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Analysis of the image in system with negative-index lens

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There are analysed the necessary conditions of image's existence for an optical system with one-lens. In this system light passes through the lens made by negative-index medium with effective negative refractive($n_{eff} = -1$) from air ($n = 1$). Based on the one-lens imaging system, similar discussions are applied to a two-lens ($n_{eff} = -1$) system and the condition of image existence in two-lens system is also demonstrated. It was found that when just the problem of image's position is considered the two-lens system can be treated equally to the one-lens system since both systems have the same necessary conditions of image's existence. This conclusion can be extended to an N-lens system, which means an N-lens system can be treated equally to a one-lens system too. A FDTD method is used to simulate such one-lens and two-lens systems, and lenses with $n_{eff} = -1$ are produced by two-dimension photonic crystal slabs.

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1. Introduction

A flat slab of an effective medium with a refractive index of -1 can focus electromagnetic waves [1]. Such a negative-index lens can partly overcome the diffraction limit and achieve subwavelength focusing since evanescent waves are restored inside the negative-index medium(NIM) [2]. Since the medium with negative refractive do not exist naturally, it is usually constructed from artificially arranged composites. One way is to arrange a periodic lattice of split-ring resonators, which has an effective negative permeability, and metallic wires, which have an effective negative permittivity [3-5]. A negative-index lens also can be implemented by means of a planar slab consisting of a grid of printed metallic strips over a ground plane, loaded with series capacitors and shunt inductors [6]. A third choice is the use of photonic crystals (PhC), which has the added values of low absorption losses and straightforward scalability up to visible and infrared wavelengths [7].

Notomi [7] studied light propagation in strongly modulated 2D-PhC, and a negative n_{eff} for a frequency range was found where the equifrequency contours (EFCs) become rounded. If the EFC shrinks with increasing frequency then the group velocity points inwards and a phenomenon of negative refraction can be expected at the interface between the PhC and air(or a dielectric medium)[8] and a PhC can be behave as a flat negative-index lens. Further, Luo et al [9] showed that the propagation of light in a square 2D-PC in the first photonic band leads to an all angle negative refraction (AANR), which leads to superlensing. One can then define an effective refractivity of the PhC for all angles as that of the conventional material.

Recently, several authors have reported theoretical and experimental studies related to 2D-PhC [7,10-11]. And PhC slab with $n_{eff} = -1$ can be produced by different types of lattices, for example, triangular lattice and honeycomb lattice photonic crystal slabs [12,13].However, the effect of optical system consisting of two or more NIM or PhC slabs with negative refractive $n_{eff} = -1$ to point imaging has not been discussed. In this paper, we analyze an optical system consisted of two NIM-slabs with $n_{eff} = -1$ theoretically and the effect of this system to the image position is discussed. A finite-difference time-domain (FDTD) method [14] with perfectly matched layer boundary conditions [15] is used in our numerical simulations in system consisting of PhC-slab lens.

2. Discussion and results

We consider the issue about image's position in the case of one-lens slab with effective negative refraction index -1, which means when light passes through the air-NIM interface, the light's incident angle and the refractive angle have the same absolute values but reverse signs. This conclusion can be easily deduced from the Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$, so, the light rays beside the air-NIM interface are symmetric. Generally speaking, the symmetry of light rays to two different materials' interface appears when the two media possess refractive indexes which have reverse signs but equal absolute value, e.g. $n_1 = -n_2$.

For a point source, we define the distance between the source and the left air-NIM interface and the thickness of slab lens as D and L, respectively, along the x axis (shown in Fig. 1). The NIM-slab is infinite in z and y directions but finite in x direction. The relationship between D and L can be divided in three cases: i) D>L, ii) D<L, iii) D=L. In the first case, when D>L, the point image can not be formed because of radiating active transmitted light rays in the right area of the lens. We can find a contrary situation in the second case, where D<L, the point image exists in the right area of the lens.

