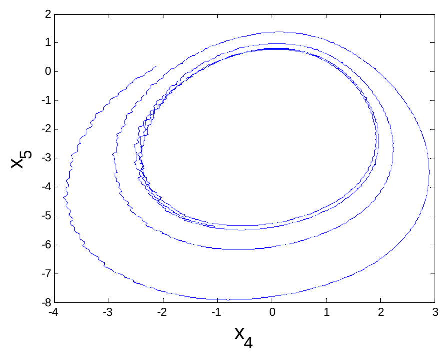


Fig. 4. x_4, x_5 waveform of slow part for Eq.(1).

3. Analysis of noise robust system

In this section the mechanism for noise robust of the system 2 is explored.

Seen from section 2, the chaotic system is consists of a fast part (Rossler chaotic system) coupled to a slow part. It has been shown that increasing the time scale separation between fast and slow parts increases the ability of these systems to resist added noise[13]. Herein, we will offer the reason for its noise robust.

Notice that the slave system of Eq.(1) in Ref. 14 is described by:

$$\begin{cases} x_d = x_2 + n(t) \\ \dot{y}_1 = -(c_1y_1 + 0.5y_2 + y_3) \\ \dot{y}_2 = -(-y_1 - c_2x_d + y_4) \\ \dot{y}_3 = -(-g(y_1) + y_3) \\ \dot{y}_4 = -c_3(c_4y_4 + 0.5y_5 + y_6 + c_5|y_1|) \\ \dot{y}_5 = -c_3(-y_4 + c_6y_5) \\ \dot{y}_6 = -c_3(-g(y_4) + y_6) \end{cases} \qquad (2)$$

$$g(y) = \begin{cases} 15(y-3), y > 3 \\ 0, y \le 3 \end{cases}$$

The parameters in Eq. (2) are chosen to match the parameters in Eq. (1). The term $n(t)$ in Eq. (2) is an additive white noise term. Here, Eq.(2) can be rewritten as:

$$\begin{cases} x_d = x_2 + n(t) \\ c_3 \dfrac{dy_1}{d\tau} = -(c_1 y_1 + 0.5 y_2 + y_3) \\ c_3 \dfrac{dy_2}{d\tau} = -(-y_1 - c_2 x_d + y_4) \\ c_3 \dfrac{dy_3}{d\tau} = -(-g(y_1) + y_3) \\ \dfrac{dy_4}{d\tau} = -(c_4 y_4 + 0.5 y_5 + y_6 + c_5 |y_1|) \\ \dfrac{dy_5}{d\tau} = (-y_4 + c_6 y_5) \\ \dfrac{dy_6}{d\tau} = -(-g(y_4) + y_6) \end{cases} \tag{3}$$

$$g(y) = \begin{cases} 15(y-3), y > 3 \\ 0, y \le 3 \end{cases}$$

where $\tau = c_3 t$ represents the slow time scale. When $c_3 \to 0$, the fast time part $y_1 \sim y_3$ could be rewritten as

$$\begin{cases} c_1 y_1 + 0.5 y_2 + y_3 = 0 \\ -y_1 - c_2 x_d + y_4 = 0 \\ -g(y_1) + y_3 = 0 \end{cases} \tag{4}$$

Substituting the solution of Eq.(4) into the slow equations ($y_4 \sim y_6$ part in Eq.(3)) yields

$$\begin{cases} \dfrac{dy_4}{d\tau} = -(c_4 + c_5) y_4 - 0.5 y_5 - y_6 + c_2 c_5 |x_d| \\ \dfrac{dy_5}{d\tau} = (-y_4 + c_6 y_5) \\ \dfrac{dy_6}{d\tau} = -(-g(y_4) + y_6) \end{cases} \tag{5}$$

The Jacobian matrix for the slow system defined by Eq.(5) is

$$J = \begin{pmatrix} -(c_4 + c_5) & -0.5 & -1 \\ -1 & c_6 & 0 \\ \dfrac{\partial g(y_4)}{\partial y_4} & 0 & -1 \end{pmatrix} \tag{6}$$

In this way, the values of the fast variables $y_1 \sim y_3$ do not appear in the Jacobian of the slow system. That means the slow part does not affect by the additive noise (noise is added in fast part x_2 term) .

