

# An electro-magneto-optical resonance method for determination of the value and sign of the flexoelectric coefficients in nematics

H. P. HINOV<sup>a\*</sup>, Y. MARINOV<sup>b</sup>

*Institute of Solid State Physics, Bulgarian Academy of Sciences, 72 Tzarigradsko Chaussee Blvd., 1784 Sofia, Bulgaria*

<sup>a</sup>Liquid Crystal Laboratory, <sup>b</sup>Biomolecular Layers Laboratory

A novel method for determining of the flexoelectric coefficients of nematics has been proposed. It is based on the electro-optical behaviour of the surface-induced flexoelectric domains in an additional applied magnetic field in the Z direction. The electro-magneto-optical curves have a resonance character due to competition between the flexoelectric, dielectric and magnetic torques. From the resonance, where the intensity of the transmitted light is maximal, one can obtain formulae which permit calculation of a relation between the flexoelectric coefficients of bend  $e_{3x}$ , splay  $e_{1z}$ , and the values of the electric and magnetic fields. As a second relation, we choose the sum of the two flexoelectric coefficients. The obtained quadratic equations have been solved numerically with the aid of computer, for all permitted negative and positive values of the sum of the two coefficients. Finally, the values of the coefficients of bend and splay are discussed. Their variations for small values of the total flexoelectric coefficient around zero are the most interesting.

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## 1. Introduction

The electro-magneto-optical behaviour of surface-induced flexoelectric domains (flexo-dielectric walls) can give, first, valuable information about the exact place (near the anode or cathode) of a possible inhomogeneity of the electric field in the nematic cell [1] and second information on the value and sign of the flexoelectric coefficients of bend  $e_{3x}$  and splay  $e_{1z}$  [2]. One of the authors (H.P.H.) calculated the value of the flexoelectric coefficients for the case of the nematic MBBA [2] on the basis of previously performed experiments, in [1]. These calculations had a preliminary character, since only a positive sign of the sum of the flexoelectric coefficients ( $e_{1z} + e_{3x}$ ) was used. In this work, we repeated the calculations performed in [2] by computer, for all possible (negative and positive) values of the total flexoelectric coefficient,  $p$ .

## 2. Theory and results

It is well known that relations exist between the dielectric constants of a “free” nematic which is able to show its flexoelectric properties, the dielectric constants of a “clamped” nematic which is unable to show its flexoelectric properties and the quantities  $4\pi(e_{1z}^2/K_{11})$  and  $4\pi(e_{3x}^2/K_{33})$  as follows (we have used the simpler relations proposed by Helfrich [3], rather than those proposed by Derzhanski and Petrov [4]):

$$\varepsilon_{||}^f = \varepsilon_{||}^c + 4\pi(e_{1z}^2 / K_{11}) \quad (1)$$

$$\varepsilon_{\perp}^f = \varepsilon_{\perp}^c + 4\pi(e_{3x}^2 / K_{33}) \quad (2)$$

where  $f$  and  $c$  are abbreviations for “free” and “clamped”. Furthermore, we obtained an important relation between the dielectric anisotropy of a “free” and a “clamped” nematic:

$$|\Delta\varepsilon|^f = |\Delta\varepsilon|^c + 4\pi\left(\frac{e_{3x}^2}{K_{33}} - \frac{e_{1z}^2}{K_{11}}\right) \quad (3)$$

It is clear that  $|\Delta\varepsilon|^f$  can be equal to, bigger than, or smaller than  $|\Delta\varepsilon|^c$ , depending on the value and the sign of the second term in Eq. (3) [1,2].

From [1,2] we know that the magnetic field can enhance the flexoelectric deformations up to the following equality for a “free” nematic:

$$-\frac{|\Delta\varepsilon|^f}{4\pi}E_f^2 + |\Delta\chi|H_c^2 = 0 \quad (4)$$

From Eqs. (3) and (4), we finally obtain the following important relation:

$$\left(\frac{e_{3x}^2}{K_{33}} - \frac{e_{1z}^2}{K_{11}}\right) = -\frac{|\Delta\varepsilon|^c}{4\pi} + |\Delta\chi|\frac{H_c^2}{E_f^2} \quad (5)$$

where with  $H_c$  is the value of the magnetic field which enhances maximally the flexoelectric deformations (see Fig. 1):

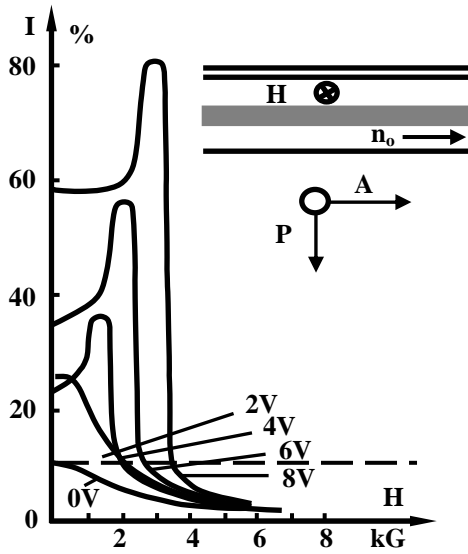


Fig.1. Electro-magneto-optical transmittance curves of the surface-induced flexoelectric domains obtained in a nematic MBBA layer with a thickness of  $15 \mu\text{m}$ . The data in the theory are extracted from the maxima of the curves.

