JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 9, No. 4, April 2007, p. 1065 - 1070

Analysis of noncooperative targets using a diode-pumped Nd:YAG microchip laser with frequency-shifted optical feedback

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It is described an experimental setup with a diode-pumped Nd:YAG microchip laser with frequency-shifted optical feedback and we present an experimental study of the characteristics of the reinjected laser, which we use for determining the reflectivity of noncooperative targets. It is demonstrated the very high sensitivity of this laser to frequency-shifted optical feedback. In order to shift the frequency of the output laser beam, we use two acousto-optic modulators. It is shown experimentally that the reinjection of a frequency-shifted beam into the laser cavity induces fluctuations of the laser output intensity and spectra. The laser output presents a strong amplification and also irregular spiking oscillations. By observing the amplification of the laser signal for different relaxation frequencies and frequency-shifts, the maximum amplification was obtained when the frequency-shift corresponded to the relaxation frequency of the laser in the absence of feedback. In this case, the response of the laser cavity to the optical reinjected field is maximized. The laser output in the presence on frequency-shifted optical feedback consists of alternative stable output and chaotic pulsation. The power spectrum of the laser with optical feedback contains the typical relaxation frequency and also the shifted frequency. The high sensitive response to optical feedback makes this laser very useful in practical applications. Based on the existing experimental methods and theoretical models, we show that the reflectivity of noncooperative targets can be determined and we also calculate the ultimate sensitivity of the technique. All the experimental results are in good agreement with the theoretical models.

(Received November 2, 2006; accepted February 28, 2007)

Keywords: Microchip laser, Frequency-shift, Optical feedback, Noncooperative target

1. Introduction

The dynamical behavior of lasers subjected to optical feedback is of current interest for many applications. The first study of the dynamics of lasers submitted to optical feedback was realized by Lang and Kobayashi [1]. There are recent studies in this subject in order to control the effects of optical feedback and to use them for practical purposes.

The sensitivity of lasers to the optical feedback increases with the ratio $\frac{\gamma_c}{\gamma_1}$, where γ_c is the damping rate of the laser cavity and γ_1 is the damping rate of the population inversion. The sensitivity of diode-pumped microchip lasers to the optical feedback is determined by the fact that the photon lifetime in the laser cavity is much smaller than the fluorescent lifetime in comparison with the case of laser diodes. The high sensitivity of these lasers to external feedback makes them good candidates for practical use.

Because of their utilization in practical applications, it is imperative to characterize the diode-pumped microchip lasers dynamics in the presence of optical feedback. Because of the difference in the time scales between the microchip lasers and the laser diodes, their dynamics is also different. The dynamical response of a diode-pumped microchip laser with optical feedback cannot be explained by the model of low-frequency fluctuations (LFF) in laser diodes (described by Lang-Kobayashi). While LFF occurs when the laser is biaset near the threshold and in the presence of a moderate feedback, the instabilities induced by the optical feedback in microchip lasers occurs when the laser is pumped at a high level (far from the threshold) and also for week feedback regimes. The first study of the changes of the temporal laser spiking pattern of a class-B laser submitted to optical feedback was demonstrated by Kleinman [2].

In this work we realize a diode-pumped microchip Nd:YAG laser system and we present an experimental study of laser dynamics in the presence of frequency-shifted optical feedback, which we use to analyse the noncooperative targets. This method is based on the high sensitivity of this type of laser to the reinjection of the beam reflected by an object and relies on an existing imaging technique (LOFI-Laser Optical Feedback Imaging) [3].

The laser system that we constructed is a class B laser, caracterized by the following relationship between the damping rate of the laser cavity and the damping rate of the population inversion: $\gamma_c >> \gamma_1$. The intensity of this laser presents oscillations at a characteristic frequency, named relaxation frequency.

