Invited paper

BCS variations

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This year is the golden anniversary of the theory of superconductivity of Bardeen, Cooper and Schrieffer (BCS theory). The BCS theory finds wide applicability in quantum field theory including high energy particle physics, nuclear matter, quantum chromodynamics, superfluid 3He and unconventional superconductors including high Tc superconductors. Here we shall review a few aspects of unconventional superconductors. Very recent development on half-quantum vortices (HQV) and Majorana fermions are briefly touched.

1. Introduction

The BCS theory [1] is one of the most beautiful theories in the last century. It not only has described most of experimental facts about s-wave superconductors, the basic concept is applied to particle physics, nuclear physics etc. Also after the discovery of ^{3}He in 1972, most of its characteristics are interpreted in terms of p-wave BCS superconductivity [2–5].

Since 1979 there appears a new class of superconductors: heavy fermion superconductors, organic molecular superconductors, high T_c cuprate superconductors, Sr_2RuO_4 etc. An early review on these new superconductors," unconventional superconductors" is found in [6]. Around 1994, d-wave symmetry of high T_c cuprate superconductors is established through the powerful angle resolved photoelectron spectra (ARPES) [7] and the elegant Josephson interferometry [8,9]. Also E_{2u} or f-wave symmetry of superconductivity in UPt_3 is deduced from the thermal conductivity and the shift in NMR [10–12]. Further a few people started to analyze the BCS theory of d-wave superconductivity. In particular, Volovik [13] has succeeded in calculating the quasiparticle density of states in the vortex state of d-wave superconductors. The striking √H dependent specific heat is subsequently detected in the optimally doped YBCO [14,15], LSCO [16] and Sr_2RuO_4 [17,18]. Note contrary to the claim made in [19], the specific heat data in Sr_2RuO_4 is fully consistent with the chiral f-wave superconductor [20]. Volovik's method is extended into a variety of directions, a) scaling relation [21,22], b) thermal conductivity, c) for arbitrary field orientation and d) for a variety of unconventional superconductors [20]. Since 2001, in a brilliant series of experiments Izawa et al. has succeeded in identifying the gap symmetries of Sr_2RuO_4 [23], $CeCoIn_5$ [24], $_\kappa$-$(ET)_2Cu(NCS)_2$ [25], YNi_2B_2C [26], $PrOs_4Sb_{12}$ [27,28] and UPd_2Al_3 [29,30]. These gap symmetries are shown in Fig. 1. From this and others we construct the list of known symmetries of nodal superconductors in Table 1. More details on Volovik's approach and its applications see [20].

Table 1.

quasi 1D systems		
Bechgaard salts $(TMTSF)_2X$, $(TMTTF)_2X$ with X=PF_6, ClO_4, AsF_6	Triplet [31]	chiral f-wave?[32]
quasi 2D systems		
Sr_2RuO_4	triplet [19]	chiral f-wave [23]
$CeCoIn_5$, $_\kappa$-$(ET)_2X$ with X=$Cu(NCS)_2$, $CuCN(CN)_2Br$, high T_c cuprates	singlet	d-wave
UPd_2Al_3	singlet	g-wave [29,30]
URu_2Si_2 UNi_2Al_3 $CePt_3Si$	triplet(most likely)	?
3D systems		
UPt_3 [12]	triplet	E_2u or $Y_{3,\pm2}(\theta,\varphi)$
YNi_2B_2C	singlet	s+g-wave
$PrOs_4Sb_{12}$[27,28]	triplet	p+h-wave

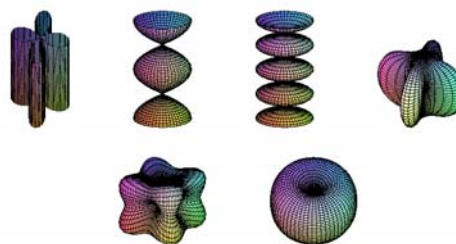In the following we shall review a few topics on the further development of the BCS theory.

Fig. 1. Order parameters from top left: d-wave - high-T_c cuprates,$CeCoIn_5$, $_\kappa$-$(ET)_2Cu(NCS)_2$; chiral f-wave - Sr_2RuO_4;g-wave - UPd_2Al_3; s+g-wave - YNi_2B_2C; p+h-wave, $PrOs_4Sb_{12}$ A-phase; p+h-wave, $PrOs_4Sb_{12}$ B-phase.

