Anomalous test particle transport in turbulent MHD fields

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Anisotropic stochastic magnetic fields lead to significant changes in the test particle transport as compared to isotropic cases. The analysis of the Langevin equations for the test particles, when considering both the magnetic anisotropy and the collisions, lead to anisotropic transport ranging from subdiffusion to diffusion. In absence of collisions, the transport is diffusive in all directions but with different diffusion coefficients. In the case of MHD produced fields, the transport in the plane perpendicular to the main magnetic field is strongly reduced, and appears to be almost diffusive while the parallel transport is still in the ballistic regime.

1. Introduction

Fusion plasmas are never totally quiescent. Their dynamics rely on turbulence due to electromagnetic micro-instabilities and on stochastic processes such as the magnetic perturbations of the confining magnetic surfaces. The edge localized mode (ELM), of the former instance, leads to high heat and particle fluxes to the plasma edge inducing strong erosion of plasma facing components (e.g. the divertor) [1]. It is therefore an absolute necessity to protect the plasma facing components of the fusion reactors (like ITER under construction in France) either by ergodization of the magnetic field, which is a method tested successfully in Tore Supra (Cadarache, France) and in Textor (Julich, Germany), or by seeding impurities as was done in Textor or in JET (Culham, UK). The latter method may lead to an enhanced confinement mode, the radiative improved mode (RI-mode), which is very beneficial to the bulk plasma behaviour [2]. However, this mode of operation often terminates by an impurity accumulation to the plasma centre. This effect is sometimes slowed down when the perturbed confining magnetic field periodically reconnects leading to plasma mixing and plasma temperature sawtoothing [3]. Cyclotron heating of the seeded impurity sometimes also leads to a reduction of the impurity accumulation [4]. These methods modify the impurity transport in a way that still needs to be understood.

The impurity transport is often analysed using the drift kinetic equation (DKE) that provides us with the "neoclassical" transport coefficients [5] which take into account the in-homogeneities and curvatures of the strong confining magnetic field. These coefficients are bigger than their "classical" counterpart obtained for straight magnetic fields. Comparisons with experiments show that this neoclassical theory needs to be improved to account for the electrostatic turbulence and the magnetic stochasticity.

We shall be concerned by the test particle transport in anisotropic stochastic magnetic fields. The field may be given analytically or be produced for example as in the case considered below by a three dimensional spectral MHD solver. Predictions on the test particle transport will be obtained from theoretical considerations as well as from numerical simulations. Our goal is to analyse the influence of the anisotropy of the stochastic magnetic field on the test particle transport.

2. Test particle transport derived from Langevin equations

We consider a magnetic field
$$\vec{B} = \vec{B}_0 \left(\vec{e}_z + \beta \sum_j b_j(x, y, z, t) \vec{e}_j \right)$$
with $j = (x, y, z)$, $\beta << 1$ and satisfying the constraint $\nabla \cdot \vec{b} = 0$. The random fluctuating components b_j are given Gaussian stochastic processes characterized by two different spatial scales (parallel and perpendicular correlation length) λ_z, $\lambda_x = \lambda_y$.

Obviously,

$$B = B_0 \sqrt{(1 + \beta b_z)^2 + \beta^2 b_x^2 + \beta^2 b_y^2} = B_0(1 + \beta b_z) + O(\beta^2)$$

thus

$$\vec{b} = \vec{B}/B \approx \frac{(1+\beta b_z)\vec{e}_z + \beta b_x \vec{e}_x + \beta b_y \vec{e}_y}{(1+\beta b_z)} =$$
$$= \vec{e}_z + \beta b_x \vec{e}_x + \beta b_y \vec{e}_y + O(\beta^2)$$

A test particle is singled out from the plasma. Its motion in the stochastic magnetic field, approximated by its guiding centre, is interrupted by the collisions with the particles of the background plasma. This effect is modelled by a random variation of the velocity field.

The equations of motion for a test particle in the guiding centre approximation are the following Langevin equations: $d\vec{x}/dt = v_z(t)\vec{b}(x,y,z,t)$ with $\vec{x} \equiv (x, y)$ and $dz/dt = v_z(t)$ where, due to the collisions, the velocity $v_z(t)$ is an independent zero-mean Gaussian stationary stochastic process for which we assumed the following autocorrelation: $\langle v_z(t_1)v_z(t_2)\rangle_{v_z} = 2^{-1}v_T^2 \exp(-\nu|t_1 - t_2|)$ where v_T is the thermal velocity and ν is the collision frequency. In absence of collisions, v_z is constant (extreme sub-diffusion) and normal diffusion is observed in the plane perpendicular to the main magnetic field component. A formal solution of the Langevin equations is

$$\vec{x}(t) = \vec{x}_0 + \beta \int_0^t dt' v_z(t')\,\vec{b}(x(t'), y(t'), z(t'), t'),$$
$$z(t) = z_0 + \int_0^t dt' v_z(t')$$