If the laser beam is reflected on a noncooperative target, the reinjected light is only partially coherent and the interference effects between the electric field inside the cavity and the reinjected field are drastically reduced. In order to increase these effects it is necessary to perform a maximum excitation of the laser (which acts as a source and detector in the same time), by shifting the frequency of the reinjected beam and adjusting it to be equal with the relaxation frequency of the laser.

2. Theoretical model

In order to study the dynamics of the laser with frequency-shifted optical feedback it is necessary to add to the rate equations the term corresponding to the reinjected field after a round-trip laser-object of time τ [3]:

$$E_{reinj}(t) = E_c(t-\tau) e^{i\Phi_c(t-\tau)} e^{i(\omega+\omega_m)t} e^{-i\left(\omega+\frac{\omega_m}{2}\right)\tau} \quad (1)$$

where $E_c(t)$ is the amplitude, $\Phi_c(t)$ is the optical phase, ω_m is the pulsation corresponding to the frequency-shift, τ is the photon round-trip between the laser and the object, ω is the optical running laser frequency and ω_c is the laser cavity frequency.

If we suppose that $\omega_m \tau \ll 0$ we can approximate

$$E_c(t-\tau) \cong E_c(t) \quad (2a)$$

$$\Phi_c(t-\tau) \cong \Phi_c(t) \quad (2b)$$

With the assumption the Langevin's noise terms are negligible, the behavior of the laser with frequency-shifted optical feedback is described by the following set of equations:

$$\frac{dN}{dt} = \gamma_1(N_0 - N) - BN|E_c|^2 \quad (3a)$$

$$\frac{dE_c}{dt} = \frac{1}{2}\left[BN - \gamma_c + 2\gamma_{ext}\cos(\omega_m t - \omega_c \tau)\right]E_c \quad (3b)$$

$$\frac{d\Phi_c}{dt} = \gamma_{ext}\sin(\omega_m t - \omega_c \tau) \quad (3c)$$

where N is the population inversion, $\gamma_1 N_0$ is the pumping rate, $B \approx 2\cdot 10^{-4} s^{-1}$ is the Einstein coefficient and γ_{ext} represent the coupling rate (which depends on the cavity damping rate and of the reflectivity of the object). With the assumption that the multiples reflections laser-object are negligible, the coupling rate is given by [3]

$$\gamma_{ext} = \gamma_c \sqrt{R_{object}} \quad (4)$$

where R_{object} is the reflectivity of the object which realize the optical feedback.

From Eqs. (3) it can be seen that the frequency-shifted optical feedback induces a modulation at the shifted-frequency of the net gain and also of the optical phase (therefore we will also name the shifted frequency *modulation frequency* F_m).

For a week optical feedback, the Eqs. (3) are solved by linearization, considering small fluctuations around the steady states of the form

$$\begin{aligned} N(t) &= N_s + \Delta N(t) \\ E_c(t) &= E_s + \Delta E_c(t) \\ \Phi_c(t) &= 2\pi + \Delta\Phi_c(t) \end{aligned} \quad (5)$$

In Eqs. (5) N_s, E_s are the steady-state solutions given by:

$$N_s = \frac{\gamma_c}{B} \quad (6)$$

$$|E_s|^2 = E_{sat}^2(\eta - 1) \quad (7)$$

with $\eta = \frac{BN_0}{\gamma_c}$ the normalized pumping rate and $E_{sat}^2 = \frac{\gamma_1}{B}$ the saturation intensity. It can be seen that the phase is randomly distributed over 2π.

From Eqs. (3), (5) and considering only the terms of the first order, the linearized equations are:

$$\frac{d\Delta N}{dt} = -\left(\gamma_1 + B|E_s|^2\right)\Delta N - 2BN_s E_s \Delta E_c \quad (8a)$$

$$\frac{d\Delta E_c}{dt} = \frac{1}{2}BE_s\Delta N + \gamma_{ext}\cos(\omega_m t - \omega_c\tau)E_s \quad (8b)$$

$$\frac{d\Delta\Phi_c}{dt} = \gamma_{ext}\sin(\omega_m t - \omega_c \tau) \quad (8c)$$