2. Bogoliubov de gennes (bdg) equation

The BCS theory for unconventional superconductors is most readily formulated in terms of the quantum field theory [33] or the Nambu-Gor'kov Green's function [34,35]. This method is exploited in 20. The Bogoliubov de Gennes equation is introduced in 1964 [36–38]. In particular Caroli et al [37] have calculated the bound state spectrum around a single vortex in s-wave superconductor.

As is well known, the theory of type II superconductors with quantized vortices is developed by Abrikosov [39] in the frame work of the phenomenological Ginsburg Landau theory [40]. After the BCS theory Gorkov and de Gennes [38,41] derived the Ginsburg Landau theory in the vicinity of $T = T_c$, where $|\Delta(r)|/T_c \ll 1$. As it turns out that all nodal superconductors appear to belong to the type II superconductors and then Abrikosov's vortex is the most ubiquitous topological defects in nodal superconductors [42]. The Bogoliubov de Gennes equation for unconventional superconductors is written as

$$Eu(\mathbf{r}, \mathbf{k}) = \xi k u(\mathbf{r}, \mathbf{k}) + \Delta(\mathbf{r}) f(\mathbf{k}) v(\mathbf{r}, \mathbf{k}) \qquad (1)$$

$$Ev(\mathbf{r}, \mathbf{k}) = -\xi k v(\mathbf{r}, \mathbf{k}) + \Delta^*(\mathbf{r}) f(\mathbf{k}) u(\mathbf{r}, \mathbf{k}) \quad (2)$$

where $\xi_k = -1/2m(\Box\nabla + ieA)^2 - \mu$ and $f(k) = p_F^{-2}(\partial_x^2 - \partial_y^2)$ for d-wave superconductors [43].
Now let us consider the bound states around a single vortex in unconventional superconductors. For a singlet superconductor, we can assume that

$$\Delta(r) = e^{i\phi}\tanh(r/\xi) \qquad (3)$$

in good approximation where $\xi \approx v/\Delta$ and v is the Fermi velocity. Then following Caroli et al [37], the bound state spectrum of singlet superconductors is given by

$$\varepsilon_n = \omega_0(n + 1/2) \qquad (4)$$

with n = 0, ±1, ±2, ... and

$$\omega_0 = \frac{p_F^{-1}(\int_0^\infty dr\Delta(r)/rf(k)e^{-2K(r,k)}}{(\int_0^\infty dr\Delta(r)f(k)e^{-2K(r,k)}}, \qquad (5)$$

where

$$K(r,k)=$$
$$v_F^{-1}f(k)\int_0^r \left|\Delta(r)\right|dr \approx v_F^{-1}f(k)\int_0^r \Delta\tanh(r/\xi)dr = v_F^{-1}\Delta\xi f(k)\ln(\cosh(r/\xi)) \qquad (6)$$

Inserting K(r, k) into Eq. , we obtain

$$\omega_0 = \frac{2}{\pi}\frac{\Delta}{p_F\xi} = \frac{1}{\pi}\frac{\Delta^2}{E_F} \quad . \qquad (7)$$

Here (...) means average over the Fermi surface. Eq. 4 for singlet superconductors has been obtained by Kopnin and Volovik [44]. These are fully consistent with the numerical solution of BdG equation for d-wave superconductors given in [45,46]. On the other hand, for triplet superconductors, Kopnin and Salomaa [47] find

$$\varepsilon_n = \omega_0 n \qquad (8)$$

with n = 0, ±1, ±2 and $\omega_0 = \Delta^2/E_F$. For a triplet superconductor, we have the zero mode with $\varepsilon_0 = 0$ for $n = 0$. First of all the zero mode is common to the triplet superconductors with equal spin pairing (ESP) like superfluid ^{3}He-A, UPt$_3$ (E$_{2u}$) and Sr$_2$RuO$_4$ (chiral f-wave). More recently Ivanov [48] has shown that the zero mode is associated with the Majorana fermion [49–51]. These bound state associated with vortices and their role in the vortex dynamics are well handled in a text book by Kopnin [52]. Unfortunately, however, Kopnin appears to neglect the extended states or the delocalized states in the vortex state in unconventional superconductors discussed for example in [20]. Indeed it is well known that the low temperature properties of nodal superconductors are dominated by delocalized states. Therefore we should conclude that the treatment of the transport properties in [52] is inadequate and unrealistic.

