Application of square-stacked Fresnel zone plate in Cassegrain concentrator

S. LI, H. YANG*, P. JIANG, W. CAIYANG, F. XU
School of Physics, University of Electronic Science and Technology of China, Chengdu 610054, China

Due to the secondary mirror blockage of Cassegrain Concentrator , the energy distribution of the Cassegrain Concentrator is not uniform, which will influence on the lifetime of the photovoltaic (PV) receiver. A square-stacked Fresnel zone plate is designed in this paper, which is applied to the square Cassegrain Concentrator at the primary mirror open position, eliminating the central dark area and obtaining a uniform square spot that is suitable for a square PV receiver. The simulation results show that the spot uniformity can reach up to 84.56%.This research result possesses potential application value in solar collectors.

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Keywords: Fresnel zone plate, Cassegrain concentrator, spot uniformity

1. Introduction

The zone plate is one of the hotspots in the field of diffractive optics. Most studies of the zone plate are based on the circular Fresnel zone plate. Although there are researches on the square Fresnel zone plate [1-5], they are mainly focused on related theoretical and experimental studies to improve resolution and diffraction efficiency. In the solar energy system, the Cassegrain concentrator is the most widely used [6-10], but the PV receivers are mostly square, which is different from the shape of the convergent spot obtained by the Cassegrain solar concentrator. This cannot only make full use of the solar energy, but also cause spot energy uneven distributed on the PV, which seriously affects the lifetime of the PV receiver. In practical solar energy system applications, Fresnel zone plates are lightweight, easy to process [11], and can be made very small for integration and arraying, which reduces the cost of the entire solar energy system.

In this paper, a square-stacked Fresnel zone plate is designed based on the traditional square Fresnel zone plate, and applied to a square Cassegrain concentrator as shown in Fig. 1 in order to obtain a square and uniform spot that is adapt to the PV receiver. This design is very significant for improving the utilization of solar energy and prolonging the lifetime of the PV receiver.

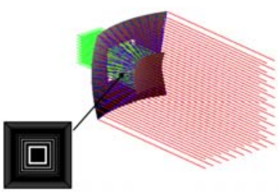

Fig. 1. Schematic diagram of 3D optical path of Cassegrain concentrating System

2. Design of square stacked Fresnel zone plates

The Fresnel zone plate is composed of alternating transparent and opaque rings, which are used to form the even or odd bands of Fresnel half-wave bands. Under point light illumination, it can obtain high intensity spots. When a square Fresnel zone plate is vertically irradiated with a plane wave, the radius of the zone plate needs to satisfy that the optical path difference from the adjacent ring to the focus point is an integral multiple of the half wavelength, that is, $n\lambda / 2$ $(n = 1,2,3, ...)$. As is shown in Fig. 2.

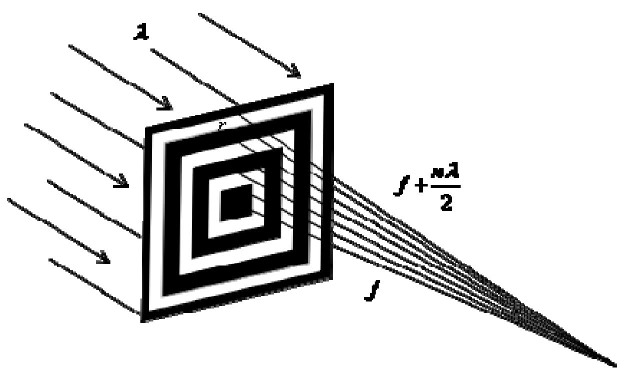

Fig. 2. Plane wave vertically irradiated square Fresnel zone plates

According to the geometric relationship we can know:

$$f^2 + r_n^2 = (f + \frac{n\lambda}{2})^2 \quad (1)$$

where λ is the wavelength of the plane wave, f is the focal length of the zone plate, r_n is the radius of the n-th ring. For the paraxial approximation, $f \gg \lambda n_{max}$, Eq.1 can be approximated as:

$$r_n \approx \sqrt{n\lambda f} \tag{2}$$

If $n = 1$, then $r_1 \approx \sqrt{\lambda f}$, that is $r_n = \sqrt{n} r_1$, so the focal length can also be expressed as

$$f \approx \frac{r_n^2}{n\lambda} = \frac{r_1^2}{\lambda} \tag{3}$$

When the square Fresnel zone plate is vertically irradiated with a plane wave, a diffraction image is obtained at a distance of z from zone plate. The diffraction field obtained from Huygens-Fresnel integral diffraction formula is:

$$\begin{aligned} U_0(x_0, y_0) &= \frac{1}{i\lambda z} e^{ikz} e^{ik\frac{1}{2z}(x_0^2+y_0^2)} t(x, y) \\ &= \frac{1}{i\lambda z} e^{ikz} e^{i\frac{k}{2z}\left((x_0^2+y_0^2)\right)} \sum_{k=1}^{n} (-1)^{(k+1)} b_k^2 \sin c\left(\frac{b_k x_0}{\lambda z}, \frac{b_k y_0}{\lambda z}\right) \end{aligned} \tag{4}$$

here $b_k = \sqrt{k} b_1$, which is the length of the k-th waveband of the Fresnel zone plate, and $t(x, y)$ is the transmissibility function of the square zone.

$$t(x, y) = \sum_{k=1}^{n} (-1)^{(k+1)} rect(\frac{x}{b_k}, \frac{y}{b_k}) \tag{5}$$

Therefore, the diffracted light intensity at the diffraction screen is:

$$\begin{aligned} I_0(x_0, y_0) &= U_0(x_0, y_0) U_0^*(x_0, y_0) \\ &= \left(\frac{b_1^2}{\lambda z}\right)^2 \left[\sum_{k=1}^{n} (-1)^{(k+1)} \varepsilon_k^2 \sin c\,(\varphi x_0, \varphi y_0)\right]^2 \end{aligned} \tag{6}$$

where $\varepsilon_k = \frac{b_k}{b_1} = \sqrt{k}, \phi = \frac{b_k}{\lambda z} = \frac{b_1 \varepsilon_k}{\lambda z}$.

According to the light intensity diffraction formula obtained above, we can get highly concentrated light spots at the focal point. In order to uniform the energy, the designed square-stacked Fresnel zone plate is a stack of four Fresnel zone plates, whose structure is shown in Fig. 3.

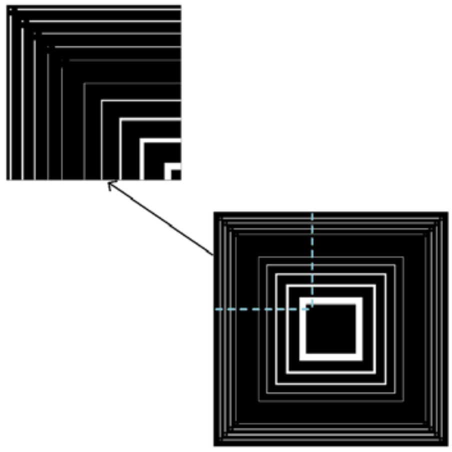

Fig. 3. Square stacked Fresnel zone plates

The transmissibility function of the square-stacked Fresnel zone plate is:

$$\begin{aligned} t_m(x, y) &= \sum_{k=1}^{n} \left\{(-1)^{(k+1)} rect(\frac{x-tk}{b_k}, \frac{y-tk}{b_k}) + (-1)^{(k+1)} rect(\frac{x-tk}{b_k}, \frac{y+tk}{b_k})\right\} \\ &+ \sum_{k=1}^{n} \left\{(-1)^{(k+1)} rect(\frac{x+tk}{b_k}, \frac{y-tk}{b_k}) + (-1)^{(k+1)} rect(\frac{x+tk}{b_k}, \frac{y+tk}{b_k})\right\} \end{aligned} \tag{7}$$

where b_k is the length of the k-th waveband of the square Fresnel zone plate and t_k is the distance between adjacent zone plates. Combining Eq. 4 with Eq. 7 and considering the superposition principle of light intensity, when the new type of stacked square Fresnel zone plate is vertically illuminate, the diffracted intensity obtained at a distance of $z = f$ (f is the focal length) from the zone plate will be the superposition of the diffracted light intensity of each individual zone plate.