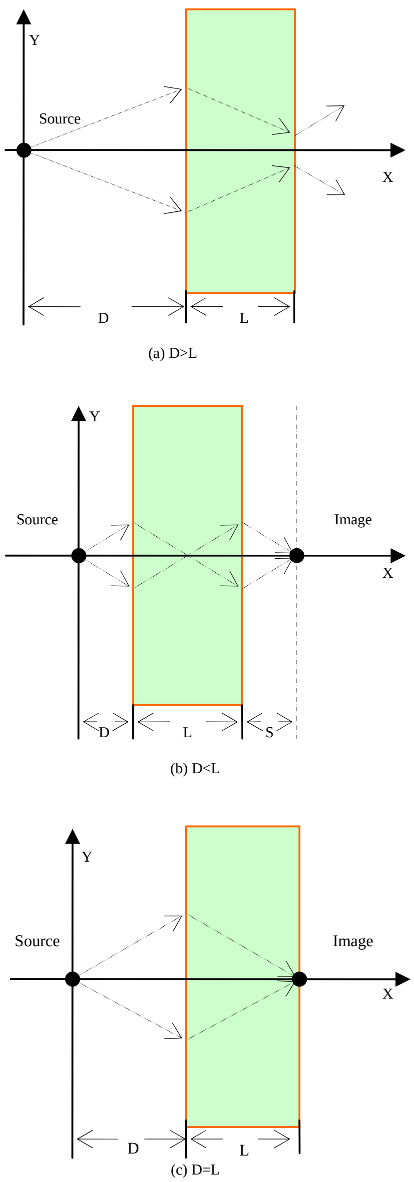


Fig. 1. (a), (b),(c) The schematic diagrams of the imaging system formed by NIM with $n_{eff} = -1$ *under the conditions D>L, D<L and D=L respectively.*

The third case is the transition state between case 1 and case 2. Here the image is just on the right interface of the air-NIM, which represent a critical state. Three different cases corresponding to D>L, D<L, D=L are shown in Fig.1 (a), (b), (c), respectively. Apparently, only when the imaging system with one negative refractive index lens satisfied the condition

$$D<L \qquad (1a)$$

the system can transform a point light source to a point image and the equation

$$D+S=L \qquad (1b)$$

also exists here, where S, the distance between the right interface of lens and the image can be obtained from equation (1b).

Next we consider a system consist of two lenses with negative refractive index -1. Several parameters are defined here. D1 represents the distance between the source and the left interface of the lens 1 and D2 is the distance between the two lenses. We use L1 and L2 to mark the length of lens 1 and lens 2 respectively. S here is used to represent the distance between the right interface of lens2 (right) and the image. The track of light which starts from the point source is shown in Fig. 2a; in this schematic map, two different light ray groups are labeled with 1 to 5 and 1' to 5', where the two groups are symmetric to the x axis.

Firstly, the case of D1>L1 is discussed. When the light reaches the left interface of lens 1, refraction happens and the light ray will follow the path 2 to continue its travel. Because of D1>L1, the internal focus can not be formed in lens 1 and after passing the right interface of lens 1, the light would transmit as ray 3 does. If the right interface of lens 1 doesn't exist, light rays would intersect in point B (shown in Fig. 2) since ray 1 and ray 2 are symmetric to the left interface of lens 1. Point A produced by reverse prolong line of ray 3 and ray 3' denotes the corresponding point of B to the right interface of lens 1, it is easy to find out that A and B are symmetric to the right interface of lens 1 and AC=BC where point C is the intersection point of x axis and right interface of lens 1. Therefore, rays 3 and 3' can be treated to radiate from point A, the problem of image's position for light passing through the optical system consisted of two different thickness slabs is equal to a simpler one, we just need to consider about this problem: where the image position is when light radiates from point A and then passes through lens 2. Applying equation (1a) to point A and lens 2, we can get following equation which assures the existence of image behind lens 2:

$$AC+D2<L2 \qquad (2)$$

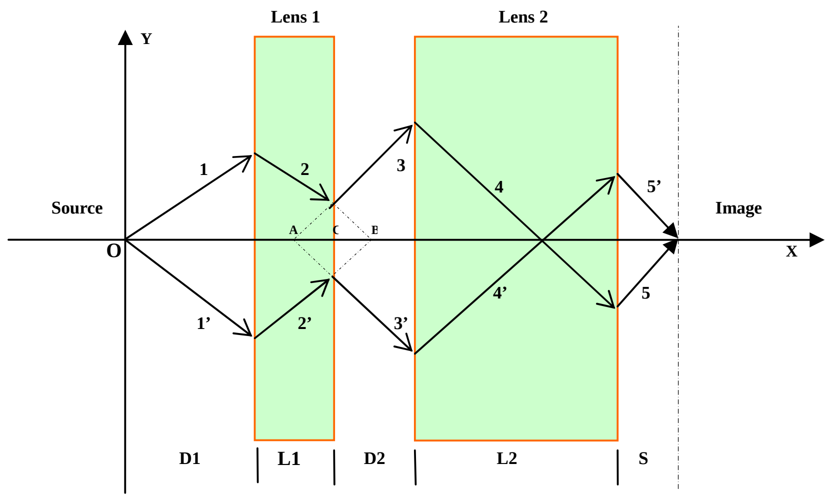
since the symmetry of source O and point B, we get D1=L1+CB, so AC=BC=D1-L1. Then the equation (2) becomes to be D1-L1+D2<L2, also this equation can be rewritten as:

$$D1+D2<L1+L2 \qquad (3a)$$

Similarly, if D1+D2>L1+L2, the image can not be formed and if D1+D2=L1+L2, the image will just on the right interface of lens 2. Here, if we define D=D1+D2 and L=L1+L2, the precondition of imaging system with two lenses can be written as D<L too, which is equal to equation (1a). Also, basis on the equation (1b), we can use another equation to get the information about the image's position which is AC+D2+S=L2. By replacing the AC with AC=D1-L1, we find that:

$$D1+D2+S=L1+L2 \tag{3b}$$

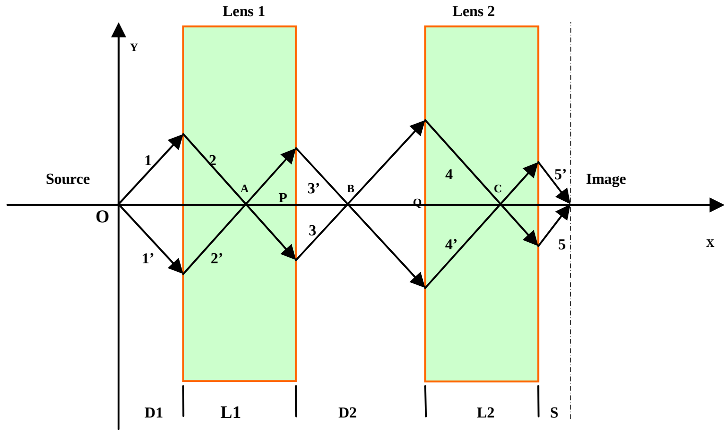
Here if we define D=D1+D2 and L=L1+L2 and equation (3b) can be written as D+S=L, which is equal to equation (1b).



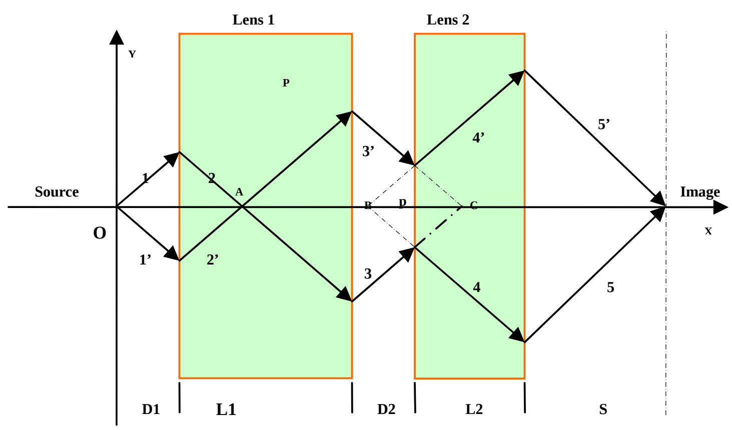
D1>L1

Fig. 2. Schematic diagram of the imaging system with two-lens ($n_{eff} = -1$) under the conditions D1>L1 and D1+D2<L1+L2.