To summarize the bound state spectrum around a single vortex in unconventional superconductors is very similar to the one found in s-wave superconductors [37,38] and appears to depend little on the direction of the magnetic field. In other words the specific heat associated with the bound states depend little on field orientation. Therefore the angle dependent magnetospecific heat should be dominated by the extended states as discussed in [20]. More recently we have analyzed the entropy carried by vortices (Nernts effect) in unconventional superconductors and found that it is very similar to the one in s-wave superconductors [53].

3. Half quantum vortices (hqv)

The half quantum vortices (HQV) in superfluid ^{3}He-A has been speculated since 1976 [54-56]. The order parameter of superfluid ^{3}He-A is characterised by two unit vectors: **l** the chiral vector and **d** spin vector [2–4]. In order to realize HQV in superfluid ^{3}He-A the uniform **l** texture is required, which is realized in a principle by the parallel plate geometry. In particular when the gap D between 2 plates is less than 2.3$\xi_D \sim$ 23 μm, where ξ_D (~10μm) is the dipole coherence length, there will be no **l** texture between the parallel plates [57,58]. In spite of the intensive search of HQV by the Helsinki group no HQV has been seen until recently [59,60]. Recently Yamashita et al 61 has reported a surprising NMR satellite seen in the rotating superfluid ^{3}He-A in the parallel plate geometry at LT24 conference at Orlando, Florida. Unlike all earlier experiments, the gap between their parallel plates is D~10μm, which is adequate to realize the uniform **l** texture. Indeed we have succeeded in describing the NMR

satellite in terms of HQV [62]. The HQV in triplet superconductors have been invoked in order to interpret the strong flux pinning observed in some of triplet superconductors like UPt$_3$, Sr$_2$RuO$_4$ and U$_{1-x}$Th$_x$Be$_{13}$ by Mota et al [63–66]. We would like to present a microscopic analysis of HQV in UPt$_3$, Sr$_2$RuO$_4$ [67]. First of all the superconducting order parameters in the B phase of UPt$_3$ are characterised by **l** and d as in superfluid ^{3}He-A [12,19]. Further **l** is fixed parallel to the crystalline c axis. Also in the absence of magnetic field **l** ∥ **d** very similar to the superfluid ^{3}He-A, though it is believed that this equilibrium configuration is due to the spin orbit interaction and not due to the nuclear dipole interaction. Nevertheless we use ξ_D ~1μm to refer this length scale. Similarly in a high magnetic field **d** ⊥ **H** is realized. For the related length scale we use ξ_H, which is very analogous to superfluid ^{3}He-A [42]. The HQV are most readily described in terms of the texture free energy [62,66]

$$ F = \frac{1}{2}\chi\, Nc^2 \int dxdy \left\{ K(\nabla\phi)^2 + \sum_{i,j}\left| \partial_i d_j \right|^2 + \xi_{D'}^{-2}(d_x^2 + d_y^2) \right\} $$

(9)

where

$$ K = \frac{\rho_s(t)}{\rho_{sp}(t)} = \frac{1 + 1/3F_1}{1 + 1/3F_1^\alpha} \frac{1 + 1/3F_1^\alpha(1 - \rho_s^0(t))}{1 + 1/3F_1(1 - \rho_s^0(t))} $$

(10)

and $\xi^{-2}{}_{D'} = \xi^{-2}{}_D - \xi^{-2}{}_H$, and χN, c, ρ_s, ρ_{sp} and ρ_s^0 are the triplet spin susceptibility in the normal state, the spin wave velocity, the superfluid density, the spin superfluid density and the unrenormalized superfluid density. Further F_1 and F_1^a are Landau's Fermi liquid coefficients. Here we limit ourselves to the case for **H** ∥ z, though the present formalism can be extended for arbitrary field orientations.