3. Design of Cassegrain concentrator

The Cassegrain concentrator reflector is composed of three main parts: the primary reflector, the secondary reflector and the PV receiver. In the design of this paper, the primary reflector and the secondary reflector are both parabolas. The schematic diagram of the design principle is shown in Fig. 4.

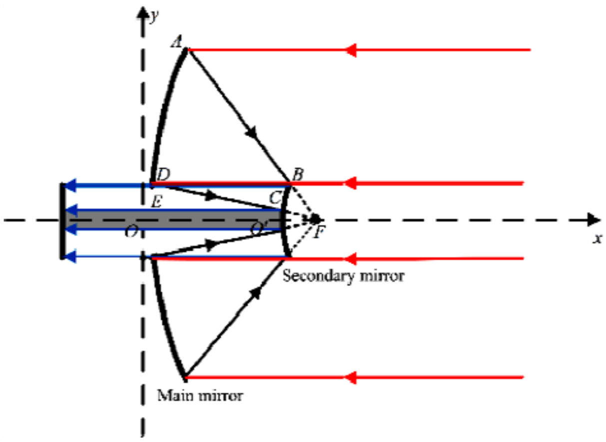

Fig. 4. Schematic of Cassegrain concentrator reflector

According to the above schematic diagram, the parabolic primary reflector equation is:

$$y_0^2 = 2p_0x_0 \tag{8}$$

where $p_0$is the focal length of the parabola. Therefore, the coordinate of the focal point F is $(\frac{p_0}{2}, 0)$.

Assuming that the aperture size of the parabolic primary reflector is 2L, the coordinate of point A is $(\frac{L^2}{2p_0}, L)$.

As is shown in Fig. 3, assuming that the distance between the center $O$of the primary reflector and the center $O'$of the secondary reflector is d, since the primary reflector and the secondary reflector have the same focus, the equation of the secondary reflector can be expressed as:

$$y_1^2 = 2(p_0 - 2d)(x_1 - d) \tag{9}$$

The equation of the straight line l_{AF}obtained from the coordinates of points A and F is:

$$y_{l_{AF}} = \frac{2p_0L}{L^2 - p_0^2}\left(x - \frac{p_0}{2}\right) \tag{10}$$

The ordinate of point B can be obtained from Eq. 9 and Eq. 10:

$$y_B = \frac{(p_0 - 2d)L}{p_0} \tag{11}$$

It can be seen from the Fig. 3 that the ordinate of point B is the diameter of the secondary reflector. In the design of the Cassegrain concentrator, the opening size of the primary reflector and the size of the secondary reflector are often the same, so it can be obtained as follows:

$$y_D = y_B = \frac{(p_0 - 2d)L}{p_0} \tag{12}$$

Combining Eq. 12 with Eq. 8 we can get the coordinates of point D is $(\frac{(p_0-2d)L^2}{2p_0{}^3}, \frac{(p_0-2d)L}{p_0})$, so the equation of the straight line l_{FD} is:

$$y_{l_{FD}} = \frac{2(p_0 - 2d)p_0^2L}{(p_0 - 2d)L^2 - p_0^4}\left(x - \frac{p_0}{2}\right) \tag{13}$$

From Eq. 9 and Eq. 12, we can get the ordinate of point C as:

$$y_C = \frac{(p_0 - 2d)^2L}{p_0{}^2} \tag{14}$$

The concentration ratio of the concentrator is the ratio of the area of the incident light spot to the area of the exit light spot. Since a square Cassegrain concentrator reflector is used in the design, the concentration ratio K can be expressed as

$$K = \frac{L^2}{y_D^2 - y_E^2} = \frac{p_0^4}{4d\left(p_0 - 2d\right)^2\left(p_0 - d\right)} \tag{15}$$

In the design of this section, the aperture of the primary reflector of the Cassegrain concentrating reflector is $2L = 150mm$ and $p_0 = 185$mm. During the design of the Cassegrain concentrating reflector, the opening size of the primary reflector and the size of the secondary reflector are generally the same. Assuming that the shielding ratio is$NC = 20\%$and the distance between the primary reflector and secondary reflector is $d = 80mm$, we can get the concentration ratio of the concentrator is $K = 55.78$.