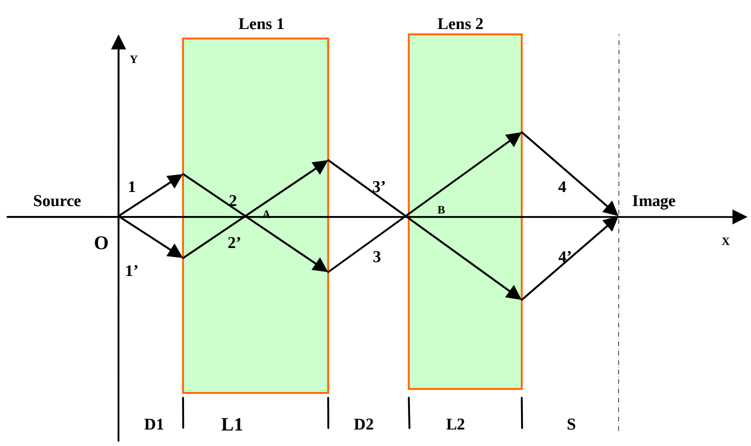
Secondly, we consider the system with two lenses under the condition D1<L1. In this case, the situation becomes more complex and the relationship of L1 and D2 need to be discussed in detail. Three cases are analyzed here: i) D1+D2>L1 ii) D1+D2<L1 iii) D1+D2=L1. Fig. 3a shows how light travels under the conditions D1<L1 and D1+D2>L1. From the map we can see that since D1<L1, a internal focus (point A) exists in lens 1 and a external image (point B) also appears in the space between the two lenses because of D1+D2>L1. Using equation (1b), we can obtain PB=L1-D1. The light from point source firstly focuses at point B by passing through lens 1 and then continues its travel to lens 2. The precondition of image formed by lens 2 from focus point B becomes a one-lens imaging system problem again. By using equation (1a), we find if BO<L2 the image can be formed. From Fig. 3a, we can see that BO=D2-PB=D2-L1+D1, so the condition of forming image becomes D2+D1<L1+L2, which is just the equation (3a). For finding the position of the image, we have equation S+D1+D2=L1+L2, which can be obtained from Fig.3a and apparently no different to equation (3b). Fig.3b shows the imaging system in the case of D1+D2<L1, similar analyse is used here. Since D1<L1 and D1+D2<L1, an internal focus (A) exists in lens 1 and no focus would be formed in the space between the two lenses. We use letter C to present the inverse prolong lines of ray 3 and ray 3', and the inverse prolong lines of ray 4 and ray 4' is denoted by B.. D1+D2<L2+L1 is right since the precondition D1+D2<L1 exists. Also, because of the symmetric of point B and the image point to the right interface of lens 2, this is shown in Fig.3b, the image exists and we have BP+L2=S. Point C will be the image formed by lens 1 to the point source if no lens 2 exist and C and B is symmetric to the right interface of lens 2, so BP=PC is easily obtained. By using equation (1b) we will get PC+D2+D1=L1. So we have L1-D1-D2+L2=S, it can be rewritten by D1+D2+S=L1+L2 which is just the equation (3b) exactly. For the case of D1<L1 and D1+D2=L1, the same conclusion can be obtained and we don't discuss this situation here.



(a) D1>L1, D1+D2>L1



(b) D1>L1, D1+D2<L1



(C) D1>L1, D1+D2=L1

Fig. 3. (a),(b),(c) The schematic diagram of the imaging system with two-lens ($n_{eff} = -1$) under the conditions D1+D2>L1, D1+D2<L1 and D1+D2=L1respectively. Here, D1>L1and D1+D2<L1+L2 are treated as preconditions for all three cases.

At last, a special situation under the condition of D1=L1 can be treated as the middle state between the cases of D1>L1 and that of D1<L1. In Fig. 4, we find that the first focus (A) in lens 1 is just on the right interface of lens 1 because the condition D1=L1. For light rays traveling through lens2 after passing lens1 can be treated to radiate from the point A. After applying the equations (1a) and (1b), D2 needs to satisfied D2<L2 in order to get a point image and equation S+D2=L2 also can be obtained. So the condition of forming focus on the right space of lens 2 becomes D1+D2<L1+L2, and S meets the equation S+D1+D2=L1+L2.

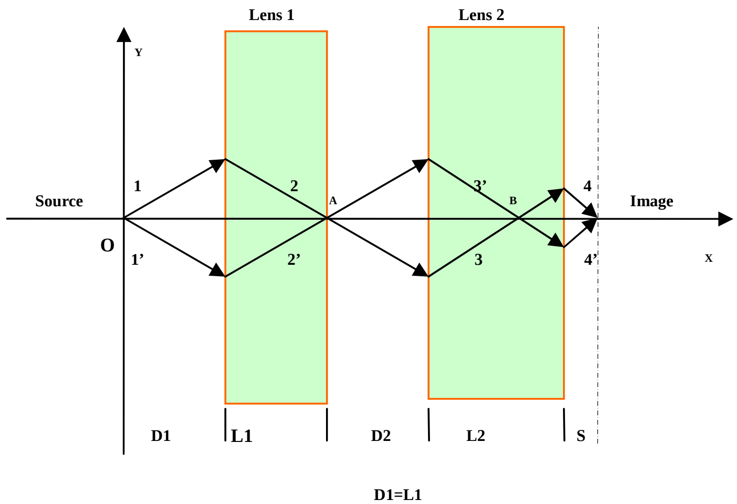


Fig. 4. Schematic diagram of the imaging system with two-lens ($n_{eff} = -1$) under the conditions D1=L1 and D1+D2<L1+L2.