The values of the magnetic constants for the case of the nematic MBBA (utilized experimentally [1]) can be found, for example, in [5-7]. The value of  $|\Delta\epsilon|^c/4\pi$  exactly coincides with that extracted from the data for  $(E_c/H)^2$  for the case of MBBA, considered in [7]. We, however, prefer to use the data for the case of the nematic MBBA (already utilized in [2]) as follows:

$|\Delta\epsilon|^c = 0.6$ ,  $\Delta\chi = 0.9 \times 10^{-7}$ ,  $K_{11} = 5.6 \times 10^{-7}$  dynes,  $K_{33} = 8.2 \times 10^{-7}$  dynes, obtained at  $T = 294^0$  K. Following the procedure in [2] and utilizing relation (5) and  $p = (e_{1z} + e_{3x})$ , we have obtained a quadratic equation which permits the determination of the flexoelectric coefficients, for instance, the coefficient of bend  $e_{3x}$ :

$$e_{3x}^2 + (b/a)e_{3x} + (c/a) = 0 \quad (6)$$

where:

$$(b/a) = -\frac{2K_{33}p}{K_{33} - K_{11}}$$

$$(c/a) = \frac{K_{33}K_{11}}{K_{33} - K_{11}} \left\{ \left[ \frac{|\Delta\chi|H_c^2}{E_f^2} - \frac{|\Delta\epsilon|^c}{4\pi} \right] + p^2 \right\}$$

$$p = (e_{1z} + e_{3x}) \neq 0$$

From the resonance in the curves presented in Fig. 1, we find the ratio  $H_c/E_f = 0.52$ .

The numerical solution of Eq. (6), performed by computer with the aid of the data given above, for the two solutions  $e_{3x1}$  and  $e_{3x2}$  and positive and negative signs of  $p$ :

is shown in Fig. 2. The allowed values of the  $e_{3x}$ , according to the inequalities [3]:

$$e_{3x}^2 < K_{33} \frac{\epsilon_{\perp}^f - 1}{4\pi}, \quad e_{1z}^2 < K_{11} \frac{\epsilon_{\parallel}^f - 1}{4\pi} \quad (7)$$

are also shown in Fig. 2 by the dotted lines.

It is clear that for the case of  $p > 0$ ,  $e_{3x1}$  (dotted curve) is an unphysical solution while the  $e_{3x2}$  solution is a physical one. In a similar manner, the unphysical and physical solutions for the case  $p < 0$  are presented by dotted and solid curves. The real solution for the flexoelectric coefficient of bend  $e_{3x}$  is shown in Fig.2. The most interesting behaviour of  $e_{3x}$  is around zero, where  $p = (e_{1z} + e_{3x})$  changes its value between  $-1.8 \times 10^{-4}$  and  $+1.8 \times 10^{-4}$  dynes. In this region,  $e_{3x}$  changes its sign from positive to negative for  $p > 0$  and from negative to positive for  $p < 0$  (see Fig. 2). In the rest of the region,  $e_{3x}$  is either positive when  $p = (e_{1z} + e_{3x})$  is positive or negative when  $(e_{1z} + e_{3x})$  is negative. Furthermore, the value of  $e_{3x}$  is within the limits imposed by Helfrich [3].

Analogous curves for the flexoelectric coefficient of splay  $e_{1z}$  are shown in Fig. 3. The only difference is that around zero, where  $p = 0$ ,  $e_{1z}$  does not change its sign and has a minimum or maximum, depending on the sign of  $p$ .

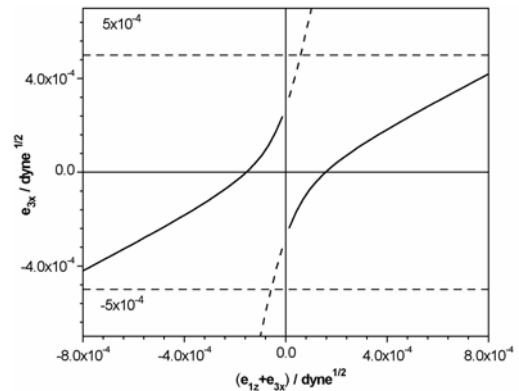


Fig. 2. The solution of the quadratic algebraic Equation(6), including physical (solid) and unphysical (dotted) values of the flexoelectric coefficient  $e_{3x}$  for the nematic MBBA.