By solving Eqs. (8) we obtain $\Delta E_c(t)$ and the relative laser output power of the modulation is [4]

$$\begin{aligned} \frac{\Delta P_{out}}{P_{out}} &= \frac{2\Delta E_c}{E_s} = \\ &= 2\gamma_c\sqrt{R_{object}}\frac{\sqrt{(\gamma_1\eta)^2 + \omega_m^2}}{\sqrt{\left(\omega_r^2 - \omega_m^2\right)^2 + (\gamma_1\eta)^2\omega_m^2}} \\ &\cdot\cos(\omega_m t - \omega_c\tau + \Phi_r) \end{aligned} \quad (9)$$

where

$$P_{out}(t) = \gamma_c |E_c(t)|^2 \quad (10)$$

is the photon output rate, $\omega_r = 2\pi F_r$ is the relaxation pulsation and Φ_r is the dynamical phase shift given by

$$\Phi_r = \arctan\left\{\frac{\omega_m\left[\left(\omega_r^2 - \omega_m^2\right) - (\gamma_1\eta)^2\right]}{\gamma_1\eta\omega_r^2}\right\} \quad (11)$$

From Eq. (9) it can be seen that the signal amplitude has a maximum value when the relaxation frequency is equal with the modulation frequency (Fig. 1). We also see that by knowing the amplitude of the signal it is possible to determine the reflectivity of the object R_{object}.

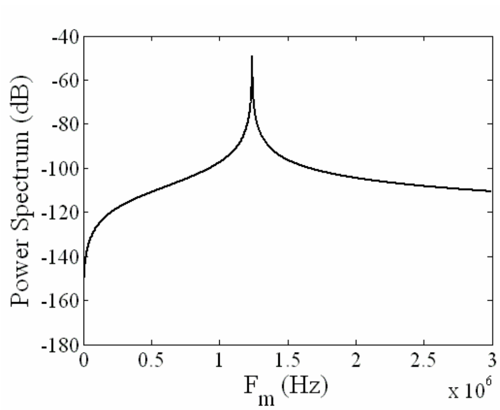

Fig. 1. Power spectrum of the square of the relative amplitude of the modulation versus the shifted-frequency (modulation frequency) $\eta = 2.94$, R_{object}= 10^{-17}, F_r=1200kHz, $\gamma_1 = 4.3 \cdot 10^3 s^{-1}$, $\gamma_c = 0.7 \cdot 10^{10} s^{-1}$.

The power density spectrum of the fluctuations of the output power due to the Langevin quantum noise (A_{noise}) is obtained from the rate equations of the laser without optical feedback ($\gamma_{ext} = 0$), with the addition of the Langevin forces, which describe the quantum fluctuations of the laser population ($F_N(t)$) and the electric field ($F_E(t)$).

$$\begin{aligned} \frac{dN}{dt} &= \gamma_1 (N_0 - N) - BN |E_c(t)|^2 + F_N(t) \\ \frac{dE_c}{dt} &= \frac{1}{2} [BN - \gamma_c] E_c + F_E(t) \end{aligned} \tag{12}$$

A linear analysis of the system gives the power density spectrum of the fluctuations of the output power determined by the Langevin quantum noise

$$A_{noise}(\Omega) = \frac{P_{out}\gamma_c^2}{\pi} \frac{\eta\gamma_1^2 + \Omega^2}{\left(\omega_r^2 - \Omega^2\right)^2 + (\eta\gamma_1)^2 \Omega^2} \tag{13}$$

From Eq. (9) is obtained the power density spectrum of the signal

$$A_{signal}(\Omega) = P_{out}^2 \gamma_{ext}^2 \cdot \frac{(\eta\gamma_1)^2 + \Omega^2}{\left(\omega_r^2 - \Omega^2\right)^2 + (\eta\gamma_1)^2 \Omega^2} \delta(\Omega - \omega_m) \tag{14}$$

For a detection bandwidth situated around the modulation pulsation ω_m and narrower than the resonance width ($\Delta\Omega \ll \eta\gamma_1$) the power signal to noise ratio for the amplitude of the modulation is

$$\frac{S}{N} = \frac{P_{out}}{2\Delta F} \frac{\gamma_{ext}^2}{\gamma_c^2} \frac{\eta^2\gamma_1^2 + \omega_m^2}{\eta\gamma_1^2 + \omega_m^2} \tag{15}$$

where $2\Delta F = \frac{1}{T}$ and T is the integration time.