After analyzing the two-lens imaging system, we find obviously that when we use D=D1+D2 to present the air length along x axis and L=L1+L2 to present the total thickness of two lenses, we can get the same preconditions which are obtained in one-lens imaging system. When a point source is placed in air before a system consists of one or two lenses with effective negative refractive -1, the precondition is the same that can be described by D<L and the position of image behind the last lens (the most right one) can be forecasted by S=L-D. This conclusion means if we don't consider the intensity of image and just wants to know the information about the image's position a two-lens system is equal to a one-lens system. Also this conclusion can be extended to that: a system with N-lenses can be equal to an one-lens system after several effective steps, where N denotes the natural number.

3. Numerical experiment

In order to confirm our conclusions, we use the FDTD method to simulate the wave propagation through the two-dimensional photonic crystal slab. The two-dimensional photonic crystal slab (infinite in y-direction and finite in x-direction) we considered here is a triangular lattice of air holes in a dielectric material $\varepsilon = 12.96$, with a lattice constant a and a hole radius $r = 0.4a$. The lattice extends over the x-y plane and is infinite in the y dimension. To assure all-angle negative refraction, here we choose the frequency $\omega = 0.305(a/\lambda)$, at which the effective refractive index of the photonic crystal is $n_{eff} = -1$ .Only transverse magnetic (TM) modes are considered here. It should be noted that the optical property of a photonic crystal with effective negative refractive index $n_{eff} = -1$ is different from that of a negative index material with refractive index $n = -1$, light can go through an air-NIM interface without reflection [2].

Fig. 5a gives the electric field distribution for D=6h, L=8h, where $h = \sqrt{3}/2a$. According to our conclusions, we can find that the image exists just on the right of the PhC slab with S=2h and this result agrees with our discussion about one-slab system and equation (1b). From Fig. 5b to Fig. 5f, all five maps represent the electric field distribution in a two-slab system where the total thickness of slab is fixed at L=L1+L2=8h and the total air length along x axis is also a unchanged value D=D1+D2=6h. This choice of parameter are helpful to compare the one-lens system and two-lens system. Since more air-PhC interfaces lead to more refractions, the intensity of image is relatively weaker than that in one-slab system. Several useful methods can be used to enhance the image intensity [16,17], but we have not optimized the termination to maximize the lens resolution since our aim is to obtain the information of the image position. In Fig. 5b, we assume L1=L2=4h while D1=5h and D2=1h, corresponding the case of D1>L1. In this map, we can clearly find the position of image is no different from that of Fig. 5a if we do not consider the intensity of the image. Similarly, Fig. 5c shows the image system in the case of D1=L1 where we have D1=L1=L2=4h and D2=2h. In Fig. 5d, the parameters become D1=2h and L1=L2=D2=4h, which represent the case of D1<L1 but D1+D2>L1. In Fig. 5e, the parameters become D1=4h, D2=2h, L1=6h and L2=2h, which represent the case of D1+D2=L1. In Fig. 5f, the parameters become D1=4h, D2=2h, L1=7h and L2=h, which represents the case of D1+D2<L1. In all of six pictures, the D=6h and L=8h do not change with position of the point source, so, all images in those six pictures appear at the same place, which confirms that a two-lens system can be equal to a one-lens system.

