JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 10, No. 8, August 2008, p. 2174 - 2180

A numerical method to analyse the thermal phenomena involved in phase transformations at laser beam irradiation

M. PEARSICĂ, S. BĂLUȚĂ[a], C. CONSTANTINESCU, C. STRÎMBU
”Henri Coandă” Air Force Academy, IT and Electronic Department, 160 Mihai Viteazu St, 500183 Brasov, Romania
[a]The Military Equipment and Technology Research Agency, Aeroportului Street, No. 16, CP 19, OP Bragadiru, 077025, Ilfov, Romania

---

The first part of this paper deals with the differential equations system, which governs the laser processing of materials in the presence of the O_2 jet, occurring on the mobile irradiation. It deals also with the hypotheses basing the equations system obtained. Spatial and temporally distribution of temperature inside the material is governed by the full source of temperature, which has been modelled by taking into consideration the fact that the irradiation is made with a CO_2 laser with a “Gaussian distribution” of photons beam’s intensity and oxidizing energy. The second part of paper deals with the determination of separation frontiers equations and graphical representations of radial distribution of the temperature of the three phases: solid, liquid and vapours. The results thus obtained allow the determining of the technological parameters of processing and the material constants.

(Received December 21, 2007; accepted June 30, 2008)

Keywords: Laser, Thermal phenomenon, Temperature distribution

---

1. Introduction

The mathematic model analysed derives from the thermal transfer equation in a homogeneous medium heathen by a laser beam. In time, the medium submitted to the actions of the laser presents the solid, liquid and vapour state separated by previously unknown frontiers. A simplifying model taking into consideration these frontiers, by considering them as having a cylindrical symmetry, was proposed.

By specifying the pattern D, the initial conditions of the temperature and the conditions on D pattern frontiers, one can have the solution of thermal equation, T(x,y,z,t) for a certain substantial. It must mention here that the very same substantial has different reactions when in solid, liquid or vapour state and in order to obtain thermal distribution it is necessary to solve simultaneously thermal equation for each of the phases with specific initial and boundary conditions. It was considered in the paper three phases of evolution in time of these phenomena: phase 1 for the solid state of the substantial, phase 2 that includes both solid and liquid state, phase 3 in which all three solid, liquid and vapour states are to be found.

The irradiated area being much smaller than the size of the object, the pattern D dealing with thermal equation may be approached as a semi-space.

2. Equations on mathematical model

Because the print of the laser beam on the surface of the material is circular, thermal phenomena produced within the substantial, have a cylindrical symmetry. Oz is considered as symmetry axis of the laser beam, the object surface equation is z = 0 and the positive sense of Oz axis is from the surface to the inside of the object.

The thermal equation within cylindrical co-ordinates (θ,r,z) will be [1,3,4]:

$$\frac{1}{K}\frac{\partial T}{\partial t}=\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2} \tag{1}$$

Due to cylindrical symmetry, $\frac{\partial^2 T}{\partial \theta^2}=0$ and equation (1) changes to:

$$\frac{1}{K}\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2} \tag{2}$$

Pattern D within thermal phenomenon are produced is $z \geq 0$.

In time, successions the phases the object suffers while cut by the laser beam are the following:

- phase 1, for $0 \leq t < t_{top}$;
- phase 2, for $t_{top} \leq t < t_{vap}$;
- phase 3, for $t \geq t_{vap}$, where t_{top}, is the time moment when the melting of the material starts and t_{vap} is the time moment when the process of vaporising of the material starts.

The surfaces separating solid, liquid and vapour state are previously unknown and will be determined using the conditions of continuity of thermal tide on separation surfaces of two different substantial, knowing the temperature of the separation surface and the speed of separation surface.

Referring back to the three time equations, these are as follows:

Phase 1

$$\frac{1}{K_1}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \tag{3}$$

$$T(r,z,0) = T_a, \qquad z>0 \tag{4}$$

$$T(r_\infty, z, t) = T_a \tag{5}$$

$$T(r, z_\infty, t) = T_a \tag{6}$$

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_1}\phi_S(r,0,t), \quad r \in [0, d] \tag{7}$$

$$\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = 0, \quad r > d \tag{8}$$

where: d is the ray of the laser beam, k_1 is thermal conductivity that belongs to solid state, T_a is environment temperature, φ_S - the power tide on the surface to work on and K_1 is the diffusivity of the material that belongs to solid state.

