JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS, Vol. 10, No. 10, October 2008, p. 2528 - 2531

Application of higher-order variational method to an optical rib waveguide

V. A. POPESCU*
Department of Physics 1, University "Politehnica "of Bucharest, Splaiul Independentei 313, 060042 Bucharest, Romania

We have obtained closer results to the exact value of the normalized propagation constant for the fundamental mode of the optical rib waveguide by using the second-order variational method. The lateral and depth confinements of the light in the guiding layer can be optimized by the variation of the height and the refractive index of the rib.

(Received August 26, 2007; accepted August 14, 2008)

Keywords: Waveguides, Quantum mechanics, Optical rib waveguide, Refractive index

1. Introduction

The higher-order variational method has been used to determine the propagation constant in a planar optical graded-index waveguide [1-2].

In this paper we use the second-order variational method to improve the accuracy of the propagation constants for an optical rib waveguide where an exact analytical solution is not possible.

2. Application to an optical rib waveguide

The scalar wave equation

$$\frac{\partial^2\psi(x,y)}{\partial x^2}+\frac{\partial^2\psi(x,y)}{\partial x^2}+k^2n^2(x,y)\psi(x,y)=\beta^2\psi(x,y) \quad , \quad (1)$$

for a rib waveguide which is shown in Fig. 1, is solved with a function [3-4]

$$\psi(x,y)=X(x)Y(y) \quad (2)$$

to determine the propagation constant β for the fundamental mode TE_0 from the relation

$$\beta^2=\frac{\int\left\{-(Y^2X'^2+X^2Y'^2)+k^2n^2(x,y)X^2Y^2\right\}dxdy}{\int X^2Y^2dxdy}, \quad (3)$$

where X and Y are functions of the same even parity,

$$X_I(x)=\begin{cases}X_1(x), & \text{for } x<-b,\\ X_2(x), & \text{for } -b<x<b,\\ X_3(x), & \text{for } x>b,\end{cases}$$

$$X_{II}(x)=\begin{cases}X_4(x), & \textit{for } x<-b,\\ X_5(x), & \textit{for } -b<x<b,\\ X_6(x), & \textit{for } x>b,\end{cases} \quad (4)$$

$$Y_I(y)=\begin{cases}Y_1(y), & \text{for } y<0,\\ Y_2(y), & \text{for } 0<y<d,\\ Y_3(y), & \text{for } y>d,\end{cases}$$

$$Y_{II}(y)=\begin{cases}Y_4(y), & \textit{for } y<0,\\ Y_5(y), & \textit{for } 0<y<d,\\ Y_6(y), & \textit{for } y>d,\end{cases} \quad (5)$$

$$X_1=\cos(k_3b)\exp(\gamma_3(x+b)),\ X_2=\cos(k_3x),\ X_3=\cos(k_3b)\exp(\gamma_3(-x+b)), \quad (6)$$

$$X_4=\cos(k_{33}b)\exp(\gamma_{33}(x+b)),\ X_5=\cos(k_{33}x),\ X_6=\cos(k_{33}b)\exp(\gamma_{33}(-x+b)), \quad (7)$$

$$Y_1=\exp(\gamma_1y),\ Y_2=\cos(k_1y)+A\sin(k_1y),\ Y_3=B\exp(-\gamma_2(y-d)),\ A=\gamma_1/k_1, \quad (8)$$

$$Y_4=\exp(\gamma_{11}y),\ Y_5=\cos(k_{11}y)+A_{11}\sin(k_{11}y),\ Y_6=B_{11}\exp(-\gamma_{22}(y-d)), \quad (9)$$

$$\gamma_3=k_3\tan(k_3b),\gamma_{33}=k_{33}\tan(k_{33}b), \quad (10)$$

$$B=\cos(k_1d)+A\sin(k_1d),B_{11}=\cos(k_{11}d)+A_{11}\sin(k_{11}d),\ A_{11}=\gamma_{11}/k_{11}, \quad (11)$$

$$\gamma_2=\left(k_1\left[\sin(k_1d)-A\cos(k_1d)\right]\right)/B,\ \gamma_{22}=\left(k_{11}\left[\sin(k_{11}d)-A_{11}\cos(k_{11}d)\right]\right)/B_{11}. \quad (12)$$

