

A numerical approach for designing multi-wavelength plate

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Wave plates made of birefringent materials can only be used at certain single wavelength, since the phase retardation produced is approximately inverse proportional to the wavelength. In this article, we introduced a geometric numerical method to design multi-wavelength wave plates with the plot of the curves of $kd_o(\lambda)$ ($k=1, 2, \dots$), where $d_o(\lambda)$ is the thickness of the zero-order $\lambda/4$ waveplate. The two common cases in designing are discussed and simulated with quartz crystal as application examples. In the first case, the thickness is specified. A quartz plate with thickness of 1 mm is designed, which operates as $\lambda/4$ waveplate at 866 nm, as $\lambda/2$ waveplate at 777 nm, and as λ waveplate at 606.8 nm. In the second case, the two wavelengths are specified. Two dual-wavelength quartz plates for 460 nm and 520 nm are designed. One is 282 μm thick, which operates as a $\lambda/4$ waveplate at 460 nm and a λ waveplate at 520 nm. The other one is 380 μm thick, which operates as $\lambda/4$ plate at both 460 nm and 520 nm.

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1. Introduction

Polarized light and polarization technology are essential in cryptography, tissue characterization, ellipsometry, spectroscopy, and other laser technology [1,4]. Polarization technology is the basis of linear polarization, circularly polarization and elliptically polarization of polarized light. Circularly polarized light and elliptically polarized light are produced by linear polarized light going through the optical phase retarders. The emergent light is circularly polarized light if the incident linearly polarized light passing through a quarter wave plate with azimuth of exactly 45° respect to incident polarization direction. The emergent light is usually elliptically polarized light if the azimuth of incident light is not exactly 45° , and the ellipticity can be regulated through the changes in azimuth.

Therefore, wave plate is essential in design of optical instruments and optical measurement technology. There are mainly three types of optical phase retarders, Babinet–Soleil compensator [5, 6], Fresnel rhomb-type retarders [7-8] and birefringent crystal plates [9-14]. Babinet–Soleil compensator and Rhomb-type retarders have nice achromatic quality, but also have large volume and large beam translation. The most commonly used phase retarders are made of birefringent crystals.

Higher precision of the thickness of wave plate is required due to higher birefringence. Therefore, most current wave plates are commonly made of material of

mica, quartz or MgF_2 . Mica can be naturally split into thin slices, which is generally used to make true zero-order wave plates. While quartz and MgF_2 are generally used to make true multi-order ones.

Single crystal retarders are non-achromatic, which can only work at particular wavelength. Composite wave plates which are consist of two or more pieces with same or different materials, are achromatic in a certain spectral range, but the accurate range is limited. It is difficult to achieve accurate retardation at any two particular laser wavelengths at the same time. With the method introduced below, we can design exactly phase retarders operate at any two or more wavelengths.

2. Theory

Uniaxial crystals like quartz have one axis of symmetry in the index ellipsoid, the optic axis, and display two distinct indices of refraction, i.e. n_o (the ordinary refractive indices) and n_e (the extraordinary refractive indices). Light which incident on a uniaxial crystal will split into two rays of polarized light with indices of n_o and n_e , which vibrate in perpendicular directions. The retardation δ introduced by a birefringent plate between the two orthogonal components of emergent light is given by [15]:

$$\delta(\lambda) = \frac{2\pi}{\lambda} (n_e(\lambda) - n_o(\lambda))d \quad (1)$$

where λ represents the incident light wavelength, d represents the thickness of the waveplate. The retardation $\delta(\lambda)$ is obviously a function of λ , because the principal refractive index, n_o and n_e , are functions of λ , which are determined by the dispersion equation. And the birefringence of the material:

$$\Delta n(\lambda) = n_e(\lambda) - n_o(\lambda) \quad (2)$$

is also function of λ .