The power tide on the processed surface belonging to solid state and is given by the relation [2]:

$$\phi_S(r,0,t) = C_S \cdot e^{-\left(\frac{r}{d}\right)^2} \tag{9}$$

where: $C_S = (A_S \cdot P_L) / \pi d^2$ (A_S - absorbability of solid surface; P_L - the power of laser beam).

Equation (4) represents the initial condition, equations (5) and (6) are limit conditions of Dirichlet type, and equation (7) and (8) are limit conditions of Newman type (thermal tide imposed on the object surface).

Because it was assumed that the area of thermal influence neighbouring the cut is comparable to the width of the cut it may consider that $r_\infty \cong 6d$. Values bigger than the thickness of the substantial will be taken into consideration for z_∞.

Phase 2 - initial conditions are:

$$T(0,0,t_{top}) = T_{top} \tag{10}$$

$$T(r,z,t_{top}) = T_1(r,z,t_{top}), \qquad z>0 \tag{11}$$

where: T_1 is the equation solution (3) and T_{top} is the melting temperature of the substantial.

In order to determine limit conditions it is stipulated the areas taken by the liquid state, respectively the solid one. The frontier equation between liquid and solid state is considered as $r = f_1(z,t)$. By intersecting this frontier with plan $z = 0$ derives the curve $r = f_1(0,t)$.

Identically, by intersecting the frontier with the line $r = 0$, it results $z_1(t)$ as solution of the equation $f_1(z,t) = 0$. The following areas are the results: D_2 area taken by the liquid state is defined by $r < f_1(z,t)$ for $0 < z < z_1(t)$; D_1 area taken by the solid state is defined by $r \in [f_1(z,t), r_\infty]$, $0 < z < z_1(t)$ and $r \in (0, r_\infty)$, $z > z_1(t)$.

In D_2 area thermal equation is:

$$\frac{1}{K_2}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \tag{12}$$

and in D_1 area thermal equation is (3).

K_2 represents the diffusivity of the substantial corresponding to liquid state.

On the frontier separating the two phases, the limit conditions are:

$$T_1(r,z,t)\big|_{r=f_1(z,t)} = T_2(r,z,t)\big|_{r=f_1(z,t)} = T_{top}, \quad z > 0 \tag{13}$$

$$k_1 \left.\frac{\partial T_1}{\partial n}\right|_{r=f_1(z,t)} - k_2 \left.\frac{\partial T_2}{\partial n}\right|_{r=f_1(z,t)} = \rho_l \cdot L_{top} \cdot v_{Ln} \tag{14}$$

where: T_2 is the equation solution (12), k_2 is thermal conductivity corresponding to liquid phase, ρ_l is mass density corresponding to liquid phase, v_{Ln} is the normal movement speed of the separation frontier between liquid and solid state, L_{top} is the latent melting heat and n is the normal on the separation surface.

For $z = z_\infty$, respectively $r = r_\infty$, the limit conditions are conditions (5) and (6).

For $z = 0$ situations a) $r_l \le d$ or b) $r_l > d$ can be obtained.

For situation a) the limit conditions are:

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_2}\phi_L(r,0,t), \quad r \in [0, r_l] \tag{15}$$

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_1}\phi_S(r,0,t), \quad r \in (r_l, d] \tag{16}$$

$$\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = 0, \quad r > d \tag{17}$$

The next relation gives the power tide on the processed surface corresponding to the liquid state:

$$\phi_L(r,0,t) = C_L \cdot e^{-\left(\frac{r}{d}\right)^2} \tag{18}$$

where: $C_L = (A_L \cdot P_L) / \pi d^2$ (A_L - absorbability on liquid surface).