Now let us divide the 2D space perpendicular to the magnetic field into circles with radius a, which enclose a single Abrikosov's vortices. Then the upper critical field H$_{c2}$(t) is related to the coherence length $\xi(t)$ [38]

$$ H_{c2}(t) = \frac{\phi_0}{2\pi\xi^2(t)} $$

(11)

with $\varphi_0 = 2.07 \times 10^7 Txm^2$ the flux quantum. Further, the radius of the circle at H = H$_{c2}(t)$ is given by a=√2ξ(t). In other words, a > √2ξ(t) in the vortex state. Perhaps it is important to notice that K ≫1 in heavy fermion superconductors except in the vicinity of T ~ T$_c$. If we ignore F_1^a for simplicity, $K(t)$ is given approximately by

$$ K(t) = \frac{m^*}{m}\left\{ 1 + (\frac{m^*}{m} - 1)(1 - \rho_s^0(t)) \right\}^{-1}, $$

(12)

where m*/m = 20 and 16 (γ band) for UPt$_3$ and Sr$_2$RuO$_4$ [68,69]. Further $\rho_s^0(t)$ for the E$_{2u}$ state (UPt3) and chiral f-state (Sr$_2$RuO$_4$) are well approximated by the one for a d-wave superconductors[70]:

$$ \rho_s^0(t) \approx 1 - 0.647t - 0.355t^2 + 0.02t^3 . $$

(13)

Then with the help of Eq. (13), K(t) for UPt$_3$ and Sr$_2$RuO$_4$ are obtained, which are shown in Fig. 2. Here we took the model of chiral f-wave superconductors [20,42]. If we had taken instead, the chiral p-wave superconductor model as in19, the temperature dependence of $K(t)$ would be very different. Now the energy of a single Abrikosov vortex in a circle with radius a is given by
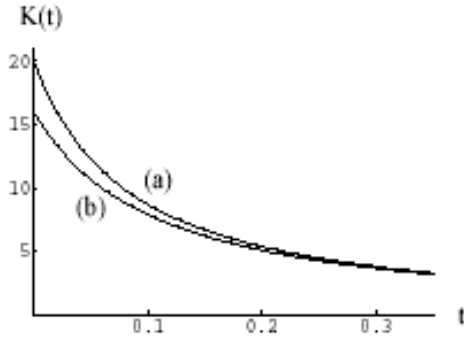
$$ F_A = \pi\chi Nc^2 \ln(a/\xi) . $$

(14)

Fig.. 2. The K(t) function is shown for UPt$_3$ (a) and Sr$_2$RuO$_4$ (b).

On the other hand, a bound pair of HQV separated by R within the same circle is given by

$$ F_{BP} = \frac{\pi}{2}\chi Nc^2 (K \ln(\frac{a + \sqrt{a^2 - R^2/4}}{2\xi}) + \ln(a/\xi) - \frac{R}{4a}\sin^{-1}(\frac{R}{\sqrt{a^2 - R^2/4}}) + \\ + \frac{1}{4}(\frac{R}{\xi_{D'}})^2 \ln(\frac{4\xi_{D'}}{R})) $$

(15)

Also for K ≫ 1, R is well approximated by R/2a = $\sqrt{2K+1}/K+1$ as in[62,67]. Inserting the R in Eq. (13), we find

$$ F_{BP} = \frac{\pi}{2}\chi Nc^2 (K \ln(\frac{a}{2\xi}\frac{2K+1}{K+1}) + \ln(a/\xi) - \frac{1}{2}\frac{\sqrt{2K+1}}{K+1}\sin^{-1}(\frac{2\sqrt{2K+1}}{K+1}) + \\ + \frac{2K+1}{(K+1)^2}(\frac{a}{\xi_{D'}})^2 \ln(\frac{2\xi_{D'}}{a}\frac{K+1}{\sqrt{2K+1}})) $$

(16)

Finally F$_A$ = F$_{BP}$ gives

$$a/\xi = \left[2(2K+1)^{\frac{1}{2}(2K+1)}(K+1)^{-(K+1)}\right]^{\frac{1}{K-1}}\left[\frac{K+1}{\sqrt{2K+1}}\frac{\xi_{D'}}{a}\right]^{\frac{2K+1}{(K+1)^2(K-1)}(\frac{a}{\xi_{D'}})^2}.$$

(17)