The particle dynamics in the perpendicular direction depends on the product of two fluctuating quantities, the magnetic field and the stochastic parallel velocity. This will lead, as shown below, to a non-diffusive transport. A substitution of the expression of the magnetic field into the Langevin equations gives

$$\delta\vec{x}(t) = \beta \int_0^t dt' v_z(t')\vec{b}_\perp\left(\vec{x}_0 + O(\beta), z_0 + \int_0^{t'} dt'' v_z(t''), t'\right).$$

For example, the mean square displacement in the Ox direction then becomes

$$\langle \delta x^2(t)\rangle =$$
$$= \beta^2 \int_0^t dt_1 \int_0^t dt_2 \langle v_z(t_1)b_x(x_0, z(t_1))v_z(t_2)b_x(x_0, z(t_2))\rangle$$

With the Fourier transform of $\vec{b}(z,t)$ expressed in the form $\vec{b}(z,t) = \int dk_z \exp(ik_z z)\vec{b}(k_z, t)$, the mean square displacement in the Ox direction becomes

$$\langle \delta x^2(t)\rangle = 2\beta^2 \int_0^t d\tau \int dk_z (t-\tau)B(k_z)Z(k_z, \tau)$$

where

$$Z(k; \tau_1, \tau_2) = \left\langle e^{ik_z \int_t^{t+\tau} v_z(\theta)d\theta} v_z(t+\tau)v_z(t)\right\rangle_V \text{ and}$$

$$B \approx \exp\left(-\frac{1}{2}\lambda_z^2 k_z^2\right) .$$ Both depend on the prescribed correlations of the magnetic field. The function $Z(k; \tau_1, \tau_2)$ can be evaluated exactly [6]. The final result is

$$\langle \delta x^2(t)\rangle = \frac{1}{[1+\gamma\psi(\nu t)]^{1/2}} e^{-|\nu t|} - \frac{\gamma}{2} \frac{\phi^2(\nu t)}{[1+\gamma\psi(\nu t)]}$$

where $\phi(\chi) = 1 - e^{-|\chi|}$; $\psi(\chi) = \chi - \phi(\chi)$ and γ is interpreted as the square of the ratio of the mean free path to the parallel correlation length of the magnetic field fluctuations. The motion in the perpendicular direction is characterized by a time dependent diffusion coefficient

$$D_\perp(t) = \frac{1}{2}\partial_t \langle \delta x^2(t)\rangle = \frac{\beta}{\nu} \frac{\phi(\nu t)}{[1+\gamma\psi(\nu t)]^{1/2}} .$$

Thus, while the dynamics along z, which is the direction of the source of magnetic anisotropy, is diffusive the transverse collisional motion is subdiffusive. This is an example of "strange" anomalous behaviour [6]. In absence of collisions, the mean square displacement reduces to $\langle \delta x^2(t)\rangle \propto t^2$ which is the ballistic short time behaviour, instead of the expected diffusive displacement in the stochastic magnetic field.

3. The decorrelation trajectory method (DCT)

This method, here adapted from [7] is expected to give the correct short time behaviour as well as to include effects due to correlated noises. To introduce the method, we consider the dominant order of the guiding centre motion and neglect the influence of the collisions. The problem then reduces to the equivalent problem of the magnetic field line transport. We consider the following magnetic field:

$$\vec{B}(x, y, z) = B_0 \vec{e}_z + \beta b_x(x, y, z)\vec{e}_x + \beta b_y(x, y, z)\vec{e}_y$$

where b_x and b_y have different correlation length in Ox and Oy directions. This introduces the ratio $\Lambda = \lambda_x / \lambda_y$ in the stochastic dimensionless magnetic field line equations

$$\frac{dx}{dz} = K_m b_x(x, y, z) \equiv w_x[\vec{x}(z); z],$$
$$\frac{dy}{dz} = \Lambda K_m b_y(x, y, z) \equiv w_y[\vec{x}(z); z]$$

The Lagrangian correlation is the fundamental quantity needed to determine the asymptotic diffusion

coefficient from the fluctuating 'velocities', i.e. $w_i[\vec{x}(z);z]$. The Lagrangian velocity autocorrelation is

$$L_{ij}(z) = \left\langle w_i[\vec{x}(0),0] \, w_j[\vec{x}(z),z] \right\rangle =$$
$$= a_{ij}(\Lambda) K_m^2 \left\langle b_i[\vec{x}(0),0] \, b_j[\vec{x}(z),z] \right\rangle$$