For situation b) the limit conditions are:

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_2}\phi_L(r,0,t), \quad r \in [0, d] \tag{19}$$

$$\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = 0, \quad r > d \tag{20}$$

Phase 3 - initial conditions are:

$$T(0,0,t_{vap}) = T_{vap} \tag{21}$$

$$T(r,z,t_{vap}) = T_2(r,z,t_{vap}), \quad (r,z) \in D_2 \setminus \{(0,0)\} \qquad (22)$$

$$T(r,z,t_{vap}) = T_1(r,z,t_{vap}), \quad (r,z) \in D_1 \qquad (23)$$

where: T_{vap} is the vaporising temperature of the substantial.

In order to determine the respective patterns it is considered the equations of the frontiers between liquid and solid state, $r = f_1(z,t)$ and $r = f_2(z,t)$ between vapour and liquid state. By intersecting these frontiers with plan $z = 0$ curves $r_1 = f_1(0,t)$ and $r_2 = f_2(0,t)$ are obtained. The same by intersecting the frontiers with the right $r = 0$, $z_1(t)$, equation solution $f_1(z,t) = 0$ and $z_2(t)$, equation solution $f(z,t) = 0$ are obtained.

The following patterns derive in this way:

- pattern D_3 taken by vapour state is defined by $r < f_2(z,t)$ for $0 < z < z_2(t)$;
- pattern D_2 taken by liquid state is defined by $f_2(z,t) < r < f_1(z,t)$ for $z_2(t) < z < z_1(t)$;
- pattern D_1 taken by solid state is defined by $r \in (f_1(z,t), r_\infty)$, $0 < z < z_1(t)$ or $r \in (0, r_\infty)$, $z > z_1(t)$.

In pattern D_3 thermal equation is:

$$\frac{1}{K_3}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \qquad (24)$$

and in pattern D_2 respectively D_1 equations are (3) and (12). K_3 represent the diffusivity of the substantial corresponding to vapour state.

On the separation frontier between D_3 and D_2 the limit conditions are:

$$\left.T_3(r,z,t)\right|_{r=f_2(z,t)} = \left.T_2(r,z,t)\right|_{r=f_2(z,t)} = T_{vap}, \quad z > 0 \qquad (25)$$

$$k_3 \left.\frac{\partial T_3}{\partial n}\right|_{r=f_2(z,t)} = k_2 \left.\frac{\partial T_2}{\partial n}\right|_{r=f_2(z,t)} = \rho_v \cdot L_{vap} \cdot v_{Vn} \qquad (26)$$

where: T_3 is the equation solution (24), k_3 - thermal conductivity corresponding to vapour state, ρ_v - mass density corresponding to vapour state, v_{Vn} - normal movement speed of separation frontier between vapour and liquid state, L_{vap} - latent heat of vaporisation.

On the separation frontier between D_2 and D_1 the limit conditions are (13) and (14). For $z = z_\infty$, respectively $r = r_\infty$, the limit conditions are conditions (5) and (6). On $z = 0$ frontier the following situations can occur a) $r_1 \le d$; b) $r_2 \le d$ and $r_1 > d$; c) $r_2 > d$.

For situation a) the limit conditions are:

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=z_{fr}} = \frac{1}{k_3}\left[\phi_V(r,z_{fr},t) - \phi_r - \phi_c\right], \quad r \in [0, r_2]$$

27)

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_2}\phi_L(r,0,t), \quad r \in (r_2, r_1] \qquad (28)$$

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_1}\phi_S(r,0,t), \quad r \in (r_1, d] \qquad (29)$$

$$\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = 0, \quad r > d \qquad (30)$$

The power tide on the surface to work on corresponding to vapour state is given by the relation:

$$\phi_V[r,z_{fr},t] = C_G \cdot e^{-\left(\frac{r}{d_V}\right)^2} \qquad (31)$$

where: $C_G = C_{G_1} + C_{G_2} = \frac{P_L}{\pi d^2} + \frac{\eta_o \cdot \varepsilon \cdot \rho_v \cdot v_S}{M}$ (η_o - oxidizing efficiency, ε - oxidizing energy on completely oxidized metal atom, v_S - the speed of vaporisation frontier, M - atomic mass of the metal, d_V - ray of the laser beam on the separation frontier between vapour state and liquid state and it is calculated with the relation (32); z_{fr} - z co-ordinate corresponding to the frontier between vapour state and liquid state).