In the second-order variational method, the value of the propagation constant β for TE_0 mode is calculated from the equation (Eq. 27 from Ref. 6),

$$\beta^2 = \frac{B_1 + \sqrt{B_1^2 - 4A_1C_1}}{2A_1}, \tag{13}$$

$$A_1 = I_{11}I_{22} - I_{12}^2,$$
$$B_1 = I_{11}J_{22} + I_{22}J_{11} - 2I_{12}J_{12}, C_1 = J_{11}J_{22} - J_{12}^2 \tag{14}$$

where

$$I_{11} = (\int_{-\infty}^{-b} X_1^2dx + \int_{-b}^{b} X_2^2dx + \int_{b}^{\infty} X_3^2dx)(\int_{-\infty}^{0} Y_1^2dy + \int_{0}^{d} Y_2^2dy + \int_{d}^{\infty} Y_3^2dy), \tag{15}$$

$$I_{22} = (\int_{-\infty}^{-b} X_4^2dx + \int_{-b}^{b} X_5^2dx + \int_{b}^{\infty} X_6^2dx)(\int_{-\infty}^{0} Y_4^2dy + \int_{0}^{d} Y_5^2dy + \int_{d}^{\infty} Y_6^2dy), \tag{16}$$

$$I_{12} = (\int_{-\infty}^{-b} X_1X_4dx + \int_{-b}^{b} X_2X_5dx + \int_{b}^{\infty} X_3X_6dx)(\int_{-\infty}^{0} Y_1Y_4dy + \int_{0}^{d} Y_2Y_5dy + \int_{d}^{\infty} Y_3Y_6dy), \tag{17}$$

$$\begin{aligned} J_{11} = &-(\int_{-\infty}^{-b} X_1^2dx + \int_{-b}^{b} X_2^2dx + \int_{b}^{\infty} X_3^2dx)(\int_{-\infty}^{0} Y_1'^2dy + \int_{0}^{d} Y_2'^2dy + \int_{d}^{\infty} Y_3'^2dy) \\ &-(\int_{-\infty}^{-b} X_1'^2dx + \int_{-b}^{b} X_2'^2dx + \int_{b}^{\infty} X_3'^2dx)(\int_{-\infty}^{0} Y_1^2dy + \int_{0}^{d} Y_2^2dy + \int_{d}^{\infty} Y_3^2dy) \\ &+(\int_{-\infty}^{-b} X_1^2dx + \int_{-b}^{b} X_2^2dx + \int_{b}^{\infty} X_3^2dx)(k^2n_3^2\int_{-\infty}^{0} Y_1^2dy + k^2n_1^2\int_{0}^{d} Y_2^2dy + k^2n_0^2\int_{d}^{\infty} Y_3^2dy) \\ &-(\int_{-b}^{b} X_2^2dx)(k^2n_0^2\int_{d}^{\infty} Y_3^2dy) + (\int_{-b}^{b} X_2^2dx)(k^2n_0^2\int_{d+h}^{\infty} Y_3^2dy + k^2n_2^2\int_{d}^{d+h} Y_3^2dy), \end{aligned} \tag{18}$$

$$\begin{aligned} J_{22} = &-(\int_{-\infty}^{-b} X_4^2dx + \int_{-b}^{b} X_5^2dx + \int_{b}^{\infty} X_6^2dx)(\int_{-\infty}^{0} Y_4'^2dy + \int_{0}^{d} Y_5'^2dy + \int_{d}^{\infty} Y_6'^2dy) \\ &-(\int_{-\infty}^{-b} X_4'^2dx + \int_{-b}^{b} X_5'^2dx + \int_{b}^{\infty} X_6'^2dx)(\int_{-\infty}^{0} Y_4^2dy + \int_{0}^{d} Y_5^2dy + \int_{d}^{\infty} Y_6^2dy) \\ &+(\int_{-\infty}^{-b} X_4^2dx + \int_{-b}^{b} X_5^2dx + \int_{b}^{\infty} X_6^2dx)(k^2n_3^2\int_{-\infty}^{0} Y_4^2dy + k^2n_1^2\int_{0}^{d} Y_5^2dy + k^2n_0^2\int_{d}^{\infty} Y_6^2dy) \\ &-(\int_{-b}^{b} X_5^2dx)(k^2n_0^2\int_{d}^{\infty} Y_6^2dy) + (\int_{-b}^{b} X_5^2dx)(k^2n_0^2\int_{d+h}^{\infty} Y_6^2dy + k^2n_2^2\int_{d}^{d+h} Y_6^2dy), \end{aligned} \tag{19}$$