where $\langle ... \rangle$ is the ensemble average over the realizations of the fluctuating magnetic field and the tensor $a_{ij}(\Lambda)$ has the components $a_{xx} = 1$; $a_{xy} = a_{yx} = \Lambda$; $a_{yy} = \Lambda^2$. Assuming that the stochastic field is 'stationary' then $D_{ij}(z) = \int_0^z d\chi L_{ij}(\chi)$. The celebrated Corrsin approximation leads to both the quasilinear and the Bohm limits:

$$L_{ij}(z) = a_{ij}(\Lambda) K_m^2 \int d\vec{x} \left\langle b_i[\vec{x}(0);0] \, b_j[\vec{x}(z);z] \, \delta(\vec{x} - \vec{x}(z)) \right\rangle$$
$$\approx a_{ij}(\Lambda) K_m^2 \int d\vec{x} \left\langle b_i[\vec{x}(0);0] \, b_j[\vec{x}(z);z] \right\rangle \left\langle \delta(\vec{x} - \vec{x}(z)) \right\rangle$$

This introduces the probability distribution function $\left\langle \delta(\vec{x} - \vec{x}(z)) \right\rangle$. To set up the DCT method it is necessary to define a sub-ensemble S of realizations of the stochastic components of the magnetic field as follows: At initial position $x = 0$ and initial "time" $z = 0$ the quantities $\psi(\vec{0};0) = \psi^0$, $b_i(\vec{0};0) = b_i^0$ for $i = (x,y)$ are given deterministic quantities. The global Lagrangian correlation is then represented as a sum over all sub-ensembles of the correlations $L_{ij}^S(z)$:

$$L_{ij}(z) =$$
$$= a_{ij}(\Lambda) K_m^2 \int d\psi^0 \int db_x^0 \int db_y^0 P(\vec{b}^0, \psi^0) b_i^0 b_j^0 \left[\vec{x}^S(z); z \right]$$

where the probability density to get $(\vec{b}^0, \psi^0)$ at the initial position $x = 0$ and initial 'time' $z = 0$ is

$$P(\vec{b}^0, \psi^0) = P(b_x^0) P(b_y^0) P(\psi^0) =$$
$$= (2\pi)^{-3/2} \Lambda^{-1} \exp\left[-\frac{1}{2}\left((\psi^0)^2 + (\Lambda^{-1} b_x^0)^2 + (\Lambda^{-1} b_y^0)^2 \right) \right].$$

Since the initial fluctuating fields $b_i(\vec{0};0) = b_i^0$ in the sub-ensemble S are deterministic we get $\left\langle b_i(\vec{0},0) b_j[\vec{x}(z);z] \right\rangle^S = b_i^0 \left\langle b_j[\vec{x}(z);z] \right\rangle^S$. To obtain explicit results, we first calculate the average Eulerian fields b_j in the sub-ensemble S i.e. $b_j^S(\vec{x}(z);z) = \left\langle b_j(\vec{x}(z);z) \right\rangle^S$. Then, a "deterministic trajectory" in each sub-ensemble satisfying the initial conditions $\vec{x}^S(0) = 0$ is obtained by solving the ordinary differential equations

$$\frac{dx^S}{dz} = K_m b_x^S; \quad \frac{dy^S}{dz} = \Lambda K_m b_y^S.$$
The Lagrangian correlation is calculated using the deterministic trajectories $\vec{x}^S(z)$.

With $\Lambda \neq 1$ in Figure 1 the diffusion in x differs from the diffusion in y showing in this framework of the DCT the impact of an anisotropic magnetic spectrum [8].
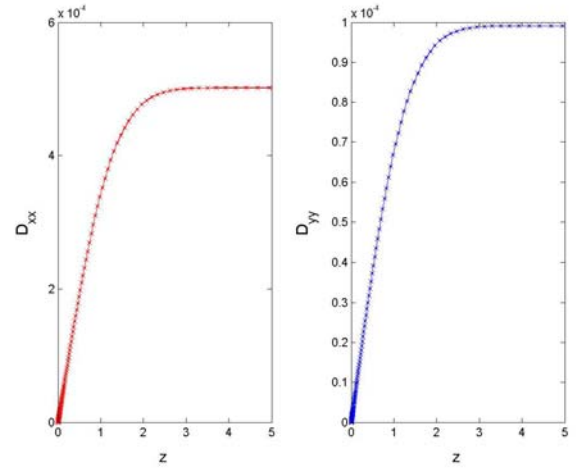

Fig. 1. The radial and poloidal diffusion coefficients for $K_m = 0.1$ and $\Lambda = 0.2$.

