A non-destructive approach for fast evaluation of elastic shear properties of marble slabs

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A comprehensive quality control of individual marble slabs within a charge used for construction purposes includes the knowledge of torsional moduli of elasticity for both orthogonal directions of slab's major surfaces. Appropriate measurement methods of the state of technology allow a reliable determination of this moduli on slabs exclusively in a destructive manner, thus only statistic statements from a strongly limited number of specimens within a charge are possible. Several damage cases have shown that such statistic statements may be insufficient. This paper introduces an eigenfrequency method which has to be applied on a measurement system with a Laser Doppler Vibrometer (LDV), in order to quickly calculate a reliable range of torsional moduli of elasticity from an oscillation transient obtained by non-destructive operation of the LDV measurement system on a marble slab. Results and advantages of the method are compared and discussed with respect to a widely used method based on ultrasonic shear waves.

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1. Introduction

A comprehensive quality control of individual marble slabs used for construction purposes becomes indispensable if its application accounts for binding safety regulations. Moreover, financial loss in consequence of damage cases [1] should be avoided.

The mechanical failure of marble slabs will be accelerated if they exhibit insufficient mechanical resistance against shear stress. This happens in particular if shear stress gains the spread of transversal microcracks in such a way that the spreading of microcracks caused by flexural stress is facilitated [2, 3].

A meaningful related elastomechanic state can be effectively determined on marble slabs by measurement of their torsional moduli of elasticity. Considering marble's anisotropic behaviour, measurements have to be executed into different suitable slab directions [13].

Reliable standardized methods [9] of the state of technology apply ultrasonic waves exclusively on rod-shaped specimens. Therefore, such methods allow the examination of slab-shaped objects only in a destructive manner.

The eigenfrequency method introduced in this paper explains how to obtain reliable ranges of torsional moduli of elasticity within marble charges from a multitude of marble varieties. To this end, a first order eigenfrequency is derived from an oscillation transient measured by a Laser Doppler Vibrometer (LDV) on a larger surface of a cantilevered marble slab. Therefore, results from application of the introduced method can be achieved by non-destructive tasks. The application of the eigenfrequency method requires the knowledge of a marble charge specific anisotropic parameter, reliability value and Poisson's ratio, which are determined through a comparative aging process, also introduced in this paper.

To assure a fast execution of the eigenfrequency method, related computations contain a Fast Fourier Transform (FFT) suitable for Graphics Processing Unit (GPU) executable without time overheads known from use of external GPU software libraries.

Measurement results and advantages obtained from application of the introduced eigenfrequency method are discussed and compared to those of a common method with ultrasonic shear waves.

2. Eigenfrequency method

Referring to the isotropic approach [6], a torsional modulus:

$$G = \frac{\omega_G^2 \cdot l^3 \cdot m}{b \cdot d^3 \cdot (1+\upsilon)} \cdot \zeta_G \pm \Delta_G \tag{1}$$

can be calculated from a first order eigenfrequency ω_G of an elastic torsional mode measured on larger surfaces (x,y) of a cantilevered slab [11;12].

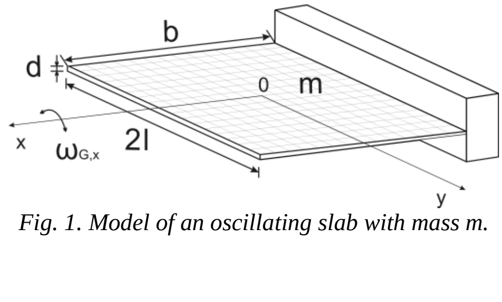

Fig. 1. Model of an oscillating slab with mass m.

Concerning the global elastomechanic behaviour of an anisotropic slab related to the model of Fig. 1, the application of (1) is subject to a spatial independency of G. Beyond geometrical and physical values as illustrated in Fig. 1, the solution of (1) requires the knowledge of an anisotropic parameter ζ_G, a Poisson's ratio υ and a reliability value Δ_G which expresses all systematic unknown deviations of G under certain conditions. ζ_G summarizes all multiplicative equation constants, the energetic distribution over the slab, and systematic known deviations of G.

