

# An useful method to analyze the single-mode 3-D optical waveguide

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A simple method is developed to analyze single-mode 3-D optical waveguide, which we call the ray approximation-effective index method. The optimum width and depth are obtained for the single-mode 3-D optical waveguide. It is easy and effective for the optimum design of the optical waveguide using this method.

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## 1. Introduction

The single-mode graded-index optical waveguides are now widely used in integrated optics[1-2]. Various numerical methods have been used for the investigation of the waveguide[3]. Until recently the most popular methods have been the finite-element method, finite-difference method, finite-difference time-domain method and different beam propagation methods. However, all these methods are complex and time consuming for the design of the waveguide.

In this paper, a simple and effective method is developed to analyze single-mode 3-D graded-index optical waveguide. The method is based on the ray-approximation method and the effective index method, which we call ray approximation-effective index method. We take the diffused LiNbO<sub>3</sub> waveguide as an example to discuss the graded-index waveguide using the ray approximation-effective index method. The optimum design is got for the waveguide.

## 2. The ray approximation-effective index method to design the single-mode 3-D optical waveguide

The 2-D graded-index waveguide is shown as in Fig.1, in which  $n_c$  is the index in the clad,  $n_s$  is the index in the substrate,  $n_f$  is the index at the surface after diffusion,  $\Delta n = n_f - n_s$ ,  $d$  is the diffusion depth. It has been discussed using the ray-approximation method in the ref.4 [4]. From

the ref. 4, the mode dispersion equation for the Fig. 1(a) is shown as the followed:

$$2V \int_0^{\xi_i} (f(\xi) - b)^{\frac{1}{2}} d\xi = (2m + \frac{3}{2})\pi \quad m=0,1,2,\dots \quad (1)$$

where  $V$  is the normalized diffusion depth,  $V = k_0 d \sqrt{n_f^2 - n_s^2}$ ,  $k_0 = 2\pi/\lambda$  is the free-space phase constant;  $\zeta = x/d$ ,  $\zeta_i = x_i/d$ ,  $x_i$  is the maximum penetration; the function  $f(\zeta)$  describes the variation in the index due to diffusion and takes on values between 0 and 1 ( $n(\zeta) = n_s + \Delta n f(\zeta)$ ),  $b$  is the normalized guided index

$$(b = \frac{N_{eff}^2 - n_s^2}{n_f^2 - n_s^2}) [5].$$

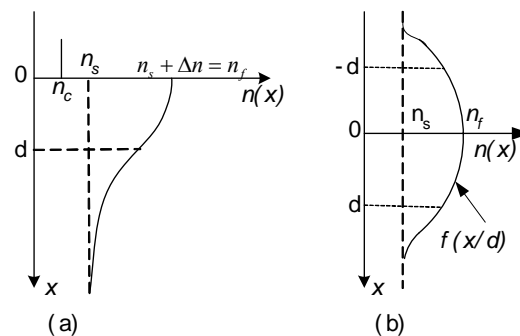


Fig. 1. The refractive-index distribution of 2-D graded-index waveguide (a) asymmetrical distribution (b) symmetrical distribution.

The cutoff of the m-th mode is easily found from the Eqs.(1), since  $b \rightarrow 0$  and  $\xi_t \rightarrow \infty$  for cutoff. Thus cutoff of the m-th mode is given by

$$V \int_0^\infty \sqrt{f(\xi)} d\xi = (m + \frac{3}{4})\pi \quad (2)$$

The refractive index profile of the diffused LiNbO<sub>3</sub> waveguide can be approximated by the Gaussian function [5-6],

$$\text{so } f(\xi) = \exp(-\xi^2) \left( \int_0^\infty \sqrt{\exp(-\xi^2)} d\xi = \sqrt{\pi/2} \right).$$

Then

$$V_m = \sqrt{2\pi} \left( m + \frac{3}{4} \right) \quad (3)$$

At the same time, for the Fig1. (b), the refractive-index distribution is symmetrical at the axis  $x=0$ , based on the previous analysis about the Fig. 1(a), the mode expression can be got for it.

$$2V \int_0^{\xi_t} (f(\xi) - b)^{\frac{1}{2}} d\xi = (m + \frac{1}{2})\pi \quad (4)$$

and the cutoff of the m-th mode is

$$V_m = \sqrt{\frac{\pi}{2}} \left( m + \frac{1}{2} \right) \quad (5)$$

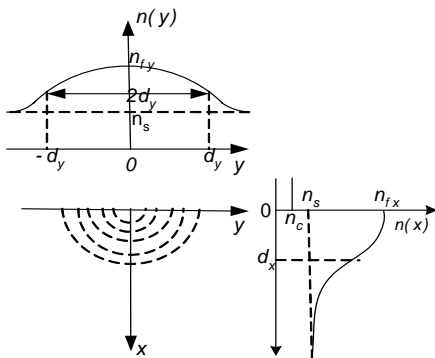


Fig. 2. The refractive-index distribution of 3-D graded-index waveguide.