The parameters of the primary reflector and the secondary reflector are shown in Table 1.

Table 1 Parameters design of the primary and secondary reflector

	caliber/mm	focal length/mm	Curve equation
Primary reflector	150	185	$y_0 = 370x_0$
Secondary reflector	30	25	$y_1 = 50(x_1 - 80)$

4. Discussion

According to the design of the previous section, the parameters used in the simulation are: the wavelength is $\lambda = 632.8nm$, the total number rings of the zone plate is N=100, the length of the outer ring of the zone plate is $b_N = 30mm$.Assuming that the distance between adjacent zone plates ist_k, the effect of $t_k$on the uniformity of the spot is analyzed, where the spot uniformity can be expressed by the following formula [12]:

$$\Delta E = \left(1 - \frac{Emean_{max}}{Emean_{max}} \times 100\%\right) \tag{16}$$

When $t_k = 0.046mm$, the diffraction image is shown in Fig. 5. Fig 5 (a) and (b) are the three-dimensional and two-dimensional diffraction images of the square-stacked Fresnel zone plate, respectively, and Fig. 5 (c) is the normalized distribution of spot energy.

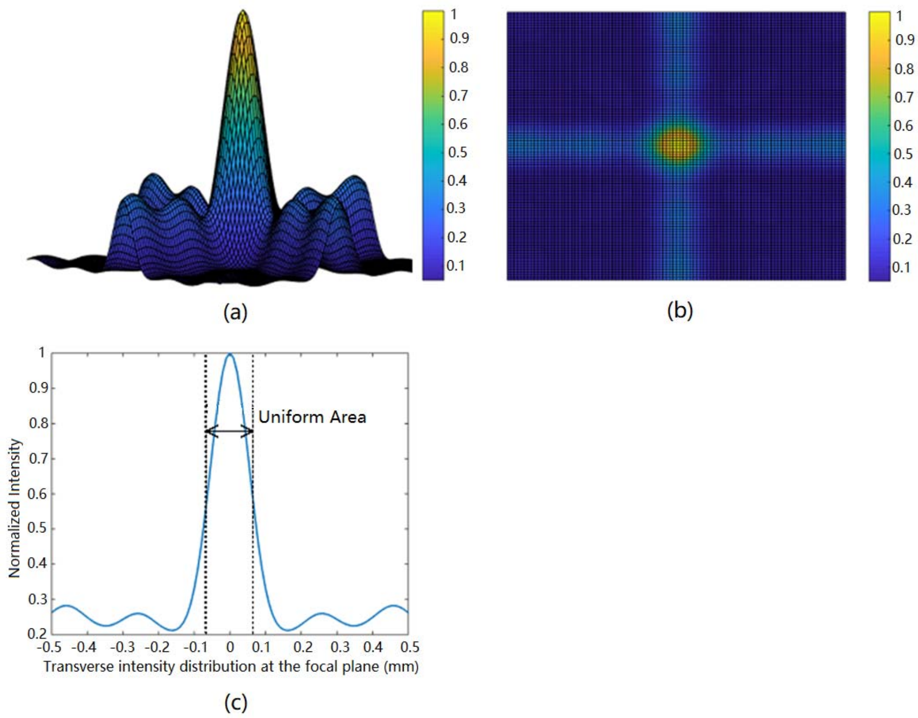

Fig. 5. The diffraction image of the square-stacked Fresnel zone plate at$t_k = 0.046mm$. (a) The three-dimensional diffraction images of the square-stacked Fresnel zone plate; (b) The two-dimensional diffraction image of the square-stacked Fresnel zone plate; (c) The normalized distribution of spot energy

When the distance between adjacent zone plates is small, the energy at the centers of adjacent spots overlap with each other resulting in the center energy being too concentrated and forming a more highly concentrated spot. In the uniform area shown in Fig. 5(c), the uniformity of its energy distribution is analyzed and its uniformity is $\eta = 59.99\%$.