In particular assuming $\xi_H = \xi_D$ at $H = H^*(t) < H_{c2}(T)$, we find

$$\frac{H^*(t)}{H_{c2}(t)} = \left[2^{-(K-2)}(2K+1)^{2K+1}(K+1)^{-2(K+1)}\right]^{\frac{1}{K-1}}.$$

(18)

Now making use of $K(t)$ in Fig. 2, we find $H^*(t)/H_{c2}(t)$ for UPt$_3$ and Sr$_2$RuO$_4$, which is shown is Fig. 3. Looking at Fig. 3, it appears that B-C phase boundary in UPt$_3$ corresponds to $H = H^*(t)$. Unfortunately, no similar phase diagram is worked out for Sr$_2$RuO$_4$. But the present analysis suggests the existence of a similar phase boundary for Sr$_2$RuO$_4$ as well. Surprisingly the curves for UPt$_3$ and Sr$_2$RuO$_4$ look exactly the same.
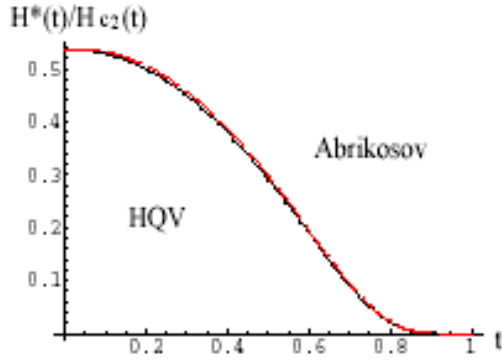

Fig. 3. $H^*(t)/H_{c2}(t)$ is plotted as a function if t, the red dashed line is for UPt$_3$.
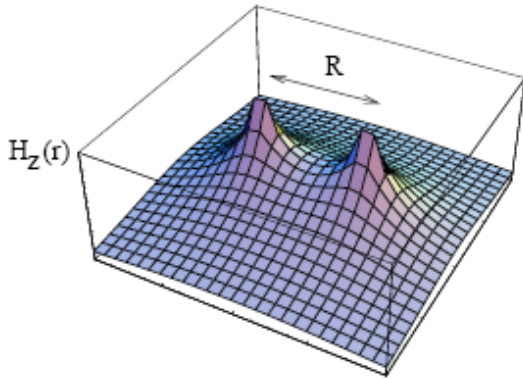

Fig. 4. The local magnetic field generated by a pair of HQV is shown.

4. Quasiparticle density of states

We have not worked out the quasiparticle density of states associated with HQV in UPt$_3$ and Sr$_2$RuO$_4$, we shall refer to Ivanov's work, where he considered the case d⊥l. Under this condition, his result is easily extended for other triplet superconductors with EPS. Then the quasiparticle density of states is given by

$$N_{pair}(r,E) = \frac{1}{2}\left\{N(r+\frac{R}{2},E) + N(r-\frac{R}{2},E)\right\}$$

(19)

where $N(r,E)$ is the one for a single vortex and $R = Rx$. For Sr$_2$RuO$_4$, $N(r,E)$ was calculated in [71]. The result is very consistent with STM data by Lupien et al [72]. On the other hand in the realistic situation d ⊥ l is broken at least in the vicinities of HQV. For example d has a tendency to become parallel to l, which will increase the core size of d-disgyration. Perhaps the vortex core size increases from ξ to $\xi_{D'}$. This will change the energy scale from Δ^2/E_F to Δ^2/E_F ($\xi/\xi_{D'}$); the much more density of states around the vortex cores. However, we do not expect Eq. (17) is much modified qualitatively. Similarly the local magnetic field generated by a pair of HQV is given by

$$H_z(r) = \frac{\phi_0}{4\pi\lambda}\left(K_0(\left|r+\frac{R}{2}\right|/\lambda) + K_0(\left|r-\frac{R}{2}\right|/\lambda)\right),$$

(20)

where $K_0(z)$ is the modified Bessel function. Eq. (18) is sketched in Fig. 4. Here the distance between 2 HQV is R ~3micron. We believe both the quasiparticle density of states and the local magnetic field should be accessible to STM[73,74], and the micromagnetometry[9,75]. However these experiments have to be carried out below 300mK, which appears to be impossible in the present moment.

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