$$d_V = d + \frac{D-d}{f} \cdot z_{fr} \qquad (32)$$

where: D is the diameter of the generated laser beam and f is the focusing distance of the focusing system. In (27), the power losses through electromagnetic radiation, φ_r and convection, φ_c were taken into account [8]:

$$\phi_r = \sigma\left(T_{vap}^4 - T_a^4\right) \qquad (33)$$

$$\phi_c = H\left(T_{vap} - T_a\right) \qquad (34)$$

where: σ is Stefan-Boltzmann constant; H - substantial thermal transfer constant. The emittance of irradiated area was considered equal to 1.

For situation b) the limit conditions are:

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=z_{fr}} = \frac{1}{k_3}\left[\phi_V(r,z_{fr},t) - \phi_r - \phi_c\right], \quad r \in [0, r_2] \qquad (35)$$

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = \frac{1}{k_2}\phi_L(r,0,t), \quad r \in (r_2\, d] \qquad (36)$$

$$\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = 0, \quad r > d \qquad (37)$$

For situation c) the limit conditions are:

$$-\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=z_{fr}} = \frac{1}{k_3}\left[\phi_V(r,z_{fr},t) - \phi_r - \phi_c\right], \quad r \in [0, d] \qquad (38)$$

$$\left.\frac{\partial T(r,z,t)}{\partial z}\right|_{z=0} = 0, \quad r > d \qquad (39)$$

3. Determination of separation frontiers equations

The problem of determining the frontiers [5,6] occurs in moment $t \geq t_{top}$. As initial moment, $t = t_{top}$ is considered. For the next moment, $t_{top} + \Delta t$, the point where the temperature is equal to T_{top} and which is the origin of co-ordinates system, transforms itself in a surface whose equation must be determined. Based on experimental results [6], the shape of this surface is known.

Therefore, this surface is governed by the equation of a rotating ellipsoid having Oz as the symmetry axis. The parameters that determine this surface (α, β) are variable in time.

In this case the cylindrical co-ordinates equation for $r \geq 0$ will be:

$$r = \frac{\alpha}{\beta}\sqrt{\beta^2 - z^2} \tag{40}$$

with: $\alpha = \alpha(t)$ and $\beta = \beta(t)$.

In order to determine the parameters α and β, there are two conditions. The relations that tie the temperature gradient on normal direction surface submit these conditions and the movement speed of the surface in this direction:

$$\frac{\partial T}{\partial n} = -\frac{\rho L}{k} v_n \tag{41}$$

where: L is the latent heat of melting or vaporisation and v_n is the normal movement speed of the frontier. Relation (41) is applicable in points $M_z(0, z(t))$ and $M_r(r(t), 0)$ in order to obtain the points of the frontier, $M'_z\,(0, z(t + \Delta t))$ and $M'_r(r(t + \Delta t), 0)$.

For point (0, z(t)):

$$\frac{\partial T}{\partial z} = -\frac{\rho L}{k} v_z \Rightarrow v_z = -\frac{k}{\rho L}\frac{\partial T}{\partial z}(M_z,t) \tag{42}$$

where: $v_z \approx \dfrac{z(t+\Delta t) - z(t)}{\Delta t}$ and results:

$$z(t+\Delta t) = z(t) - \frac{k}{\rho L}\frac{\partial T}{\partial z}(M_z,t)\,\Delta t \tag{43}$$

For point (r(t), 0):

$$\frac{\partial T}{\partial r} = -\frac{\rho L}{k} v_r \Rightarrow v_r = -\frac{k}{\rho L}\frac{\partial T}{\partial r}(M_r,t) \tag{44}$$

where: $v_r \approx \dfrac{r(t+\Delta t) - r(t)}{\Delta t}$ and results:

$$r(t+\Delta t) = r(t) - \frac{k}{\rho L}\frac{\partial T}{\partial r}(M_r,t) \tag{45}$$