The 3-D graded-index waveguide is shown as in Fig.2. We assume that the refractive index profile of the diffused LiNbO<sub>3</sub> waveguide is Gaussian function [5-6] in the x direction and y direction as shown in Fig.2. The 3-D optical waveguide should be divided into two 2-D

waveguides (2-D waveguide I and II) according to the effective index method[3].

To obtain the single-mode propagation, for 2-D waveguide I, based on the Eqs.(3), the following expression can be gotten.

$$\frac{3\sqrt{2}\sqrt{\pi}}{4} < V_I \leq \frac{7\sqrt{2}\sqrt{\pi}}{4} \quad (6)$$

where

$$V_I = k_0 d_x \sqrt{n_{fx}^2 - n_s^2} \quad (7)$$

For the 2-D optical waveguide II, the refractive-index is symmetrical distribution at the axis  $x=0$  as Fig.1(b). Then From Eqs.(5), the following expression can be obtained:

$$\frac{1}{2} \sqrt{\frac{\pi}{2}} < V_{II} \leq \frac{3}{2} \sqrt{\frac{\pi}{2}} \quad (8)$$

where

$$V_{II} = k_0 d_y \sqrt{N_I^2 - n_s^2}, \quad N_I = \sqrt{n_s^2 + b_I(n_f^2 - n_s^2)} \quad \text{is}$$

the effective refractive-index of guided mode [3].

By combining the Eqs. (7), we can get:

$$\sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{b_I} \bullet V_I} < \frac{2d_y}{d_x} \leq 3 \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{b_I} \bullet V_I} \quad (9)$$

Based on the Eqs.(1),(8) and (9), The aspect ratio versus the normalized frequency is as shown in Fig. 3. The bias is the single-mode propagation region

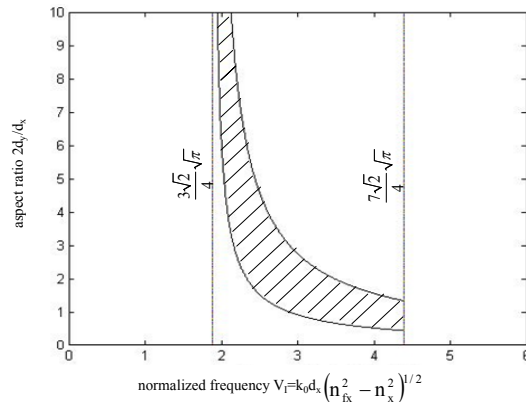


Fig. 3. The single-mode propagation region of 3-D graded-index waveguide.

### 3. Discussion

In Fig. 3, the bias is single-mode field. The single-mode waveguide is always required to attain as strong optical confinement as possible, so the waveguide parameter should be designed to be close to the upper boundary of the bias field. And, the radiation loss because of bend and branch will be higher, if the waveguide width becomes larger. So the optimum region of the single-mode area should be in the field, in which the aspect ratio is 1.5~2.5 in the Fig. 3.

If the aspect ratio is 2, we can get that the  $2.3 < V_1 < 3.5$  from Fig. 3, then we can get that  $0.26 \leq \frac{d_x}{\lambda} \sqrt{n_s \Delta n} \leq 0.39$ .

If  $\lambda = 1.06 \mu\text{m}$ ,  $n_s = 2.23$ ,  $\Delta n_x = 0.004$ , so the optimum design should be that  $2.92 \mu\text{m} \leq d_x \leq 4.38 \mu\text{m}$ , and  $5.84 \mu\text{m} \leq 2d_y \leq 8.76 \mu\text{m}$  to attain the single-mode propagation.

### 4. Conclusions

In conclusion, a simple but practical method has been developed based on the ray-approximation method and the effective index method to design single-mode 3-D waveguide. The optimum aspect ratio has been obtained for 3-D single-mode waveguide.

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