A possibly pure torsional mode is induced on anisotropic slabs, for instance, by a force pulse in proximity to the free corner [6]:

$$x \approx \frac{b}{2} \;\; ; \;\; y \approx -l \qquad (2)$$

Pronounced flexural oscillations can be measured then in proximity of the opposite free corner:

$$x \approx \frac{b}{2} \;\; ; \;\; y \approx l \qquad (3)$$

It has been mentioned by Greubel et al. [6] and confirmed by own experiences, that by choice of (2) and (3) this flexural oscillations can be detected without significant influence of other oscillation types, particularly bending oscillations. This is crucial for the efficient applicability of our eigenfrequency method, thus detected torsional oscillation transients can be transformed directly to valid torsional eigenfrequencies, whose are almost independent to other kind of frequencies. The validity of those torsional eigenfrequencies, can be proved amongst others through its invariance after a mutual exchange of (2) and (3).

Considering the anisotropic behaviour of marble slabs with respect to deformations by shear stress, a reliable statement of their elastomechanic situation can be seen as sufficiently reached, if G is calculated for each orthogonal direction x or y of slab's larger surfaces:

$$G_{x/y} = \frac{\omega^2_{G,x/y} \cdot l^3 \cdot m}{b \cdot d^3 \cdot (1+\upsilon_{x/y})} \cdot \zeta_{G,x/y} \pm \Delta_{G,x/y} \qquad (4)$$

Anisotropic parameters $\zeta_{Gx/y}$ and reliability values $\Delta_{G,x/y}$ have to be calculated through a comparative aging procedure.

As shown in Fig. 2, this procedure includes a cyclic aging process consisting of two standardized aging steps, a frost action cycle [7] followed by a drying cycle [8]. Further it is shown that the aging process has to be applied simultaneously with respect to different marble slabs and specimens which have been cut out along the edges of the same slabs.

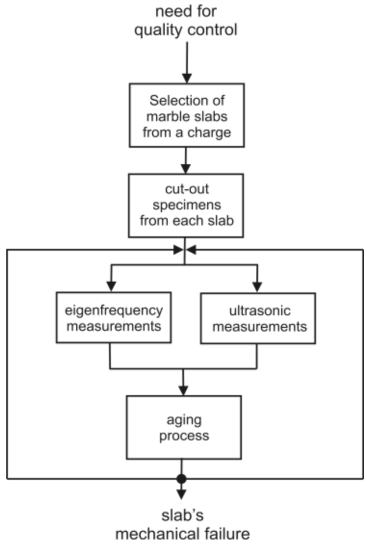

Fig. 2. Comparative aging procedure.

After each aging progress torsional moduli of elasticity $G_{x/y}$ are detected with respect to those slabs with uncertainties $\Delta_{1,x/y}$ by means of (4), and on the belonging specimens, using a standardized method of the state of technology with uncertainties $\Delta_{2,x/y}$.

Starting from a non-aged slab state, $\zeta_{Gx/y}$ is finally determined by a comparison of two result trends obtained from a number of aging cycles necessary to reach a defective slab state, which is related to a range of ultrasonic quasi-longitudinal velocities $\Delta v_{G,F}$.

This comparison consists of an overlap of result trends from eigenfrequency- and ultrasonic measurements according to the minimizing condition:

$$\left|\Delta_G^+\right| = \left|\Delta_G^-\right| =: \Delta_3 \qquad (5)$$

An overlap with a maximum deviation Δ^+_E and a minimum deviation Δ^-_E of those result trends is only reached according to (5) for a certain value of $\zeta_{Gx/y}$.

Own investigations have shown that differences between $\zeta_{G,x}$ and $\zeta_{G,y}$ remain generally low within a marble charge over all examined varieties, wherein a mean anisotropic parameter:

$$\zeta_{G,D} = \frac{\zeta_{G,x} + \zeta_{G,y}}{2} \qquad (6)$$

can be used for both directions (x,y) of slab's larger surfaces (x,y) if a deviation range $\pm\Delta_4$ is additionally taken into account.

Previous studies [10] related to the global elastomechanic behaviour of marble objects have shown that Poisson's ratios can be expressed by mean values. Obviously, the influence of related deviation ranges $\pm\Delta_5$ caused by marble's spatial anisotropy has also to be considered. Poisson's ratios can be also profitably measured on marble objects by ultrasonic systems.