With (43) and (45) it are determine:

$$\alpha = \alpha(t + \Delta t) \text{ and } \beta = \beta(t + \Delta t):$$
$$\alpha = r(t+\Delta t), \quad \beta = z(t+\Delta t) \tag{46}$$

The initial moment when applying the presented procedure is t_{top}, moment in which $z(t_{top}) = 0$ and $r(t_{top}) = 0$. Because in z = 0 and r = 0 the temperature gradient on z direction is,

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = -\frac{1}{k_2}\phi_L(0,0,t) \tag{47}$$

in equation (43) results:

$$z(t_{top} + \Delta t) = \frac{\phi_L(0,0,t)}{\rho_l L_{Top}}\Delta t \tag{48}$$

and equation (45) becomes:

$$r(t_{top} + \Delta t) = \frac{T_{top} - T(\Delta r)}{\Delta r \cdot \rho_l \cdot L_{Top}} \cdot k_2 \cdot \Delta t \tag{49}$$

where: ρ_l is the mass density corresponding to liquid phase, L_{Top} - the latent heat melting, and:

$$\frac{\partial T}{\partial r} \approx \frac{T(r+\Delta r) - T(r)}{\Delta r} \tag{50}$$

The same procedure is to be followed for $t = t_{vap}$ formula.

4. Numeric results

The equations of the theoretical proposed model to describe the way the material submitted to laser action reacts were solved numerically by finite differences method.

The variables and the unknown functions were non-dimensional and it was chosen a net of equidistant points in the pattern presented by the substantial. Because the points of the net neighbouring the frontier have the distances up to frontier different of the net parameters some digitization formula with variable steps have been used for them. Because the initial moment of interaction of the laser beam with the substantial does not depend on the parameters of the discrete net, it was necessary to replace it in initial condition on frontier z = 0, the temperature gradient reported to z with the temporal temperature gradient [2,6]:

$$z_m = 2\sqrt{K \cdot t} \tag{51}$$

where z_m represents the depth of heat penetration after a t irradiation time (the point where temperature starts to be neglected as reported to the increasing of temperature on the surface).

Taking into account the relation (51), the temperature gradient in relation to z becomes:

$$\frac{\partial T}{\partial z} = \sqrt{\frac{t}{K}} \cdot \frac{\partial T}{\partial t} \tag{52}$$

An algebraic system of equation solved at each time-step by column method was obtained after digitization and application of the limit conditions. The procedure is specific to implicit method of solving numerically the thermal equation and it was chosen because there were no restrictions on the steps in time and space of the net.

The calculus program was written in FORTRAN language and with the help of a PC there were certain numeric results obtained which permitted a comparison between the proposed method, experimental method and analytical method [6,7]. The calculated and compared parameters were the cutting speed and the cutting thickness.

The results for different powers of the laser are presented in a graphic format.

As input data there were considered:

- the substantial constant of iron (for solid, liquid and vapour state);
- the ray of the generated laser beam: D = 10mm;
- the number of intervals on direction y: N = 32;
- the number of intervals on direction x: M = 8;
- the step in time: DELTAT = 10^{-5}s;
- the maximum thickness of the substantial to be worked on: GR = 12mm.

It was denoted p the pressure of the working gas, usually oxygen or nitrogen. Parameter p influences the cutting process by means of power flow.

The temperature distributions at different depths within the material, for a laser power P_L = 400W, and a processing time t = 1ms, are presented in Figs. 1…4.

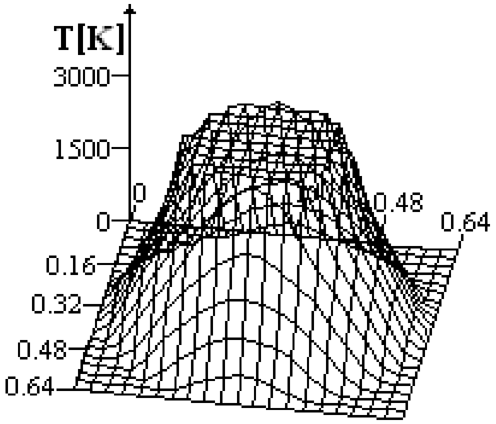

Fig.1. Temperature distribution for z = 0.