With the increase of the distance t_k between the zone plates, if we take t_k=$0.056mm$, the diffraction image is shown in Fig. 6. Fig. 6 (a) and (b) are the three-dimensional and two-dimensional diffraction images of the square-stacked Fresnel zone plate, respectively, and Fig. 6 (c) is the normalized distribution of spot energy.

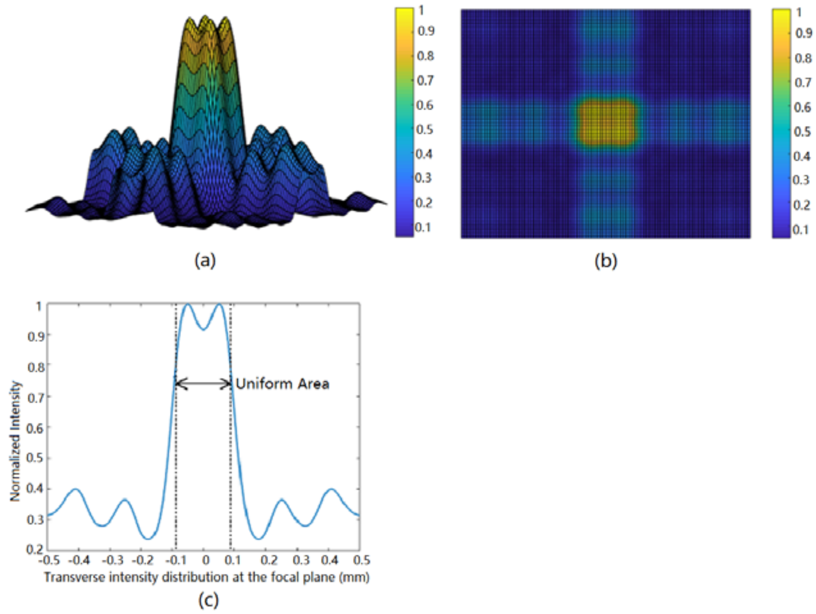

Fig. 6. The diffraction image of the square-stacked Fresnel zone plate at $t_k = 0.056mm$. *(a) The three-dimensional diffraction images of the square-stacked Fresnel zone plate; (b) The two-dimensional diffraction image of the square-stacked Fresnel zone plate;(c) The normalized distribution of spot energy*

When the distance between the adjacent zone plates increases, the energy at the edge of the zone plate overlaps with each other. The center energy of the new spot is relatively uniform. In the uniform area shown in Fig. 6 (c), the uniformity of its energy distribution is analyzed and its uniformity is$\eta = 84.56\%$ .With the further increase of the distance between the adjacent zone plates, if we take $t_k = 0.060mm$, the diffraction image is shown in Fig. 7, Fig. 7 (a) and (b) are the three-dimensional and two-dimensional diffraction images of the square-stacked Fresnel zone plate, respectively, and Fig. 7 (c) is the normalized distribution of spot energy.