In fact, the slow part of the system acts like a narrow band filter. Thus, it has the feature of noise filtering. When the slow time scale c_3 is small, the bandwidth of the low frequency is so small, it will not be strongly affected by the additive noise. If $c_3 \to 0$, the system will not be affected by noise in theory when synchronize the slave system defined by Eq.(2) to the master system defined by Eq.(1).

In all, the slow part acting like the noise filter makes the two-frequency system have the noise resistant feature when doing synchronization.

4. Noise robust method for target distance measurement

In this section a noise robust method for target distance measurement is proposed after developing the two-frequency chaotic system. Firstly, a reference sinusoidal signal is superimposed to the radar transmitted signal which is generated by the two-frequency. Then, let the radar received signal be the master signal which forces the slave chaotic system to generate the chaotic signal embedded in it. The sinusoidal signal is then recovered by chaotic synchronization even under strong noise condition. Next, compute the phase error of the two sinusoidal signals and get the flight time of the signal. Finally, the distance of the radar from the target can be obtained from the flight time. The main idea of the proposed method can be illustrated by Fig.5.

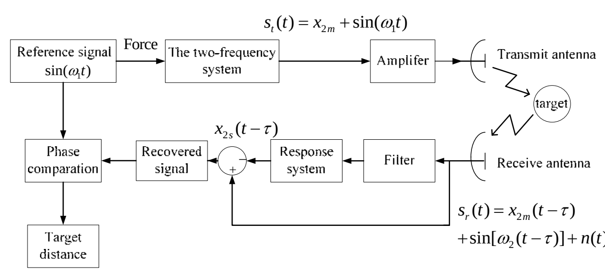


Fig. 5. The schematic of the proposed method for target distance measurement

In the proposed method, the radar transmitted signal $s_t(t)$ is described as

$$s_t(t) = x_{2m}(t) + \sin(\omega_1 t + \theta_0) \tag{7}$$

where $\sin(\omega_1 t + \theta_0)$ is an embedded reference signal. ω_1 is a constant and defined as the angular frequency, and θ_0 is defined as the initial phase. $x_{2m}(t)$ is generated by the two-frequency chaotic system which is defined as

$$\begin{cases} \dot{x}_{1m} = -\{c_1 x_{1m} + 0.5[x_{2m} + \sin(\omega_1 t)] + x_{3m}\} \\ \dot{x}_{2m} = -\{-x_{1m} - c_2[x_{2m} + \sin(\omega_1 t)] + x_{4m}\} \\ \dot{x}_{3m} = -[-g(x_{1m}) + x_{3m}] \\ \dot{x}_{4m} = -c_3(c_4 x_{4m} + 0.5x_{5m} + x_{6m} + c_5|x_{1m}|) \\ \dot{x}_{5m} = -c_3(-x_{4m} + c_6 x_{5m}) \\ \dot{x}_{6m} = -c_3[-g(x_{4m}) + x_{6m}] \end{cases} \tag{8}$$

The echo signal $s_r(t)$ which is reflected from the target is

$$s_r(t) = x_{2m}(t-\tau) + \sin[\omega_2(t-\tau) + \theta_1] + n(t) \tag{9}$$

where τ represents the time taken by the transmitted signal to return to the receiver from the target. ω_2 is the angular frequency, which is different to ω_1 due to the target Doppler shift. θ_1 represents the initial phase. $n(t)$ is the added white Gauss noise.

Let $s_r(t)$ be the master signal. The slave system which is defined by Eq.(10) , is driven by $s_r(t)$.