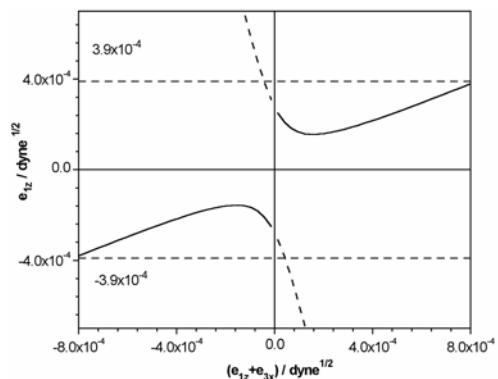


Fig. 3. The solution of the quadratic algebraic equation for the flexoelectric coefficient of splay  $e_{1z}$  of the nematic MBBA, including the physical (solid) and unphysical (dotted) values.

### 3. Discussion and conclusions

It is necessary to make here several remarks. First, the Orsay Liquid Crystal Group proposed measurements of  $e_{1z}^2$  or  $e_{3x}^2$ , or combination of them, via the thermal fluctuations of the nematic “director” [8], and Deuling via the Freedericksz transition of nematics [9]. Now, it is well known that the changes in the amplitudes of the thermal fluctuations or the value of the Freedericksz threshold are negligible. Consequently, real measurements of the flexoelectric coefficients of nematics cannot be done by these two methods. On the other hand, if these ideas are applied to zero methods, which are very sensitive to any changes, they can lead to success in the measurements of the flexoelectric coefficients. One of them is the divergence in the amplitudes of the thermal fluctuations of the nematic “director” around the Freedericksz transition [10]. The other is that proposed in this letter. The second remark concerns the value and sign of the total flexoelectric coefficient representing the sum of the two flexoelectric coefficients ( $e_{1z} + e_{3x}$ ). It is well known that this coefficient can be measured by two independent methods as follows: a) in an inhomogeneous electric field [10] and b) in a homogeneous electric [11-14] field. Since our zero method is based on the electro-magneto-optical behaviour of the dielectric flexoelectric walls in a homogeneous electric field after the redistribution of the free ions in the nematic cell, we recommend use of the value of  $(e_{1z} + e_{3x})$  for a nematic, measured in such an electric field. The third remark concerns the relation between  $(e_{1z} + e_{3x})$  and  $(e_{1z} - e_{3x})$  for the case of isotropic elasticity when  $K_{11} = K_{33} = K$ :

$$(e_{1z} - e_{3x})(e_{1z} + e_{3x}) = \left[ \frac{|\Delta\varepsilon|^c}{4\pi} - |\Delta\chi| \frac{H_c^2}{E_f^2} \right] K$$

Since, in our experiment, the right hand part of the above equation is positive, both  $(e_{1z} + e_{3x})$  and  $(e_{1z} - e_{3x})$  have equal signs (positive or negative) for the case of isotropic elasticity. Furthermore, we have assumed that  $p = (e_{1z} + e_{3x}) \neq 0$ . In the opposite case when  $e_{1z} = -e_{3x}$  and  $K_{11} = K_{33} = K$ , our method does not work since  $|\Delta\varepsilon|^c = |\Delta\varepsilon|^f$  (see the relation (3)). Finally, we show in Fig. 4 the general view of the curves for the flexoelectric coefficient of bend  $e_{3x}$ .

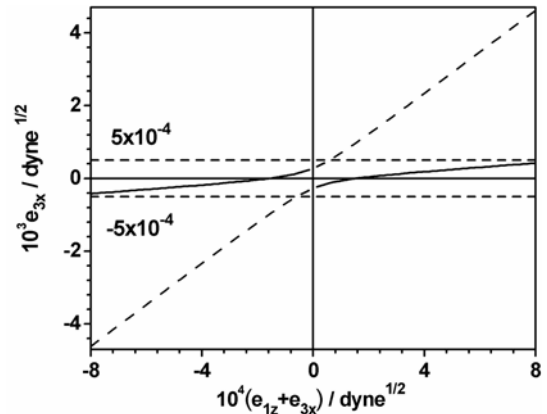


Fig. 4. Solutions of the quadratic equations for real values (solid curves) and nonphysical values (dotted curves) of  $e_{3x}$  (a general view for MBBA).

Analogous curves for  $e_{1z}$  can be drawn for MBBA, with the only difference that the unphysical values are in the second and fourth quadrants (see Fig. 3).

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\*Corresponding author: hinov@issp.bas.bg