For a quarter-wave plate (QWP), the retardation satisfies [16]:

$$\delta(\lambda) = (k-1/2)\pi, \quad k=1,2,\dots \quad (3)$$

For a half-wave plate (HWP), the retardation satisfies:

$$\delta(\lambda) = k\pi, \quad k=1,2,\dots \quad (4)$$

For a full-wave plate (FWP), the retardation satisfies:

$$\delta(\lambda) = 2k\pi, \quad k=1,2,\dots \quad (5)$$

Compare with Eqs. (1)-(5), we have:

$$d = \begin{cases} \frac{(2k-1)}{4} \cdot \frac{\lambda}{\Delta n}, & k=1,2,\dots \text{ (QWP)} \\ \frac{k}{2} \cdot \frac{\lambda}{\Delta n}, & k=1,2,\dots \text{ (HWP)} \\ k \cdot \frac{\lambda}{\Delta n}, & k=1,2,\dots \text{ (FWP)} \end{cases} \quad (6)$$

From Eq. (6), we know that the thickness of zero-order QWP is $\frac{1}{4} \frac{\lambda}{\Delta n}$, that of zero-order HWP is $\frac{1}{2} \frac{\lambda}{\Delta n}$, and of FWP is $\frac{\lambda}{\Delta n}$. Therefore, we can take the thin slice with thickness:

$$d_0 = \frac{1}{4} \cdot \frac{\lambda}{\Delta n} \quad (7)$$

as a unit block, which builds up the other multi-order wave plates.

If a certain thickness retarder is designed for different wavelengths, for example, it acts as a QWP at λ_1 and a HWP at λ_2 , then the following condition should be satisfied:

$$d = \frac{(2k_1-1)}{4} \cdot \frac{\lambda_1}{\Delta n} = \frac{k_2}{2} \cdot \frac{\lambda_2}{\Delta n}, \quad k_{1,2} = 1,2,\dots \quad (8)$$

It requires that k be only integer solution of the equation are valid.

3. Design and results

Eq.(8) is difficult to solve, so we give the numerical solves by an geometric way. The value of birefringence for the quartz is taken from Ghosh [17]:

$$n_o^2 = \frac{0.663044\lambda^2}{\lambda^2 - 0.0600^2} + \frac{0.517852\lambda^2}{\lambda^2 - 0.1060^2} + \frac{0.175912\lambda^2}{\lambda^2 - 0.1190^2} + \frac{0.565380\lambda^2}{\lambda^2 - 8.844^2} + \frac{1.675299\lambda^2}{\lambda^2 - 20.742^2} + 1 \quad (9)$$

$$n_e^2 = \frac{0.665721\lambda^2}{\lambda^2 - 0.0600^2} + \frac{0.503511\lambda^2}{\lambda^2 - 0.1060^2} + \frac{0.214792\lambda^2}{\lambda^2 - 0.1190^2} + \frac{0.539173\lambda^2}{\lambda^2 - 8.792^2} + \frac{1.8076613\lambda^2}{\lambda^2 - 19.70^2} + 1 \quad (10)$$