A worst case reliability value $\Delta_{G,w}$ has to be calculated according to the following equation:

$$\Delta_{G,w} = \sum_{i=1}^{3} \max(\Delta_{i,x}, \Delta_{i,y}) + \sum_{i=4}^{5} \Delta_i \quad (7)$$

The maximum values of $\Delta_{1,x/y}$, $\Delta_{2,x/y}$ and $\Delta_{3,x/y}$ have to be chosen from values over entire result trends. Further parameters of (4) can be measured with insignificant variances, thus their deviation influence on $G_{x/y}$ is negligible.

Considering (4), (6) and (7) with respect to the examined marble varieties, (1) has to be rewritten as:

$$G_{x/y} = \frac{\omega_{G,x/y}^2 \cdot l^3 \cdot m}{b \cdot d^3 (1+\upsilon_D)} \cdot \zeta_{G,D} \pm \Delta_{G,w} \quad (8)$$

An applicability of (8) into automatic quality control systems requires a possibly low $\Delta_{G,w}$.

Low $\Delta_{1,x/y}$ can be achieved if $\omega_{G,x/y}$ have been calculated from oscillation transients $w_{x/y}(t)$ detected by a LDV on a circular area located on (3) of slab's surface (x,y) according to:

$$w_{x/y}(t) = \frac{\lambda}{4\pi} \cdot \arctan \frac{u_2(t)}{u_1(t)} + k \cdot \pi \quad (9)$$

by means of reflected laser light with wavelength λ, which contributes to an interference beam. The fringe pattern of this interference contains necessary information for calculating k. Further, to obtain the current direction of $w_{x/y}(t)$, the interference beam is split into two phase-shifted ($\pi/2$) partial beams which are subsequently mapped to signals $u_1(t)$ and $u_2(t)$. Through adoption of an adequate demodulation technique, $w_{x/y}(t)$ can be detected with maximum resolution of few nanometre.

Fig. 3 illustrates in general manner how $\omega_{G,x/y}$ may be deduced from $w_{x/y}(t)$.

The oscillation transient $w_{x/y}(t)$ is mapped into the frequency range by an FFT.

The maximum frequency of the related amplitude spectrum magnitude corresponds to a first order eigenfrequency $\omega_{G,x/y}/2\pi$, thus $\omega_{G,x/y}$ is obtained by multiplication of this maximum frequency by 2π.

Systematic deviations caused by ultrasonic measurements ($\max(\Delta_{2,x}, \Delta_{2,y})$, Δ_5) can only be reduced by an appropriate selection of the ultrasonic measurement system.

Two approaches have been developed to profitably reduce $\max(\Delta_{3,x}, \Delta_{3,y})$ and Δ_4.

A first approach includes the assignment of appropriate anisotropic parameters to partial ranges of the eigenfrequency result trends. Concerning an improved automatic detection of defective marble slabs, further anisotropic parameters $\zeta_{G,crit}$ can be assigned charge-dependently to defined ranges of shear velocities $\Delta v_{S,crit}$ located next to ranges $\Delta v_{S,F}$ which belong to defective slab states.

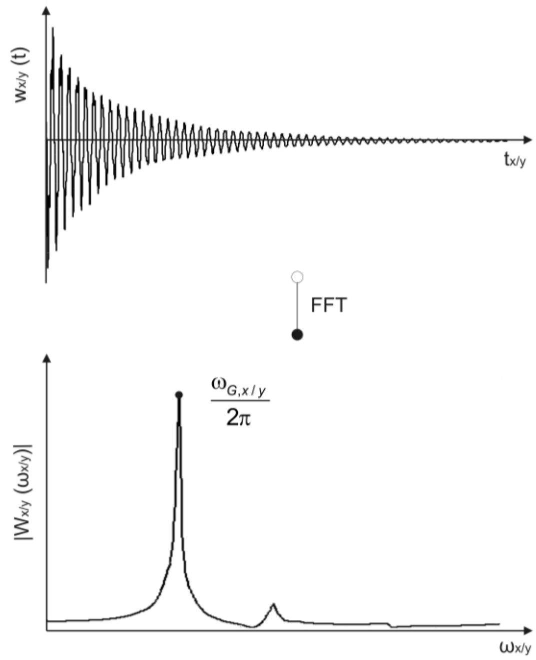

Fig. 3. Conceptual overview how torsional eigenfrequencies have been achieved by mapping of oscillation transients into frequency range.