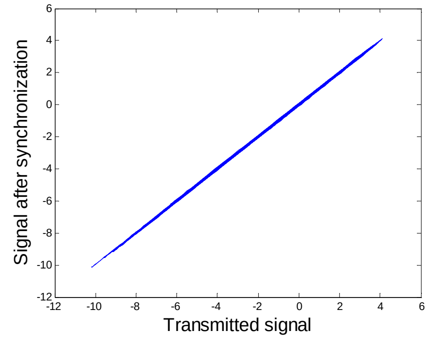
$$\begin{cases} \dot{x}_{1s} = -(c_1 x_{1s} + 0.5s_r + x_{3s}) \\ \dot{x}_{2s} = -(-x_{1s} - c_2 s_r + x_{4s}) \\ \dot{x}_{3s} = -[-g(x_{1s}) + x_{3s}] \\ \dot{x}_{4s} = -c_3(c_4 x_{4s} + 0.5x_{5s} + x_{6s} + c_5|x_{1s}|) \\ \dot{x}_{5s} = -c_3(-x_{4s} + c_6 x_{5s}) \\ \dot{x}_{6s} = -c_3[-g(x_{4s}) + x_{6s}] \end{cases} \tag{10}$$

In the ideal condition (no noise condition), the output of $x_{2s}(t-\tau)$ in Eq.(10) should be the same as that of $x_{2m}(t-\tau)$ in Eq.(8) when the two system tent to synchronize. In this case, $s_r(t)$ subtracts the signal $x_{2s}(t-\tau)$ should only have one term $\sin[\omega_2(t-\tau)+\theta_1]$. However, in practice, noise is anywhere. Then have

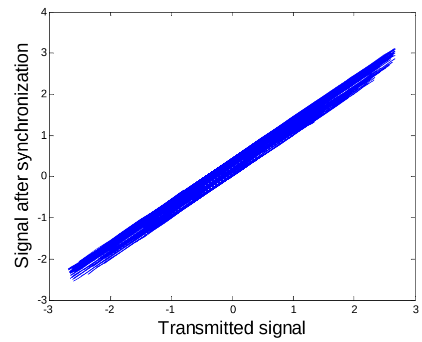
$$s_r(t) - x_{2s}(t-\tau) = \sin[\omega_2(t-\tau) + \theta_1] + \varepsilon_e \tag{11}$$

where ε_e is the synchronization error due to noise.

Herein, the chaotic synchronization performances of the proposed method and that of the method in [9] are shown in Fig. 6-Fig. 8. In Fig. 6, it is shows that when the noise is weak (SNR=40dB), the transmitted signal and the signal after synchronization are nearly along a diagonal, which means the two signals are almost same. When the noise becomes strong (SNR=20dB, in Fig. 7), the performance of the proposed method is not much affected by the noise, while the performance of the method in [9] is strongly affected by the noise. The two signals tend to diverge from the diagonal in Fig. 7(b). When SNR=0dB, the method in [9] nearly can not make the slave system synchronize to the master system, while in the proposed method the transmitted signal and the signal after synchronization is still along a diagonal, which means the synchronization error is still small. This indicates that the proposed method performs better than the method in [9] under noise condition.

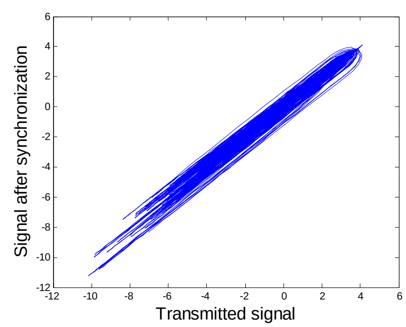


(a)

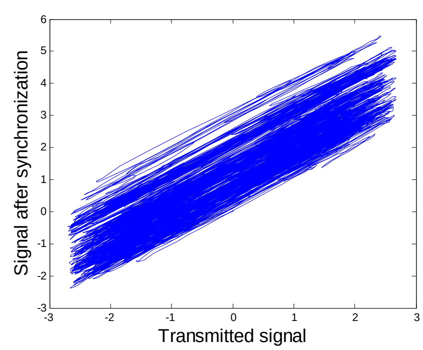


(b)