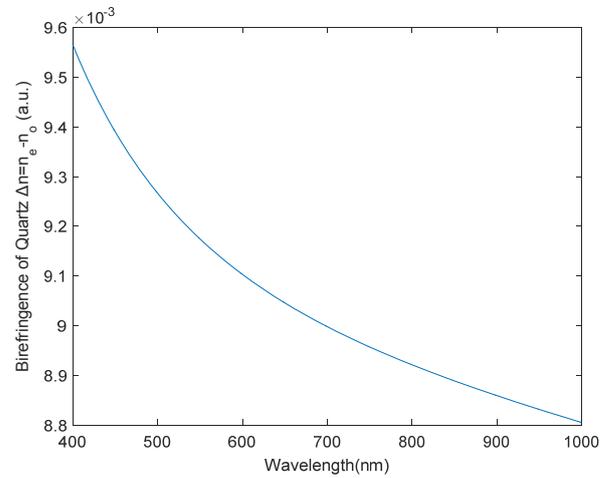


Fig. 1. Variation of birefringence of quartz with wavelength

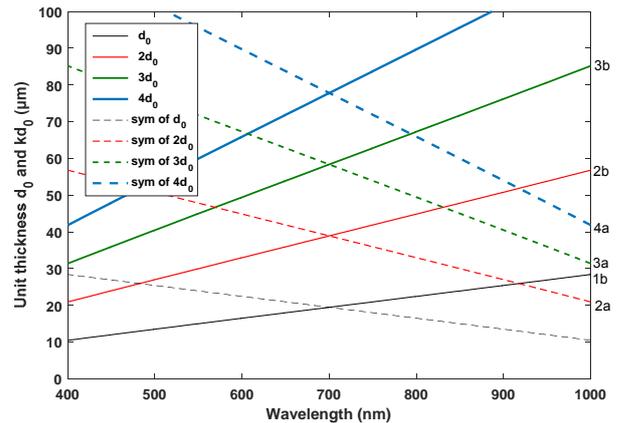


Fig. 2. Integer multiples of unit thickness with wavelength. The lowest line is the unit thickness of $d_0(\lambda)$, which varies approximately from 10 μm (denoted by a) at 400 nm to 30 μm (denoted by b) 1000 nm. The range of $d_0(\lambda)$, $2d_0(\lambda)$, $3d_0(\lambda)$, $4d_0(\lambda)$ are [a, b], [2a, 2b], [3a, 3b], [4a, 4b] respectively. The horizontal symmetry of the curves are plotted in dotted lines, which helps to figure out the overlay range of different lines

The variation of birefringence of quartz is plotted in Fig. 1. With the data of birefringence, the unit d_0 and its integer multiples are calculated and plotted with solid lines in Fig. 2. Thicker linewidths present thicker wave plate slice with higher multiples of d_0 . The mirror symmetry of the solid lines is shown in dotted lines in the same graph. With Fig. 2, it's easy to figure out the overlay range of $d_0(\lambda)$ and $2d_0(\lambda)$, or $2d_0(\lambda)$ and $3d_0(\lambda)$, or any other two curves.

If a certain thickness retarder is designed for different wavelength, for example, it acts as a QWP at λ_1 and a HWP at λ_2 . A feasible method is to select a thickness that lies in the overlap of the range between d_0 and $2d_0$, i.e. [2a, 1b] as indicated in the right vertical axis of Fig. 2. In fact this is the case $k_1=k_2=1$ in Eq. (8). We can also infer from the graph that a quartz sheet with thickness of [3a, 2b] can act as a QWP at a certain wavelength and HWP at another. And a thickness lies in [4a, 2b] corresponding to a FWP and a HWP. A thickness lies in [4a, 3b] corresponding to a FWP and a QWP.

The benefits of the numerical solutions are unambiguous and straightforward, but for case that k value is big, or the retarder in designing is thick, analytical solutions may be more convenient. Suppose the wavelength in design is 400 nm-1000 nm, and the thickness of zero-order QWP at 400 nm is a , at 1000 nm is b , which can be calculate from Eq. (7). We are only interested in the case that the range of $kd_0(\lambda)$ and $k'd_0(\lambda)$ have an overlap section. Let $k'=k+m$, and we have this condition:

$$(k+m)a < kb, \quad \text{or} \quad k > m \frac{a}{b-a} \quad (11)$$

The thickness d of designed dual-wavelength plate should satisfy:

$$d = k'd_0(\lambda_1) = kd_0(\lambda_2) \quad (12)$$

where $k' > k$ and $\lambda_2 > \lambda_1$. If k or k' is integer multiple of 4, the plate is a FWP, or else if k or k' is integer multiple of 2, the plate is QWP. Or else it's a QWP. Which can be simply write as:

$$k, k' = \begin{cases} 4m \pm 1, & m = 0, 1, 2, \dots \quad (\text{QWP}) \\ 4m \pm 2, & m = 1, 2, \dots \quad (\text{HWP}) \\ 4m, & m = 1, 2, \dots \quad (\text{FWP}) \end{cases} \quad (13)$$