4. Test particle transport in MHD fields

For a numeric simulation to be fully resolved (3-D spectral simulations) $k_j^{max} L^K \geq 1$ where L^K is the Kolmogorov length, the smallest turbulent scale in the Kolmogorov theory of turbulence defined as $L^K = (\upsilon^3 \varepsilon)^{1/4}$ with υ the kinematic viscosity and ε the total dissipation of energy. The time step is defined as $dt < \left(k^{max} |U^{max}| \right)^{-1}$. Classical fluids introduce the Reynolds number Re by comparing the nonlinear term to the viscous diffusive term υ. MHD turbulence introduces the magnetic Reynolds number Re_m by comparing the nonlinear term to the magnetic diffusivity η. The ratio of the two dimensionless numbers gives the magnetic Prandtl number $Pr = Re_m / Re = \upsilon / \eta$. For the numerical work we consider $Pr = 1$. The anisotropy in the MHD produced spectrum can be measured in terms of the Shebalin angle θ defined for a quantity Q (a vector field or a scalar) as $\tan^2 \theta(Q) = \sum k_\perp^2 |Q(k,t)|^2 / \sum k_{//}^2 |Q(k,t)|^2$ where if $k_{//} = k_z$ is a privileged direction then $k_\perp^2 = k_x^2 + k_y^2$ and we take the summation over all values of k. For isotropic spectrum $\tan^2 \theta(Q) = 2$ or $\theta = 54.74°$. For a quasi-2D spectrum, with the energy in the modes perpendicular to the source of anisotropy, $\theta = 90°$, while for a slab type of

spectrum, with the energy in wave-vectors parallel to the anisotropy source, the angle tend towards $\theta = 0°$. Starting from isotropic conditions for the velocity and magnetic fields $\theta(\vec{u}, \vec{b}) = 55.4°$, values $\theta(\vec{u}, \vec{b}) = 63°$ for an imposed strong magnetic field in the z-direction $B_0 = 1$ and $\theta(\vec{u}, \vec{b}) = 68°$ for $B_0 = 2$ are easily achieved. From Ohm's law in the MHD approximation we find that the electric field is given by $\vec{e} = -\vec{u} \times \vec{b} + \eta \vec{j}$ with $\vec{j} = \nabla \times \vec{b}$ where $\vec{j}$ is the electric current, $\vec{e}$ the electric field expressed in appropriate units ($\vec{b}$ is expressed in velocity units, the electric field $\vec{e}$ has the dimension of velocity squared). A test particle has a specific charge α and is driven by the Lorentz force: $d\vec{x}/dt = \vec{v}$ and $d\vec{v}/dt = \alpha(\vec{e} + \vec{v} \times \vec{b})$ where the parameter α couples the time evolution of the field and of the test particle. For a value 1 (used here) the time scale of the test particle and the electromagnetic field is the same. The time unit used in the Figure 2 shown below is measured in gyro time units $t_c = 2\pi\left(\alpha\langle b^2\rangle^{1/2}\right)^{-1}$.

From here on, the procedure is elementary, once the MHD fields are properly generated and statistical equilibrium is achieved, we solve, using a Runge-Kutta method, the equations of motion of 10.000 independent test particles. This large number is sufficient to obtain good statistics of the test particle transport. One of the results is shown in Figure 2. The anisotropy induced by an imposed strong magnetic field along the axis Oz leads, as expected, to a reduced transport in the perpendicular plane.
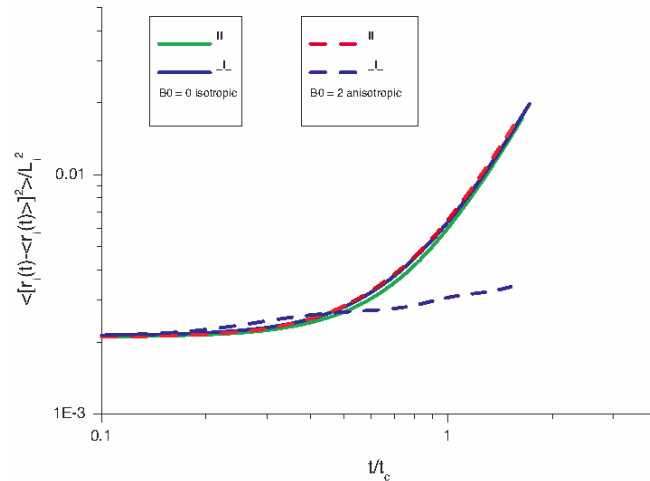

Fig. 2. Mean square displacement in the isotropic case ($B_0 = 0$) and in the anisotropic case ($B_0 = 2$) showing a strong reduction of the transport in the plane perpendicular to the anisotropy.

5. Conclusions

We have shown, using three different approaches that the magnetic anisotropy may lead to significant change in the test particle transport. The analytical work on the Langevin equations, considering both the magnetic anisotropy and collisions, lead to subdiffusion and diffusion respectively. In absence of collisions, as the DCT method shows, the transport is diffusive in all directions but with different diffusion coefficients. In the case of MHD produced fields, the transport in the plane perpendicular to the main magnetic field is strongly reduced, and appears to be almost diffusive while the parallel transport is still in the ballistic phase.

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