Fig. 6. Synchronization performances under SNR=40dB (a) the performance of the proposed method (b) the performance of the method in [9]

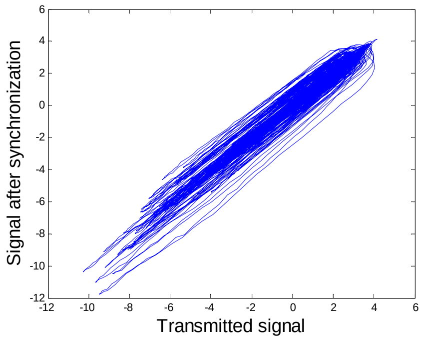


(a)

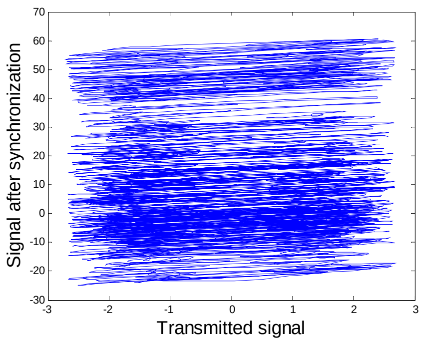


(b)

Fig. 7. Synchronization performances under SNR=20dB(a) the performance of the proposed method (b) the performance of the method in [9].



(a)



(b)

Fig. 8. Synchronization performances under SNR=0dB
(a) the performance of the proposed method
(b) the performance of the method in [9].

Besides the noise robust feature, the proposed method has another two advantages. One is the high frequency signal of the two-frequency system makes the radar transmitted signal have wide bandwidth and high distance resolution, which is suitable for high precision distance measurement. The other is the low frequency signal can be extracted after synchronization in the receiving part, which is suitable for getting the Doppler shift of moving target. The Doppler shift error is always smaller than that when using wideband signal. These two advantages can be illustrated by Fig. 9. This paper is focuses on target distance measurement. The velocity measurement will be studied latter.

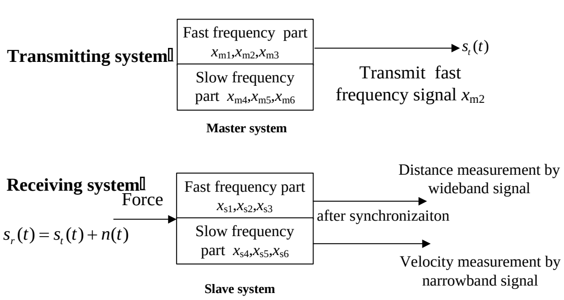


Fig. 9. The chaotic radar schematic for distance and velocity measurement.

Next, the way to obtain the target distance is given by computing the phase error of the reference sinusoidal signal and the recovered sinusoidal signal in noise condition.

Let the recovered sinusoidal signal $m(t)$ as

$$m(t) = \sin[\omega_2(t-\tau)+\theta_1]+\varepsilon_e \tag{12}$$

In order to obtain the phase error of recovered signal $m(t)$ and the reference signal $\sin(\omega_1 t+\theta_0)$, herein , a new way is given by using Hilbert transform. Unlike the method in [9] which gets the phase error of the two signals by comparing them directly in time domain, in the proposed method Hilbert transform of $m(t)$ is computed, and the phase error is obtained based on the analytic signal of $m(t)$. The reason in doing so is that under noise condition the exact phase error may not be got directly. In order to overcome this, a method of getting the phase error of the two signal by Hilbert transform is offered. It is shown as follows.

Firstly, compute the analytic signal $\psi(t)$:

$$\psi(t) = m(t) + jm'(t) = A(t)e^{j\theta(t)} \tag{13}$$

where $A(t)$ is the amplitude of $\psi(t)$, $m'(t)$ is the Hilbert transform of $m(t)$, which is defined by:

$$m'(t) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau \tag{14}$$

where $P.V.$ represents the integration of Cauchy principle value.