Because $\omega_m \gg \eta\gamma_1$ and using Eq. (4), the signal to noise ratio will be

$$\frac{S}{N} = \frac{P_{out}}{2\Delta F} R_{object} \tag{16}$$

From (16) we see that the minimum reflectivity of the object that can be determined with this method is (for a given value of signal to noise ratio):

$$R_{object,\text{minimum}} = \frac{2\Delta F}{P_{out}} \frac{S}{N} \tag{17}$$

3. Results

The laser system which we realize in our experiment is a class B Nd:YAG microchip laser, with a cavity lenght L=0.8*mm*, operating at $\lambda = 1.064 \mu m$. In order to obtain the laser emission, the active medium is pumped by a system of laser diodes emitting at 810*nm*. Using a pair of lens we collimate the pump beam, then pass it through an anamorphic prism pair, which transforms it into a circular beam. The pump beam is focused onto the laser crystal by a microscope objective.

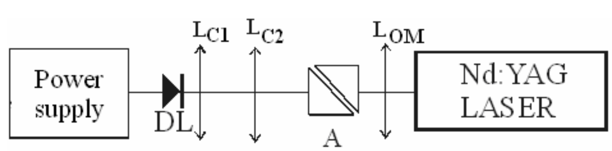

Fig. 2. Pumping system DL-Diode laser, L_{C1}, L_{C2} – collimating lens, L_{OM} - objective microscope, A-anamorphic prism pair.

This type of laser has relaxation oscillations at a characteristic frequency, named relaxation frequency F_r. For the maximum pump parameter $\eta \approx 6$ the maximum output power of the laser is $P_{out,laser} \approx 130mW$ and the relaxation frequency $F_r \approx 1MHz$. The current of the pumping system necessary to obtain the laser emission is $I_{diode\ laser} \approx 430mA$, corresponding to a pump power $P_{diode\ laser} \approx 160mW$. The power characteristics of the laser is shown in Fig. 3.

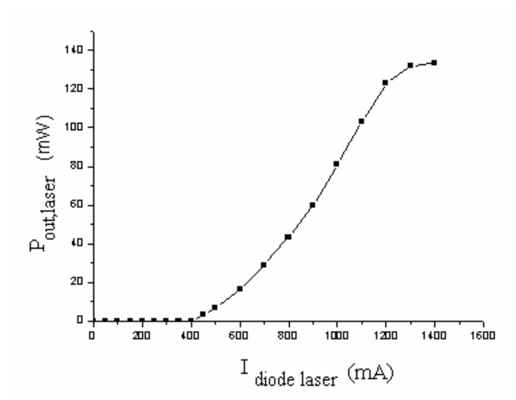

Fig. 3. Power characteristic of the laser.

The experimental setup is presented in Fig. 4.

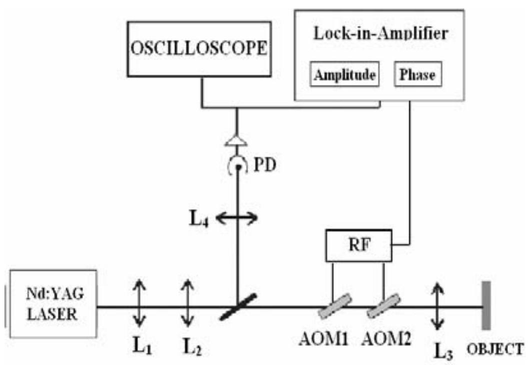

Fig. 4. Experimental setup L-lens, BS-beam splitter, AOM-acousto-optic modulator, PD-photodiode, RF-radio-frequency generator.