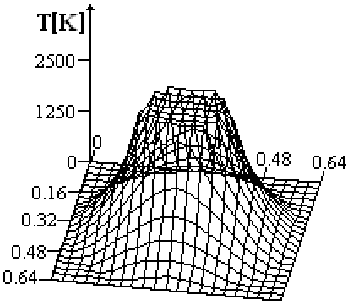

Fig. 2. Temperature distribution for z = 0.187mm.

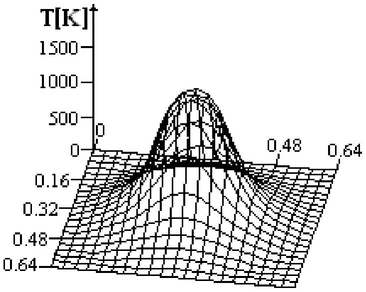

Fig. 3. Temperature distribution for z = 0.375mm.

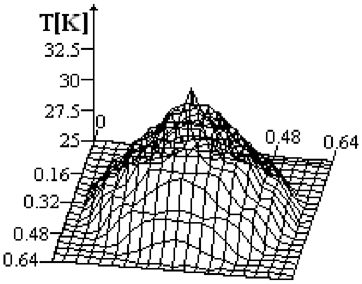

Fig. 4. Temperature distribution for z = 0.754mm.

Figs. 5 and 6 are showing the temperature distribution for a laser power P_L = 1000W and processing time t = 10ms. Temperature distribution was represented in two situations: at the material surface and at the material evaporating depth (z = 4.192mm).

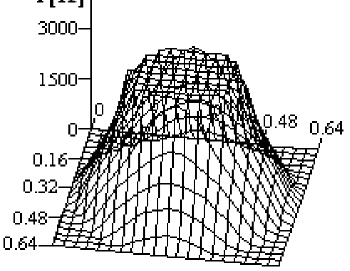

Fig. 5. Temperature distribution for z = 0 (P_L = 1kW).

The temperature distributions at material surface for $P_L = 400W$ and $P_L = 1000W$ are quite identical, as one can observe in Figs. 1 and 5.

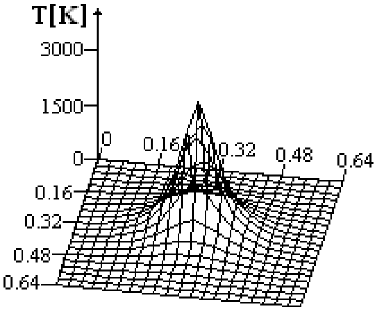

Fig. 6. Temperature distribution for z = 4.192mm.

Fig. 7 shows the vaporization depth variation as a function of laser power, for three values of the processing time, and figure 8 presents the vaporization depth variation as a function of additional gas, (O_2), pressure, for three values of the processing time. The vaporization depth has a nearly linear increasing with the laser power. For an additional gas pressure greater then 2 bars, the vaporization depth is quite constant, owed to the insignificant variation of the oxidizing efficiency in this case ($\eta_0 = 0.99\ldots1$).

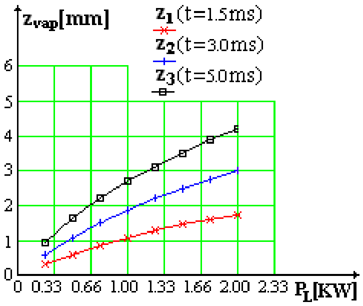

Fig. 7. Vaporization depth variation within the material, function of the laser power.

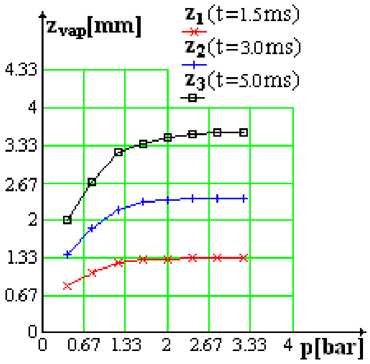

Fig.8. Vaporization depth variation within the material, function of the pressure of the additional gas.