Then, the phase of the recovered signal is

$$\theta(t) = \arctan \frac{m'(t)}{m(t)} \tag{15}$$

Thus, the time taken by the transmitted signal to return to the receiver from the target is

$$\tau = \frac{\Delta\phi}{\omega_1} = \frac{\theta(t)-\theta_0(t)}{\omega_1} \tag{16}$$

Finally, the target distance r_0 can be obtained by

$$r_0 = \frac{\tau c}{2} \tag{17}$$

where c is the light velocity.

5. Numerical simulation

In this section, numerical simulation is given to verify the effectiveness of the proposed method.

Let the angular frequency $\omega_1 = 10\text{kHz}$ and the target is 1500m far from the radar under SNR=0dB. The signal bandwidth is 100MHz. The transmitted signal is $s_t(t)$ described by Eq.(7) and Eq.(8), where $c_1 = 0.02, c_2 = 0.13, c_3 = 0.01, c_4 = 0.1, c_5 = 0.5, c_6 = 0.1$ in

Eq.(8). Fig. 10 shows the waveform of transmitted signal $s_t(t)$. $s_t(t)$ under SNR=0dB is shown in Fig. 11. It is seen clearly in Fig. 11 that $s_t(t)$ is covered by added noise. Then using the chaotic synchronization method described in section 4 in this paper to recover the signal under noise condition. The recovered receiving signal $s_r(t)$ through chaotic synchronization is shown in Fig.12. For comparing, the method in [8,16] is also used for recovering $s_r(t)$ .They are shown in Fig. 13 - Fig. 14. Seen from Fig. 12 - Fig. 14, the performance of the recovered signal is better than that of the method in [8,16]. This means the proposed method has noise robust feature.

Next, the phases of the recovered sinusoidal signal and the reference sinusoidal signal are obtained using Eq.(13)-Eq.(15). The result is shown in Fig. 15. The enlarged figure of Fig. 15 is shown in Fig. 16. Seen from Fig. 15 and Fig. 16, the phase error of the reference sinusoidal signal and the recovered sinusoidal signal is 0.1rad. Thus the time taken by the transmitted signal to return to the receiver from the target is 10^{-5} second by Eq.(16). Then, the target distance 1500m is got by Eq.(17).

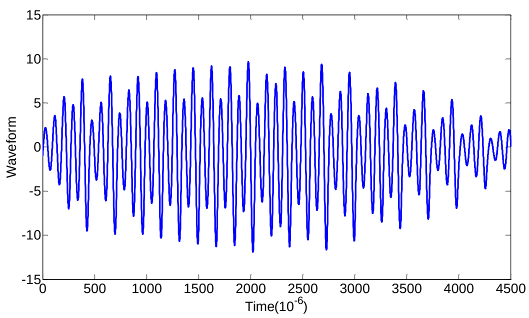


Fig. 10. Transmitted signal $s_t(t)$.

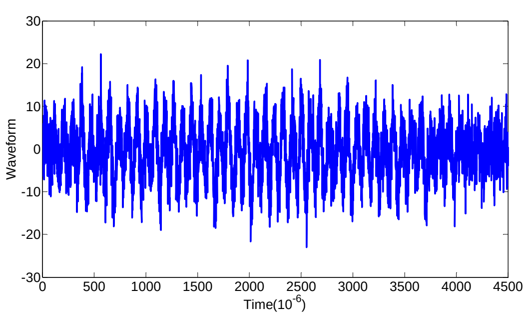


Fig. 11. Transmitted signal $s_t(t)$ *under SNR=0.*

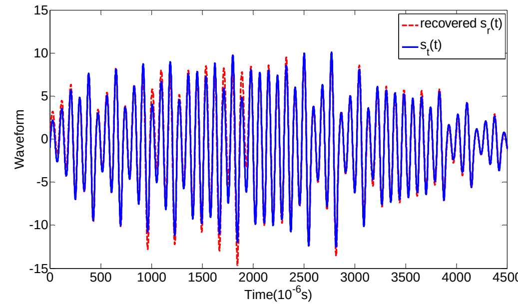


Fig. 12. $s_t(t)$ *and the recovered signal through chaotic synchronization method in this paper.*