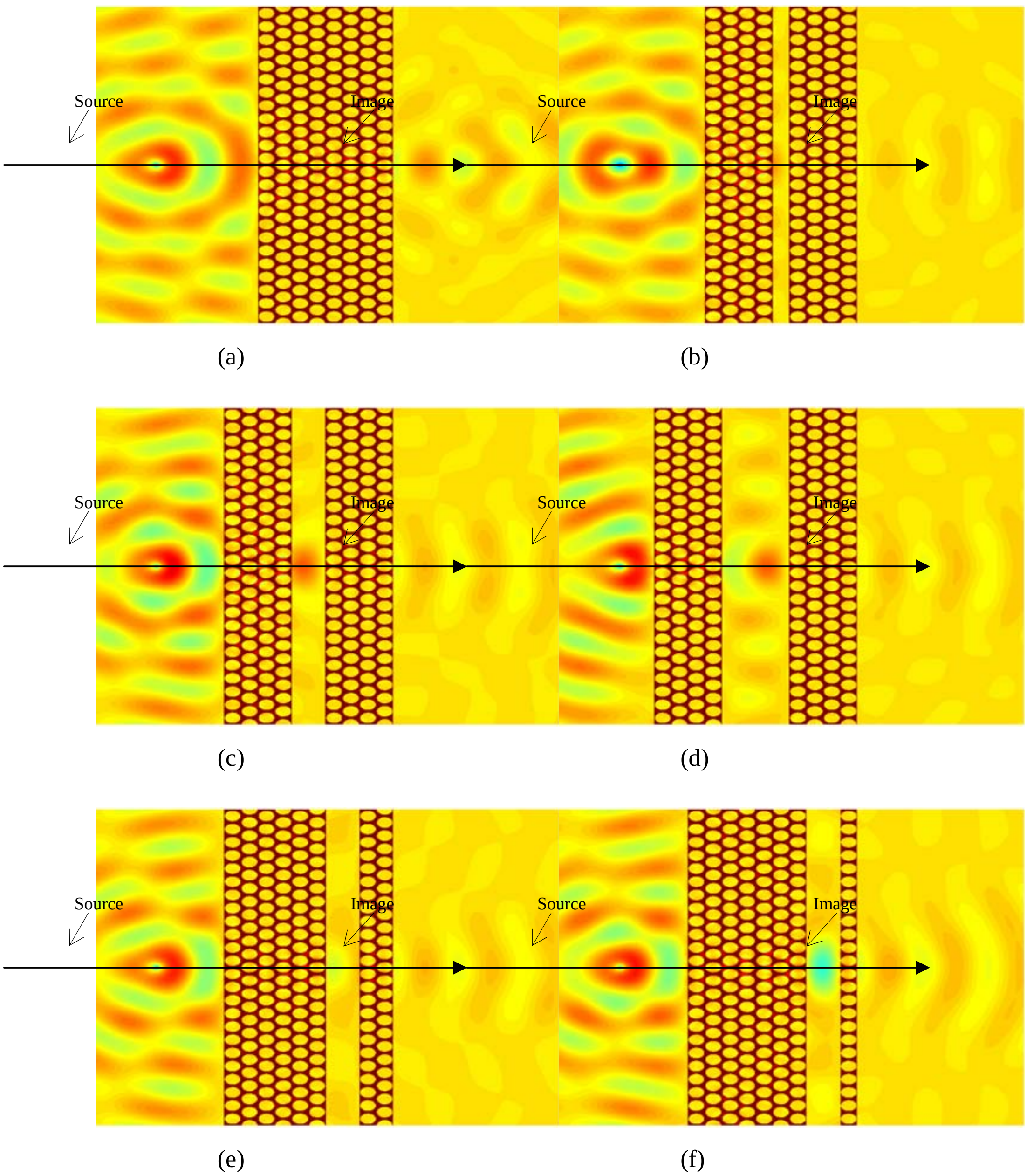


Fig. 5. The snapshots of the electric field of a point source and its image across a 2D-PhC slab lens (a)and lenses(b-f).In all six maps from (a) to (f), D=D1+D2=6h and L=L1+L2=8h are fixed. (a) the case of one-lens imaging system ; (b) the case of D1>L1 in two-lens system; (c) the case of D1=L1 in two-lens system; (d) the case of D1<L1 and D1+D2>L1 in two-lens system; (e) the case of D1+D2=L1 in two-lens system; (f) the case of D1+D2<L1 in two-lens system. in all cases shown here , the source position is fixed and the image position is unchanged as expected.

4. Conclusion

In this paper, we have discussed the optical imaging system with one-lens firstly. When light travels through a negative-index medium with $n_{eff} = -1$ from the air ($n = 1$), the necessary condition of image' existence is given by equation (1a) and the position of image is also decided by equation (1b). Based on one-lens system, we analyzed the imaging system with two-lens systematically. There we use D=D1+D2 and L=L1+L2 to present the total length of air along x axis and total thickness of the slabs, it can be found that in two-lens system comparing to one-lens system the necessary precondition of image' existence is just the same to that of one-lens system. In other words, the two-lens system can be treated equally to a one-lens system. This conclusion can be extended to a N-lens system, which means a N-lens system can be treated equally to an one-lens system, too. This equivalency between multiply-lens and one-lens is meaningful and helpful in the combination of optical imaging system where we can consider a N-lens system as a one-lens system which makes problem to be an easier one. Also, we can use several relatively thin lenses to instead a thick one without changing the position of optic image. Finally, a FDTD method is applied to simulate the wave propagating through 2D-PhC slab and slabs group and the simulant maps are in very good agreement with our conclusions.

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