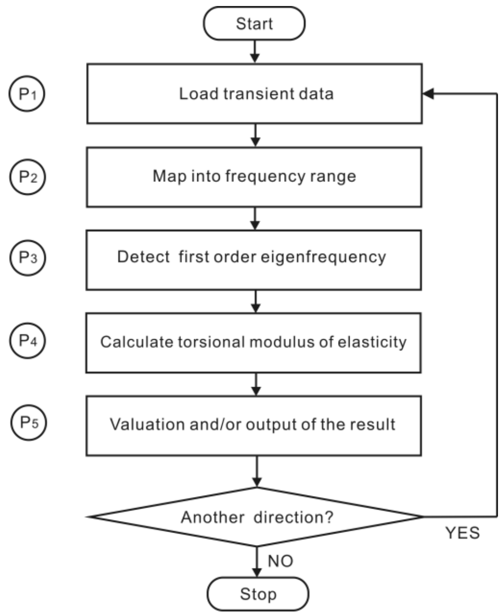

Fig. 4. Eigenfrequency method related algorithm assigned to processes P_1 to P_5.

A reduction of $\max(\Delta_{3,x}, \Delta_{3,y})$ and Δ_4 over shear velocities ranges Δv_S, corresponding to the entire aging progress, relates to a second approach, where anisotropic parameters are substituted by characteristic curves $\zeta_{G,D}(\Delta v_S)$. Such characteristic curves contain a single anisotropic parameter $\zeta_{G,D}$ for each smallest possible range Δv_S, limited by two adjacent $G_{x/y}$, which have been detected between two aging cycles.

As a basis to GPU implementation, the eigenfrequency method is finally applied to the algorithm of Fig. 4.

A significant reduction of the overall execution time related to automatic quality control of marble slabs can be achieved if the processes P_1 to P_5 are entirely executed on a GPU.

3. Results

The results and advantages of the introduced eigenfrequency method are discussed hereafter for charges of five different marble varieties (see Table 1). It has been experienced that the accuracy of $\zeta_{G,D}$ would not be significantly improved by examination of more than half a dozen slabs per charge.

Considering examinations of other charges, the amount of examined slabs has always to be empirically adapted to their anisotropic behaviour and to the used composition criteria of the charge.

Torsional moduli of elasticity and Poisson's ratios of nine related specimens per direction, have been measured for individually examined slabs, by evaluation of shear waves using a Geotron ® UKS-D ultrasonic system.

Measurements on slabs and specimens have been repeated at least a hundred times, to allow reliable statistic statements on normally distributed results.

Among others, also results from ultrasonic measurements of Poisson's ratio have shown normally distributed variances. These measurements have been executed on all specimens, independently from their spatial membership.

Poisson's ratios υ_D from charges over all varieties are shown with their uncertainties $u(\upsilon_D)$ in Table 2. Considering that magnitudes this uncertainties $u(\upsilon_D)$ are two decades lower than those of mean Poisson's ratios υ_D , Δ_5 has not to be taken into account.

Anisotropic parameters $\zeta_{G,D}$ related to the overall aging progress Δv_S and to the defective value range $\Delta v_{S,F}$ are shown for all examined varieties in Table 3. While an isotropic slab state is characterized by $\zeta_{G,D} \approx 1$, anisotropic behaviours belong to $\zeta_{G,D} > 1$.

It is further observed, that at examination over different marble varieties the achieved aging progress Δv_S can be of very different extent while the range $\Delta v_{S,F}$, which belongs to a defective slab state, remains between 1 and 7 km/s.

This relative narrow range of $\Delta v_{S,F}$, is conducive reliability referring to an application on automatic detection of defective slabs.

Due to their system-dependence, deviations between eigenfrequency- and ultrasonic measurements can not be reduced by methodical techniques.

Table 3 shows that normally distributed uncertainties of measurement points from eigenfrequency measurements are lower by a decade.

Improvements of the introduced eigenfrequency method with respect to automatic quality control, can be achieved by decreasing $\Delta_{3,x/y}$ and Δ_4 listed in Table 5. According to comparisons of Table 6, further reductions of $\Delta_{G,w}$ have to be reached for the varieties STV and THA. Obviously, a possibly low $\Delta_{G,w}$ results profitable for each variety.