$$\begin{aligned} J_{12} = &-(\int_{-\infty}^{-b} X_1X_4dx + \int_{-b}^{b} X_2X_5dx + \int_{b}^{\infty} X_3X_6dx)(\int_{-\infty}^{0} Y_1'Y_4'dy + \int_{0}^{d} Y_2'Y_5'dy + \int_{d}^{\infty} Y_3'Y_6'dy) \\ &-(\int_{-\infty}^{-b} X_1'X_4'dx + \int_{-b}^{b} X_2'X_5'dx + \int_{b}^{\infty} X_3'X_6'dx)(\int_{-\infty}^{0} Y_1Y_4dy + \int_{0}^{d} Y_2Y_5dy + \int_{d}^{\infty} Y_3Y_6dy) \\ &+(\int_{-\infty}^{-b} X_1X_4dx + \int_{-b}^{b} X_2X_5dx + \int_{b}^{\infty} X_3X_6dx)(k^2n_3^2\int_{-\infty}^{0} Y_1Y_4dy + k^2n_1^2\int_{0}^{d} Y_2Y_5dy + k^2n_0^2\int_{d}^{\infty} Y_3Y_6dy) \\ &-(\int_{-b}^{b} X_2X_5dx)(k^2n_0^2\int_{d}^{\infty} Y_3Y_6dy) + (\int_{-b}^{b} X_2X_5dx)(k^2n_0^2\int_{d+h}^{\infty} Y_3Y_6dy + k^2n_2^2\int_{d}^{d+h} Y_3Y_6dy), \end{aligned} \tag{20}$$

The integrals in equations (15-20) with the chosen trial functions are evaluated analytically to reduce the amount of numerical computation.

3. Numerical results and conclusions

We consider an optical rib waveguide (Fig.1) for which analytically solutions are not possible. Fig. 1 shows the contour plot of the field profile in an optical rib waveguide where a guiding layer with the width d and a higher refractive index n_1 is embedded between a substrate with the refractive index n_3, a rib with the width $2b$, the height h and the refractive index n_2 and the cover medium (air). The normalized propagation constants P_0 (for the fundamental mode TE_0) are calculated by using our first variational method (FVM), the scalar variational method (VP [3]), our second order variational method with (SVM) and without (SVM^r) restrictions (10), our finite element method (FEM) and the scalar finite element method (SFE) from the reference [5]. In the first-order variational method, the numerical result for the normalized propagation constant is the same in the case with the restriction (10) or without this restriction to six decimal digits (the stationary property is satisfied in both cases with the same optimized variational parameters k_3^o, k_1^o and γ_1^o). Fig. 2 shows the propagation constant β, for the fundamental mode TE_0 in an optical rib waveguide by using the second order variational method without (SVM^r) restrictions (10), as a function of the variational parameters k_3, k_{33}, k_1, k_{11}, γ_1, γ_{11}, γ_3 and γ_{33}. For $n_2 = n_1$ we obtain a larger number of modes and the maximum for the fundamental mode is localized in the middle of the rib and not in the guiding layer.