The frequency shift was obtained with two acousto-optic modulators (AOM). The first AOM is supplied by radio frequency at the frequency $F = 81.5 MHz$ and the second AOM is supplied at $\left(81.5 + \frac{F_m}{2}\right) MHz$. The first AOM is adjusted to obtain the maximum intensity for the order –1 of the diffracted laser beam; the second AOM is adjusted to obtain the maximum intensity for the order +1 of the diffracted beam, which will be frequency-shifted by the optical frequency $\frac{F_m}{2}$. Taking into account the fact that in a round-trip between the laser output and the object the laser beam passes twice through the acousto-optic modulators, the reinjected beam is shifted by the frequency F_m.

After being colimated by the lens L_1 and L_2, the laser beam is frequency-shifted with the two acousto-optic modulators, then focused on the target by the lens L_3. A part of the reflected beam is reinjected into the laser cavity, after passing again through the modulators. A small part of the laser beam is sampled from a beam splitter and focalized on a photodiode. The signal provided by the detector is sent to an oscilloscope (which gives the laser output intensity and the power spectrum) and to a Lock-in-Amplifier (which gives directly the amplitude and the phase of the signal).

First we establish that this laser is a laser of class B, which is sensitive to the optical feedback. In this purpose we use the linear dependence between the square of the relaxation pulsation and the pumping parameter $\omega_r^2 = \gamma_1 \gamma_c (\eta - 1)$ and we plot $\omega_r^2 = f(\eta)$ (Fig. 5). By a linear fit of the experimental points and knowing from the litterature the value of the damping rate of the population inversion γ_1 = 4.3 × $10^{3\text{-}1}$, we obtain $\frac{\gamma_c}{\gamma_1} \approx 1.7 \times 10^6$ >>1. This value proves that our laser is of class B and also that it can be successfully utilized for optical reinjection.

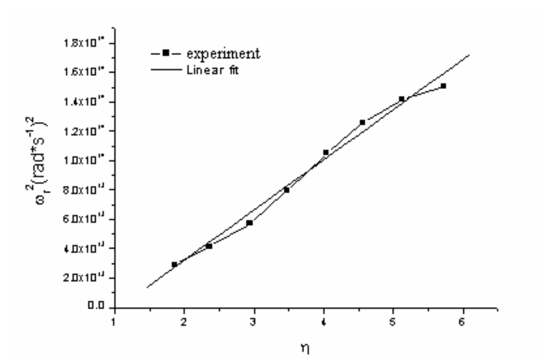

Fig. 5. Square of the relaxation pulsation fonction of the pump parameter.

In order to study the influence of the frequency-shifted optical feedback on the laser characteristics, we fix the pumping power at $P_{diode\text{-}laser}$=475 *mW*. For this value the pumping parameter is $\eta = 2.94$ and the relaxation frequency is $F_r = 1200 kHz$.

Without optical feedback the typical time evolution of the laser beam consists of a quasi-sinusoidale variation of the laser intensity at the relaxation frequency (Fig. 6).

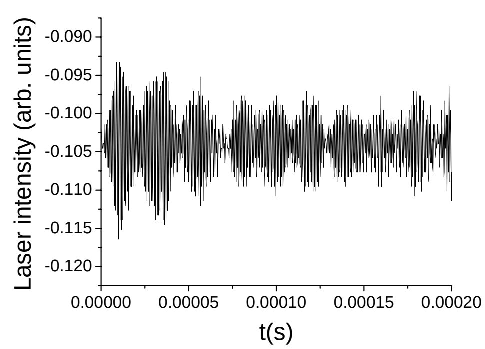

Fig. 6. Output intensity of the laser without optical feedback.