Table 1. Examined marble varieties and dimensions of selected slabs and specimens.

Variety		Slabs	Specimens
Carrara D	(CAD)	l x b x d (cm) 25 x 25 x 1	l x b x d (cm) 5 x 1 x 1
Cremo Delicato	(CRD)		
Statuario Venato	(STV)		
Thassos	(THA)		
Thassos Extra	(THE)		

Table 2. Poisson's ratios and their measurement uncertainties.

Variety	υ_D	$u(\upsilon_D)$
CAD	0.40	0.0033
CRD	0.30	0.0066
STV	0.41	0.0027
THA	0.41	0.0025
THE	0.41	0.0023

Table 3. Anisotropic parameters $\zeta_{G,D}$ of different marble varieties and their related shear velocity ranges Δv_S and $\Delta v_{S,F}$.

Variety	$\zeta_{G,D}$	Δv_S (km/s)	$\Delta v_{S,F}$ (km/s)
CAD	8.58	1.24 – 1.82	1.22 – 1.24
CRD	4.62	0.89 – 1.61	0.88 – 0.89
STV	5.44	1.05 – 1.62	0.97 – 1.05
THA	6.58	1.30 – 1.62	1.29 – 1.30
THE	8.20	1.47 – 1.63	1.42 – 1.47

Table 4. Maxima of normally distributed uncertainties from orthogonal (x,y) eigenfrequency measurements (Δ_{1x}, Δ_{1y}) and ultrasonic measurements (Δ_{2x}, Δ_{2y}) over Δv_S.

Variety	max(Δ_{1x}, Δ_{1y}) (GPa)	max(Δ_{2x}, Δ_{2y}) (GPa)
CAD	0.049	0.21
CRD	0.025	0.10
STV	0.022	0.14
THA	0.026	0.19
THE	0.025	0.14

Table 5. Δv_S related maximum orthogonal deviations (Δ_{3x}, Δ_{3y}) and Δ_4 derived respectively from application of the minimizing condition (5) and from calculations with mean anisotropic parameters $\zeta_{G,D}$ applied on different varieties.

Variety	max(Δ_{3x}, Δ_{3y}) (GPa)	Δ_4 (GPa)
CAD	1.13	0.38
CRD	0.28	0.61
STV	0.78	3.31
THA	0.69	1.80
THE	0.94	0.38

Table 6. Marble variety dependent worst case reliability values $\Delta_{G,w}$ compared to related ranges of shear moduli $G_{x/y}(\Delta v_S)$.

Variety	$\Delta_{G,w}$ (GPa)	$G_{x/y}(\Delta v_S)$ (GPa)
CAD	1.77	4.96 – 7.68
CRD	1.01	2.14 – 4.48
STV	2.68	2.68 – 5.91
THA	2.71	4.63 – 7.58
THE	1.48	4.54 – 7.58

Table 7. Additional anisotropic parameters $\zeta_{G,crit}$ of different marble varieties and their related deviations $\max(\Delta_{3x}, \Delta_{3y})$ and Δ_4.

Variety	$\zeta_{G,crit}$	$\max(\Delta_{3x,crit}, \Delta_{3y,crit})$ (GPa)	$\Delta_{4,crit}$ (GPa)
CAD	7.85	0.36	0.097
CRD	1.86	0.05	0.001
STV	5.58	0.23	0.008
THA	5.79	0.22	0.003
THE	9.14	0.36	0.090

Table 8. Worst case reliability values $\Delta_{G,crit}$ obtained along $\Delta v_{S,crit}$.

Variety	$\Delta_{G,crit}$ (GPa)	$\Delta v_{S,crit}$
CAD	0.716	1.24 – 1.39
CRD	0.176	0.89 – 0.93
STV	0.400	1.05 – 1.12
THA	0.439	1.30 – 1.40
THE	0.615	1.47 – 1.51

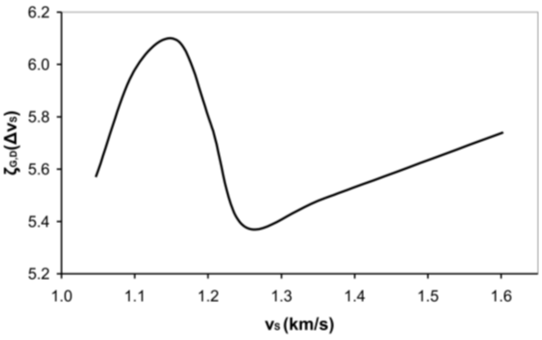

Fig. 5. STV related characteristic curve $\zeta_{G,D}(\Delta v_S)$.