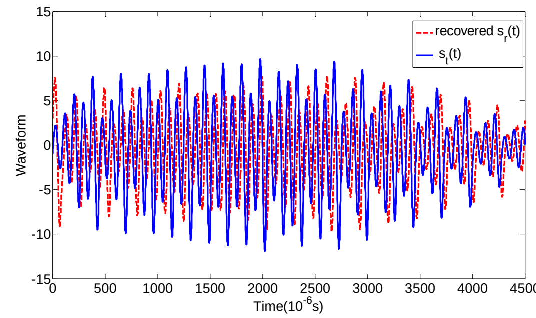


Fig. 13. $s_t(t)$ *and the recovered signal by using the method in [8].*

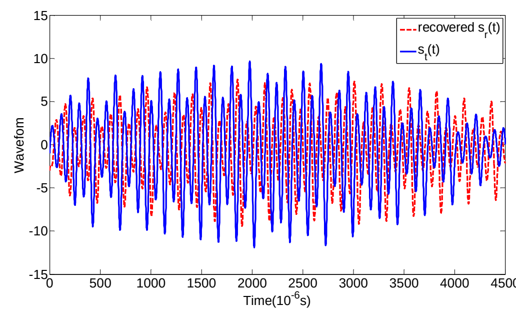


Fig. 14. $s_t(t)$ *and the recovered signal by using the method in [16].*

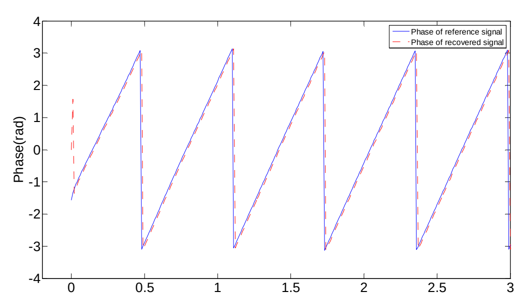


Fig. 15. The phases of the reference signal and the recovered signal.

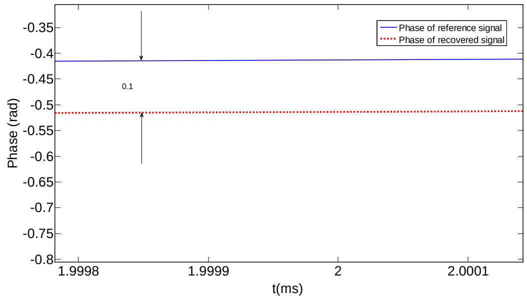


Fig. 16. The enlarged figure of Fig. 15.

Though the method in [8,9,16] can also get the target distance in no noise condition, while the proposed method is more suitable under noise condition. In [8,16], the SNR threshold should is about 40dB. When SNR is below 40dB, the target distance can not be obtained exactly. In [9] the SNR threshold is about 30dB. In this paper, the target distance can still be measured when the SNR=0dB or below. This make the proposed method has more potential in practical engineering.

6. Conclusions

This paper describes the theoretic aspect of a noise robust method of distance measurement based on chaotic synchronization. The noise robust feature of the two-frequency chaotic system is analyzed. That the fact it is the slow part acting like the noise filter makes the two-frequency system have the noise resistant feature. The noise robust feature of the chaotic system allows the recovery of the reference sinusoidal signal through chaotic synchronization. The target distance still can be accurately obtained by computing the phase error of the two signals when SNR=0dB. The noise robust feature makes the proposed method has potential application in practical engineering.

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