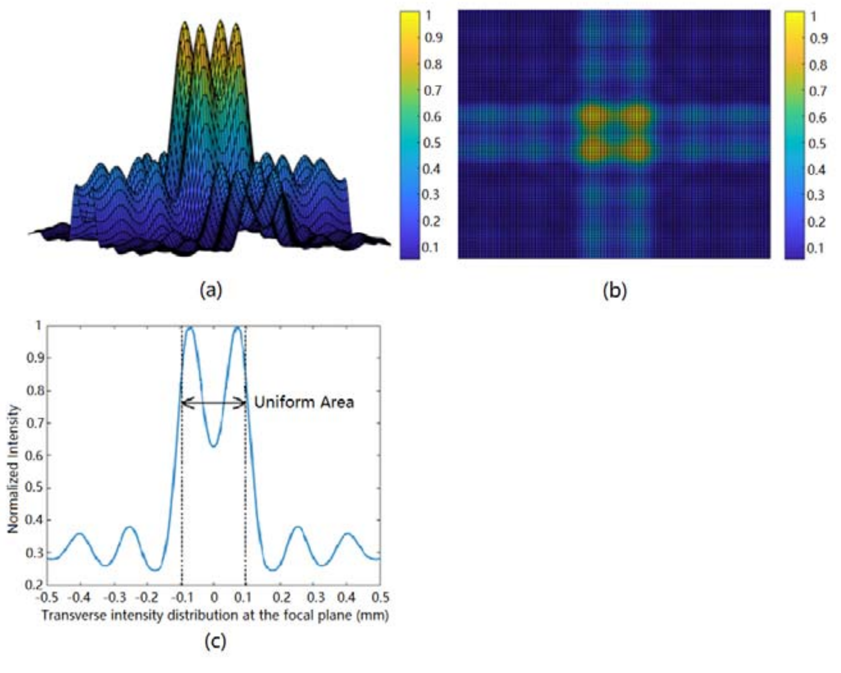

Fig. 7. The diffraction image of the square-stacked Fresnel zone plate at$t_k = 0.060mm$. *(a) The three-dimensional diffraction images of the square-stacked Fresnel zone plate; (b) The two-dimensional diffraction image of the square-stacked Fresnel zone plate; (c) The normalized distribution of spot energy*

When the distance between the adjacent zone plates further increases, the energy at the edge of the zone plate overlaps with each other, However, due to the long distance between the zone plates, the center energy of the new spot is relatively weak. In the uniform area shown in Fig. 7 (c), the uniformity of its energy distribution is analyzed and its uniformity is $\eta = 80.76\%$.

In order to further analyze the diffraction characteristics of the designed square-stacked Fresnel zone plate, the variation of the spot uniformity with t_k and the number of zone plate rings N is shown in Fig. 8.

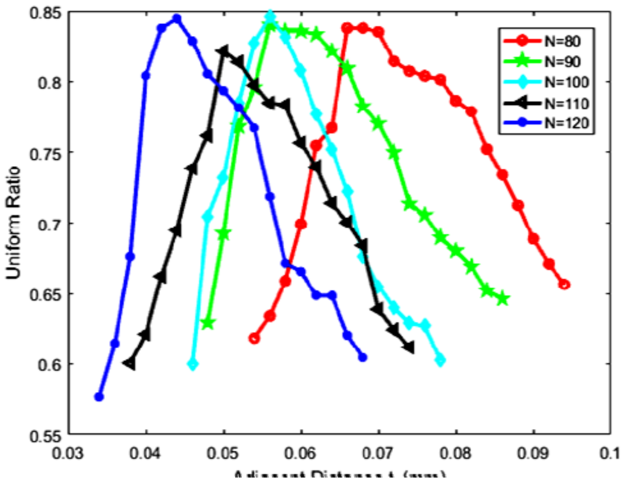

Fig. 8. The variation of the spot uniformity with t_k*and the number of zone plate rings N*

As can be seen from the Fig. 8, with the distance between zone plates increases, the spot uniformity gradually increases at first and then gradually decreases. When designing the square-stacked Fresnel zone plate, the optimum distance between the zone plates can be found according to the variation curve. On the other hand, from the figure, we can see that as the number of zone plates increases, the value of t_k decreases when the spot uniformity reaches its maximum value. Therefore, when designing the zone plate, the number of the zone plate and the optimum distance between the zone plates can be designed as required. According to Fig. 8, we can get the maximum value of the spot uniformity is $\eta = 84.56\%$ when the number of zone plates is $N = 100$ and the distance between the zone plates is $t_k = 0.056mm$.

5. Conclusion

A square-stacked Fresnel zone plate and Cassegrain concentrating reflector have been designed in this paper. The square-stacked Fresnel zone plate is obtained by superimposing multiple square Fresnel zone plates, and the Cassegrain concentrating reflector is also designed in a square shape. The simulation results show that the designed zone plate can get a square spot that suitable for the square PV receiver and the spot is relatively uniform. The simulation results show that the maximum value of the spot uniformity is $\eta = 84.56\%$. This research result possesses potential application value in solar collectors.

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*Corresponding author: yanghj@uestc.edu.cn