The depths corresponding to the melting and vaporization temperatures are: $z_{top} = 4.288mm$, respectively $z_{vap} = 4.192mm$.

The moments when material surface reaches the vaporization and melting temperatures are: $t_{Vap} = 0.181 \cdot 10^{-5}s$, respective $t_{Top} = 0.132 \cdot 10^{-5}s$.

The material vaporization depth is depending on the processing time, and the considered input parameters as well. So, for a 10 times greater processing time and a 2.5 times greater laser power, one may observe a 10.94 times greater vaporization depth, compared with the previous case.

5. Conclusions

The mathematical model proposed represents a method of determining the temperature distribution and the cutting thickness when cutting metallic substantial with concentrated laser beam assisted by an active gas jet. The method of numeric calculation used, in comparison to other known methods, has the advantage of approximating more properly the real phenomenon, considering the simultaneous existence in the substantial in the area of its interaction with the laser beam of all three solid, liquid and vapour states. In the same respect, it is taken into account the effect of de-focusing of the laser beam together with its evolution into the material as well as the losses of energy through electromagnetic and convection radiation.

The method proposed solves simultaneously the thermal equation for all three states (solid, liquid and vapour) determining the temperature distribution in the substantial and respectively, the thickness of penetrating the substantial for a certain cutting time, having been able to calculate the speed of substantial vaporising. For a better approximation of the real phenomenon, specific substantial parameters specific to each phase of the cut substantial were considered.

Among the hypothesis on which the mathematics model is based on and hypothesis that need a more thorough analysis is the hypothesis on frontiers formation between solid state and liquid state, respectively, the liquid state and vapour state, supposed to be known previously, parameters that characterise the frontiers being determined from the thermal regime prior to the calculus moment.

The indirect results obtained as such (the thickness of penetrating the substantial, the vaporisation speed) certify the correctness of the hypothesis made with frontier formula. The known considering of frontier formula necessitated the utilising of the axiom for constructing the numeric program: between the frontiers that separate two stages, the temperature is always within the value of the temperatures on to the two frontiers.

The results thus obtained are placed within the limits of normal Physics, which constitutes a verifying of the mathematics model equation.

References

[1] H. S. Carslaw, J. C. Jaeger, Conduction of Heat in

Solids, Oxford-Univ. Press, 1968.
[2] V. Draganescu, V.G. Velculescu, Thermal Cutting With Laser Beams, Academy Publishing House, Bucharest, 1986.
[3] J. Mazumder, J., Overview of Melt Dynamics in Laser Processing, Optical Engineering **30**(8), 1209 (1991).
[4] J. Mazumder, W. M. Steen, Heat Transfer Model for C. W. Laser Materials Processing, J. Appl. Phys., **51**, 941 (1980).
[5] S. Nedelcu, M. Pearsica, A. Radu, Determining Mobile Frontiers Between Solid, Liquid and Vapour State when Working on with Laser Beam, XVII-th Scientific Communication Conference - Nav-Mar-Edu 2001, Constanta, 2001.
[6] M. Pearsica, Electromagnetic and Mechanical Processes at Laser Beam Processing, Doctorate, Thesis, Politehnica University, Bucharest, 2002.
[7] M. Pearsica, S. Nedelcu, A Simulation Method of Thermal Phenomena at Laser Beam Irradiation, Int. Conf. – Applied Electronics 2005, Pilsen, Cehia, September 2005, p. 269-272.
[8] M. Pearsica, C. Constantinescu, C. Strîmbu, L. Miron, An Analytical Method to Determine the Energy Loss through Electromagnetic Radiation at Laser Beam Irradiation, 5th Int. Conf., Miklos Zrinyi Defence University, Budapest, Hungary, November 2007, p. 63-70.

---

[*] Corresponding author: sbaluta@acttm.ro