Regularly, there are two common cases in the actual wave plate designing. One case is that the thickness is specified regarding to the limitation of materials, while the two operating wavelengths are not specified. Actually, lasers of the most wavelengths are commercial available at present. Take quartz for example, let's design a quartz sheet of thickness $d=1\text{mm}$, which can act as different wave plates at multi-wavelength. Suppose the working wavelength is from 400~1000 nm, then $a=d_0(400\text{ nm})=10.5\text{ }\mu\text{m}$, $b=d_0(1000\text{ nm})=28.4\text{ }\mu\text{m}$, which can also be infer from the line of d_0 in Fig. 2. Because unit $d_0(\lambda)$ varies with wavelength λ , as shown in Fig. 2, d contains different $d_0(\lambda)$ for λ . As mentioned before, k is the number of units contained. The range of k can be

calculated from Eq. (11), (12): $d/b \leq k \leq d/a$, or $35.2 \leq k \leq 95.2$. It requires that k be only integer.

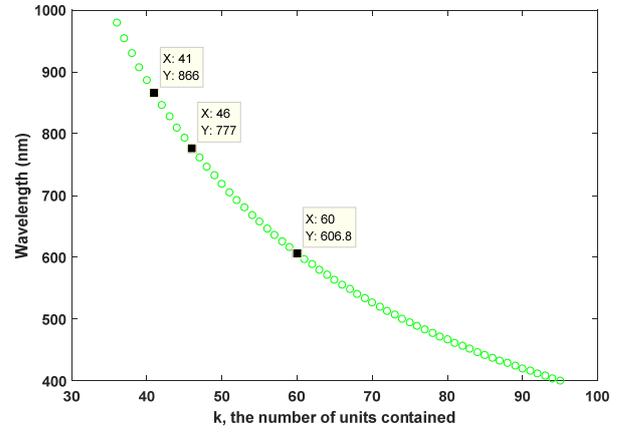


Fig. 3. Schematic diagram for designing multi-wavelength plate when the thickness is specified. A quartz sheet with thickness of 1mm can act as different wave plate at different wavelength. For examples, it's a QWP at 866 nm ($k=41$), a HWP at 777 nm ($k=46$), and a FWP at 606.8 nm ($k=60$)

The relation of wavelength and k is illustrated in Fig. 3. The quartz waveplate with 1mm thickness acts as different wave plates at certain separate wavelengths. For examples, at wavelength of 866 nm, $k=41$, as shown in the graph, it's a QWP, according to Eq. (13). In the same way, at 777 nm ($k=46$), it's a HWP. And at 606.8 nm, it's a FWP. The same goes for any other point from $k=36$ to 95 in Fig. 3.

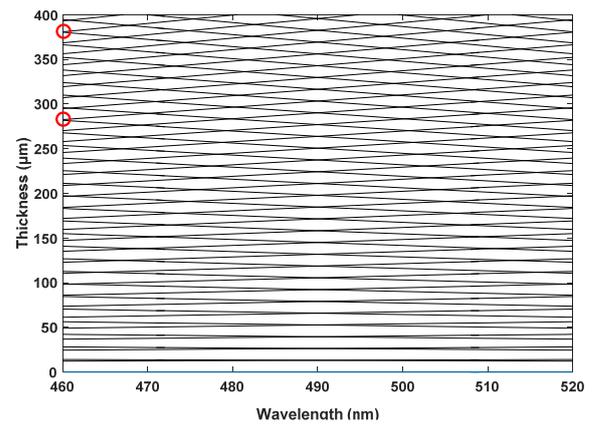


Fig. 4. Schematic diagram for designing dual-wavelength plate when the wavelength is specified. For given wavelengths of 460 and 520 nm (at the two ends of the horizontal axis), the thickness for design is located on the point of intersection (marked with circles). Wave plate with thickness of 380 μm is a QWP at both 460 ($k=31$) and 520 nm ($k=27$). Wave plate with thickness of 282 μm is a QWP at 460 ($k=23$), or a FWP at 520 nm ($k=20$)