In the presence of the frequency-shifted optical feedback, the dynamic behavior of the laser changes. We observe experimentally that the optical feedback increases the intensity of the laser beam and determines the apparition of chaotic spike oscillations (Fig. 7).

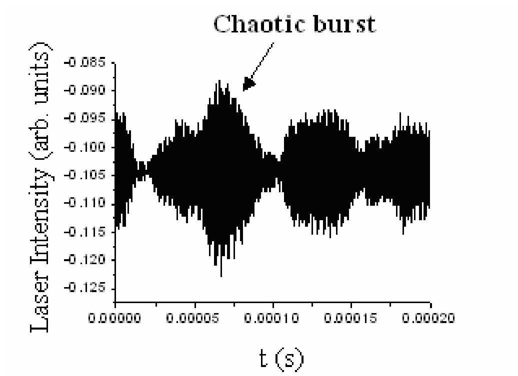

Fig. 7. Output intensity of the laser with frequency-shifted optical feedback Frequency –shift $F_m = 1190kHz$.

Fig. 8 presents the laser power spectrum in the case of the solitary laser. In this case the laser power spectrum consists of the laser quantum noise at the relaxation frequency.

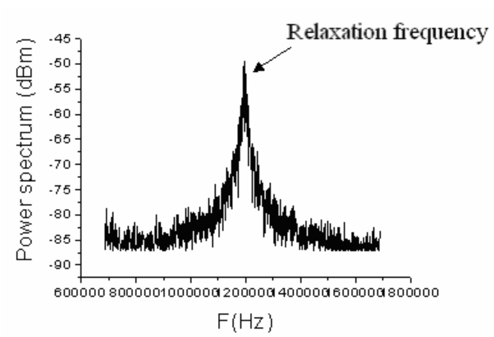

Fig. 8. Power spectrum of the solitary laser output power.

The frequency-shifted optical feedback determines the modification of the power spectrum of the laser output power (Fig. 9).

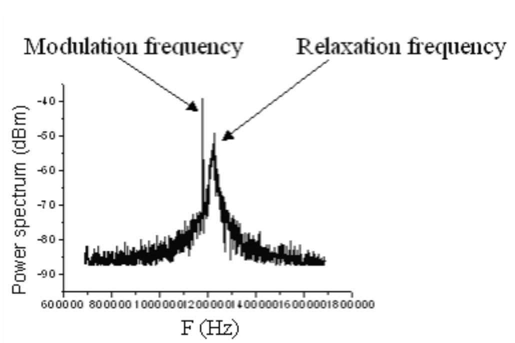

Fig. 9. Power spectrum of the laser output power with frequency-shifted optical feedback Frequency –shift $F_m = 1190kHz$.

To observe the sensitivity of the system function of the shifted frequency, we change F_m around the relaxation frequency by steps of 5kHz and we measure the amplitude of the shifted-frequency peak from the power spectrum.

By adjusting the frequency-shift at the value of the relaxation frequency ($F_m = F_r$) the sensitivity of the laser to the optical feedback is maximum. Fig. 10 presents the amplitude of the shifted-frequency peak versus the difference between the modulation frequency (the shifted frequency) and the relaxation frequency. The curve exhibits a maximum when the shifted frequency is equal to the relaxation frequency (so when $\Delta F = 0$).

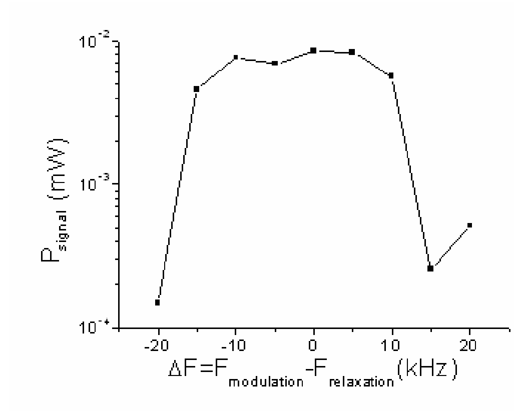

Fig. 10. Power signal versus the difference between the modulation frequency and the relaxation frequency.