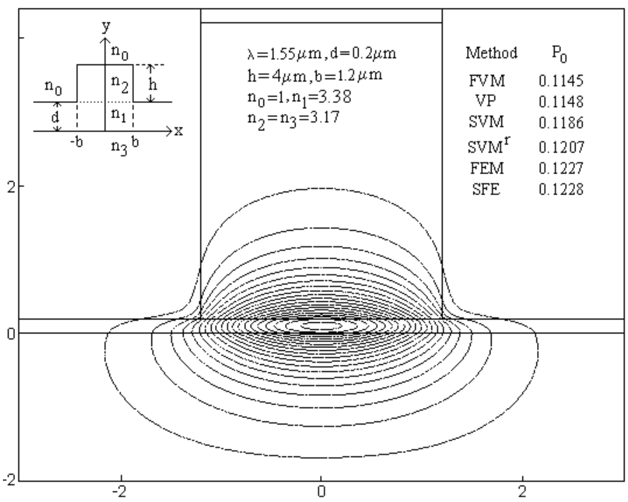

Fig. 1. The contour plot of the field profile in an optical rib waveguide where a guiding layer with the width d and a higher refractive index n_1 is embedded between a substrate with the refractive index n_3, a rib with the width 2b, the height h and the refractive index n_2 and the cover medium (air) with n_0 = 1. The normalized propagation constants P_0 (for the fundamental mode TE_0) are calculated by using our first variational method (FVM), the scalar variational method (VP[3]), our second order variational method with (SVM) and without (SVM^r) the restrictions (10), our finite element method (FEM) and the scalar finite element method (SFE) from the reference [5].

For another optical rib waveguide with the same parameters as in Fig. 1 but with a larger value of the rib width ($2b$ = 4μm) we obtain P_0 = 0.151309 in the first-order variational method with or without the restriction (10), P_0 = 0.152453 in the second-order variational method without the restriction (10) and the exact value P_0 = 0.153200 calculated with our finite element method (the stationary property is satisfied in the second-order variational method without the restriction (10)). The trial function for TE mode of a horizontal slab waveguide requires as the electric field E_x and $\partial E_x / \partial y$ to be continuous across the planes y = 0 and $y = d$. A vertical slab waveguide cannot be specified due to the geometrical shape of our rib waveguide (infinite length of the guiding and substrate layers). Thus we can to explain the better results in the absence of the relations (10) in the second order variational method but with two new variational parameters.

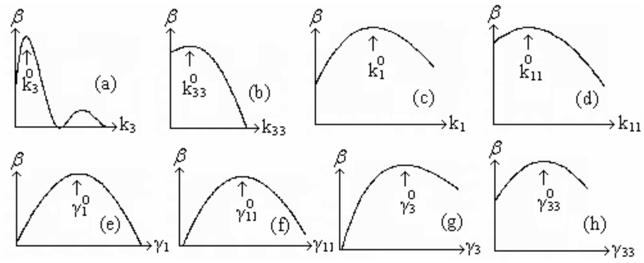

Fig. 2. The propagation constant β as a function of the variational parameters k_3, k_{33}, k_1, k_{11}, γ_1, γ_{11}, γ_3 and γ_{33}. k_3^o = 4.556959, k_{33}^o = 1.192147, k_1^o = 5.490454, k_{11}^o = 4.317781, γ_1^o = 1.986019, γ_{11}^o = 2.002621, γ_3^o = 1.864860 and γ_{33}^o = 8.546101 are the optimized variational parameters for the fundamental mode TE_0 in an optical rib waveguide (Fig.1) obtained with the second order variational method and without (SVM^r) the restrictions (10).

Fig. 3 shows the contour plot of the field profile for TE_0 ((a), (b)) and TE_1((c), (d)) modes in two optical rib waveguides with the parameters (b = 1.5μm, d = 0.6μm, h = 0.4μm, $n_1 = n_2$ = 3.44, n_3 = 3.40, n_0 = 1, λ = 1.15μm, P_0 = 0.341318, P_1 = 0.110361, (a), (c)) and (b= 1.5μm, d = 0.6μm, h = 0.4μm, n_1 = 3.44, n_2 = 3.40, n_3 = 3.40, n_0 = 1, λ = 1.15μm, P_0 = 0.238683, P_1 = 0.044141, (b),(d)). From Fig. 1 and Fig. 3, we can see how the variation of the height and the refractive index of the rib can be used to control the lateral and depth confinements in waveguide lasers with semiconductors. Thus, for a large value of n_2, the maximum of the field distributions for TE_0 and TE_1 modes is displaced towards the rib (P_0 and P_1 are large) and for a small n_2 this maximum is displaced towards the substrate (P_0 and P_1 are small).