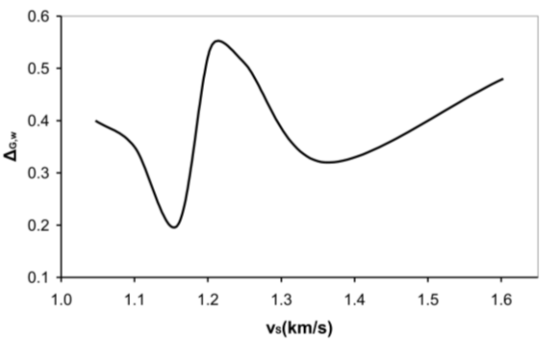

Fig. 6. Reduced $\Delta_{G,w}$ through application of $\zeta_{G,D}(\Delta v_S)$ on examination of STV slabs.

By a first approach, an additional anisotropic parameter $\zeta_{E,crit}$ related to a reduced range of velocities $\Delta v_{S,crit}$ adjacent to $\Delta v_{S,F}$ is introduced, to enhance the reliability in detection of defective slabs.

Assuming that the manufacturer will define $\Delta v_{S,crit}$ according to Table 8, through consequent reductions of $\max(\Delta_{3,x},\Delta_{3,y})$ and Δ_4 (see Table 7), related magnitudes $\Delta_{G,crit}$ will reach the same scale as the worst case deviations $\max(\Delta_{2x},\Delta_{2y})$ from ultrasonic measurements.

As mentioned afore, an extension of this profitable magnitude scale over the entire aging progress Δv_S can be achieved through a second approach by substituting single anisotropic parameters $\zeta_{G,D}(v_S)$ through characteristic curves. Fig. 5 shows such a characteristic curve for STV. The favourable reduction of $\Delta_{G,w}$ along Δv_S is illustrated in Fig. 6. Deviations remain generally low over the entire aging progress. Moreover, deviations nearby to those of ultrasonic measurements have been obtained within a shear velocity range between 1.1 and 1.2 km/s. The reducibility of $\Delta_{G,w}$ along Δv_S depends on the pitch ratio trend from result trends compared to calculate anisotropic parameters and reliability values.

The time expense for determination of aniso- tropic parameters and reliability values through the introduced comparative aging procedure corresponds approximately to that of a legally binding examination on marble slab charges, achieved by standardized ultrasonic methods.

A significant contribution to short examination time has been reached through implementation of the eigenfrequency method related algorithm of Fig. 4 by means of a GPU related three-layer concept [5]. The system integration solutions supported by this concept allows an entire execution of processes P_1 to P_5 on GPU-side with negligible processor loads on CPU-side. Therefore CPUs can be applied profitably for control tasks.

P_1 includes also measurement times of the LDV and lies between 0.8 s and 1.2 s, depending for the most on the marble variety. The FFT included in P_2 requires the largest time share. During the last decade, it has been experienced that in reference to comparable CPUs, the execution of a FFT can be sped to two decades by GPUs of consumer graphics cards.

Our own evaluations have shown that FFT execution times demand around a second. Due to the rapid development of GPUs, time savings should be further lead by application of a latest GPU.

Further processes P_3 to P_5 and control tasks have been achieved within half a second.

Overall the introduced eigenfrequency method, allows a complete quality control, belonging to a reliable range of torsional moduli of elasticity, on several hundred marble slabs from a charge of a certain variety within few hours.

4. Conclusion

A comprehensive and non-destructive quality control of each slab from a multitude of marble varieties becomes possible by additional application of the introduced eigen-frequency method on a laser measurement system with LDVs. By means of GPU-processing, the implementation of this method enables fast calculation of reliable torsional moduli ranges within a marble charge. The information included into such ranges can be profitably used inside industrial environments for automatic detection of defective slabs. Therefore, two promising approaches have been presented. In summary, the incidence of mechanical failure on constructions with marble slabs can be significantly reduced.

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