The other case is that the two wavelengths are specified by practice requirement, the thickness is need to be determined. In this case, the simulation is given in Fig. 4. Again, we use the geometric numerical method, because it's difficult to get integer solutions for k at both wavelengths within limited thickness. Suppose the working wavelength is 460 and 520 nm, which is set as the starting and ending point for calculate, as shown in the graph at the two ends of the horizontal axis, the thickness varies with k at different wavelengths is illustrated in Fig. 4. If a thickness has integer solutions for both 460 and 520 nm, there is a point of intersection on the vertical axis, either on the left axis or on the right, which is all the same due to the mirror symmetry of the lines. Comparing with calculation and instructions in Fig. 4, there is a cross point of lines $k=27$ and $k=31$ at thickness of $380 \mu\text{m}$, where a circle is marked. Which means that, at this thickness, it is a QWP at both 460 nm and 520 nm according to Eq.(13). And there is another cross point of line $k=20$ and line $k=23$ at thickness of $282 \mu\text{m}$, also marked in circle. Which means that it is a QWP at 460 nm and FWP at 520 nm according to Eq. (13).

4. Application and validation

In this part, we will compare with real commercially available dual-wavelength wave-plates [18]. Specifications of the commercial product are shown in the table below:

Table 1. Specifications of commercial dual-wavelength wave-plates

Part Number	Models	Thickness
WPDM05M-1064H-532Q	$\lambda/4$ @ 532 nm and $\lambda/2$ @ 1064 nm	303.8145 μm
WPDM05M-532H-1064Q	$\lambda/4$ @ 1064 nm and $\lambda/2$ @ 532 nm	1127.6532 μm

This is the case that wavelength are given. The simulation figure is much like the Fig. 4, except that x-axis is from 532 nm to 1064 nm, and the y-axis should larger than 1200 nm (according to the thickness of WPDM05M- 532H-1064Q), and so that the lines are much closer to each other. Hereby, only several related lines are given in Fig. 5, for the sake of simplicity and clarity. The simulation agrees with the real data very well, which shows the validation of this numerical approach.

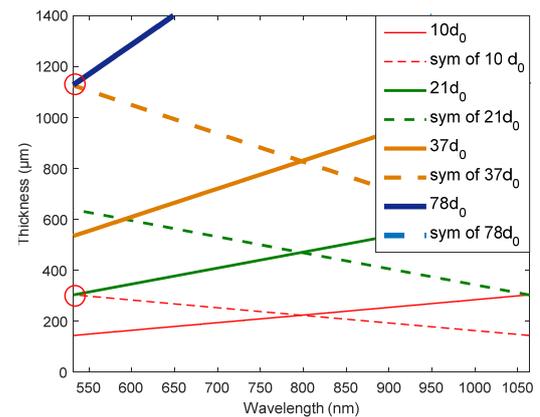


Fig. 5. Schematic diagram for designing dual-wavelength plate with wavelengths of 532 and 1064 nm (at the two ends of the horizontal axis). The thickness for design are located on the point of intersection (marked with circles). Wave plate with thickness of $\sim 300 \mu\text{m}$ is a QWP at 532 nm ($k=21$) and a HWP at 1064 nm ($k=10$). Wave plate with thickness of $\sim 1130 \mu\text{m}$ is a HWP at 532 nm ($k=78$), or a QWP at 1064 nm ($k=37$)

5. Conclusions

In this work, we discussed the condition that a retarder can work at multi-wavelength as different waveplate with a geometric numerical method. In particular, the quartz sheet with thickness of 1mm, 380 μm and 282 μm are studied. This method is intuitive and effective for designing single plate which can work in certain separated wavelengths.

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