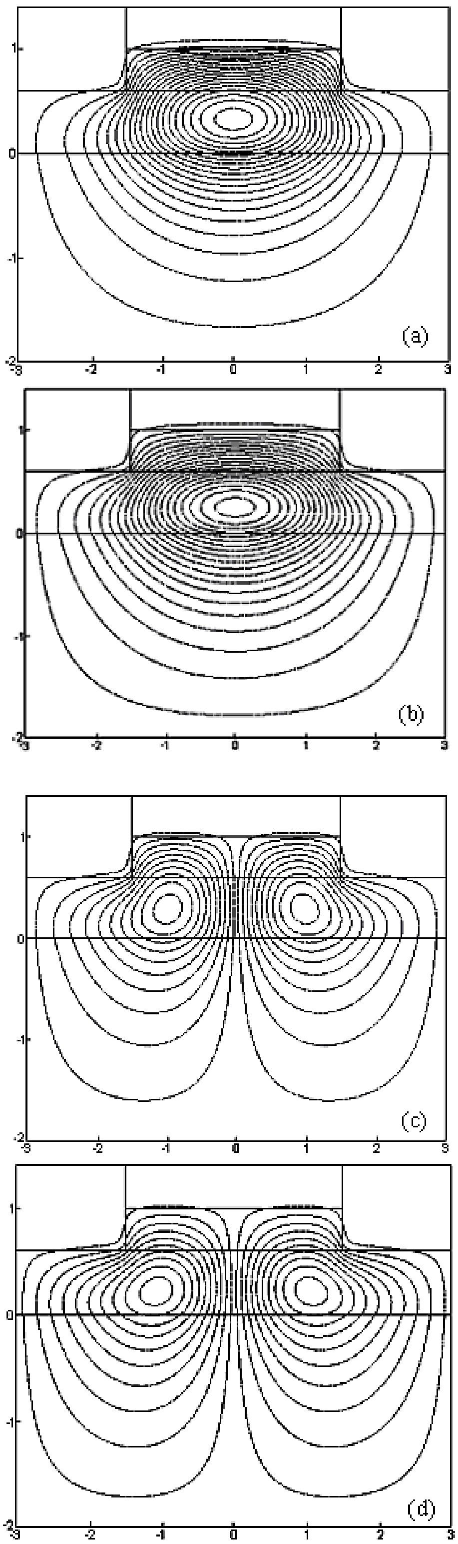

Fig. 3. The contour plot of the field profile for TE_0 ((a), (b)) and TE_1((c), (d)) modes in two optical rib waveguides with the parameters (b = 1.5μm, d = 0.6μm, h = 0.4μm, n_1 = n_2 = 3.44, n_3 = 3.40, n_0 = 1, λ = 1.15μm, P_0 = 0.341318, P_1 = 0.110361, (a), (c)) and (b= 1.5μm, d = 0.6μm, h = 0.4μm, n_1 = 3.44, n_2 = 3.40, n_3 = 3.40, n_0 = 1, λ = 1.15 μm, P_0 = 0.238683, P_1 = 0.0441407, (b),(d)).

4. Conclusions

We have obtained closer results to the exact value of the normalized propagation constant for the TE_0 mode of an optical rib waveguide by using only the second-order variational method. The lateral and depth confinements of the light in the guiding layer can be optimized by the variation of the height and the refractive index of the rib.

References

[1] V. A. Popescu, Optics Commun. **234**, 177 (2004)
[2] V. A. Popescu, J. Optoelectron. Adv. Mater. **7**, 2783 (2005)
[3] Z. Mao, W. P. Huang, IEEE J. Quantum Electron. **28**, 176 (1992)
[4] W. P. Huang and H. A. Hauss, J. Lightwave Technol. **9**, 56 (1991)
[5] Working Group I, COST 216, IEE Proc. **136**, 273 (1989)
[6] V. A. Popescu, N. N. Puscas, J. Optoelectron. Adv. Mater. **8**, 1262 (2006).

*Corresponding author: vapopescu@yahoo.com