As it was theoretically shown, by measuring the amplitude of the modulation it is possible to determine the reflectivity of the object (Eq. (9)). For the case $F_m = F_r = 1200kHz$, the value measured for amplitude of the signal allow us to determine the reflectivity of the object:

$$R_{object} \approx 7.5 \cdot 10^{-9} \tag{18}$$

In order to establish the minimum reflectivity that can be determined with this technique, we calculate (Eq. (10)) the photon output rate corresponding to the pumping parameter $\eta = 2.94$

$$P_{out}(t) = \gamma_c \left|E_c(t)\right|^2 = \frac{\gamma_1 \gamma_c}{B}(\eta - 1) \approx \approx 3 \cdot 10^{17}\ photons/s \tag{19}$$

From Eq. (17) we determine that the minimum detectable reflectivity for a signal to noise ratio $\frac{S}{N} = 1$ and for an integration time $T = 1ms$ (so $2\Delta F = 1kHz$) is

$$R_{object,\ \mathrm{minimum}} \approx 3 \times 10^{-17} \tag{20}$$

4. Discussion

Our experimental results show that the frequency-shifted optical feedback induces changes of the dynamical behavior of the laser, modifying the laser intensity and also the power spectrum of the laser output power.

The effect of the frequency-shifted optical feedback on the laser intensity was studied theoretically with the Heun method [5], by solving numerically the Lang-Kobayashi equations with Langevin noise sources. The results showed that the temporal evolution of the output of the reinjected laser consists of transitions from the stable output to the chaotic pulsations.

Experimentally we show that, if in the case of the solitary laser the typical time evolution of the laser output consists of a quasi-sinusoidal variation of the laser intensity at the relaxation frequency (Fig. 6), in the presence of frequency-shifted optical feedback the temporal behavior of the laser output consists of random chaotic bursts. As we can see in Fig. 7, the spiking oscillations are randomly distributed and they have irregular durations.

By observing the power spectrum of the laser power in the cases with/without optical feedback (Fig. 8, respectively Fig. 9), it can be seen that the power spectrum of the laser with frequency-shifted optical feedback consists of the typical relaxation frequency (F_r) (present also in the power spectrum of the solitary laser) and of the shifted frequency (modulation frequency) F_m. By increasing the optical feedback level the amplitude of the frequency peak corresponding to F_m is higher.

We also show experimentally that the optical feedback effects are strongly dependent on the modulation frequency (Fig. 10). We observe that the laser has a maximum sensitivity to the reinjected field when the frequency-shift is equal with the relaxation frequency of the laser. This result is in good agreement with the theoretical curve presented in Fig. 1.

By measuring the amplitude for the modulation of the output intensity of the laser at the shifted-frequency it is possible to determine the reflectivity of noncooperative objects; in our case the minimum reflectivity that can be detected is $R_{object,\ \mathrm{minimum}} \approx 3 \times 10^{-17}$.

5. Conclusions

It is demonstrated the very high sensitivity of a diode-pumped microchip Nd:YAG laser to frequency-shifted optical feedback. Based on the theoretical model for a class B laser with optical feedback, it is shown that the frequency-shifted optical feedback induces changes in the laser output and the power spectrum of the laser.

The intensity fluctuations are determined by the interaction between the laser radiation and the reinjected optical field. The frequency-shifted optical feedback determines the modulation of the output intensity of the laser at the shifted-frequency. Compared to the standard optical heterodyne detection, frequency-shifted optical feedback produces an intensity modulation much higher (of the order 10^6 for microchip lasers). It is investigated the sensitivity of this laser to the optical feedback and we show experimentally that the maximum modulation was obtained when the frequency-shift was adjusted to match to the relaxation frequency on the laser.

This method can be used to analyze noncooperative objects. By evaluating the ultimate sensitivity of the technique, it is shown that minimum reflectivity which can be determined with this method is of the order of 10^{-17} .

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