Chaos and chaos synchronization of semiconductor lasers with optoelectronic feedback and bidirectional couplings

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Chaos and chaotic synchronization of semiconductor lasers with optoelectronic feedback and bidirectional couplings have been investigated theoretically. The results show that chaos and chaotic synchronization of two lasers are realized by two schemes which are difference coupled and sum-coupled, and the effects of synchronization are both very good, but sum-coupled is easier. Taking coupling constant k and nonlinear gain ε into account, the influence of their variation on chaos and chaotic synchronization are analyzed, two lasers can achieve good synchronization with small tolerance of parameter mismatch, but the robustness and stability of difference coupled is better than sum coupled.

1. Introduction

Since chaos synchronization was put forward firstly by Pecora and Carroll in 1990[1], chaos and chaotic synchronization have attracted more interest because of its potential applications in the field of private communication and for the control of chaos in different dynamical systems. Recently, various methods such as occasional coupling[2,3],unidirectional coupling[5-7], bidirectional coupling[8-11] and optoelectronic feedback[12-14],have been shown to induce chaos and achieve chaotic synchronization in laser systems. A method for the synchronization of chaotic systems with occasional coupling was proposed, in which the drive and response lasers are coupled and decoupled in some intervals, synchronization can be achieved and the proposed scheme is robust with respect to noise and parameter mismatch under some mild conditions [15].And chaos synchronization in both unidirectional and bidirectional coupled laser diodes with multiple time delay and electrooptical feedback had been reported and the existence and sufficient stability conditions for the synchronization regimes were investigated[16].An excellent opportunity of semiconductor lasers for communication using chaotic waveforms, was provided and discussed the characteristics and the synchronization of two semiconductor lasers with optoelectronic feedback, and exhibited broadband chaotic intensity oscillations whose dynamical dimension generally increases with the time delay in the feedback loop[17].In particular, the optoelectronic feedback and bidirectional coupling have been widely researched for stabilizing and controlling the chaotic outputs of semiconductor lasers. The effect of delayed optoelectronic feedback on a directly modulated InGaAsP laser diode have been investigated. The results showed that the period doubling, chaos and formation of pulses with double peak structure can be completely eliminated due to the feedback with small delays, the bifurcation diagrams and time series are used to characterize the chaotic and periodic states of the laser[19].The chaotic dynamics of directly modulated semiconductor lasers with positive and negative delayed optoelectronic feedback was studied numerically, in which configuration is found to be more effective in inducing chaotic dynamics to such systems with nonlinear gain reduction factor in the practical value range[18-20].In this paper, two schemes named as difference coupled and sum-coupled based on optoelectronic feedback and bidirectional couplings, realizing chaos and chaotic synchronization of two lasers, have been investigated and analyzed in this paper. For the different values of *k*,the good synchronization and high robustness in two kinds of configurations can be observed. Finally, we concentrate on the dynamics for the different *k* and *ε*,the term governing nonlinear gain. Therefore, semiconductor lasers in two schemes can achieve chaotic synchronization.

2. Systematical configuration and theory

Fig. 1 is the schematic diagram for semiconductor lasers with optoelectronic feedback and bidirectional couplings. The light output of SL1 and SL2 is fed back to form optoelectronic feedback via a photodetector (PD) and an amplifier (A), experience the adder or substracter,and modulated SL1 and SL2 together with their injection

current,respectively.During these processes, photodetector is used to convert optical signal into electronic signal, and amplifier is used to adjust the feedback ratio and the coupling ratio. The solid lines indicate the electronic paths, while the dashed lines indicate the optical paths.
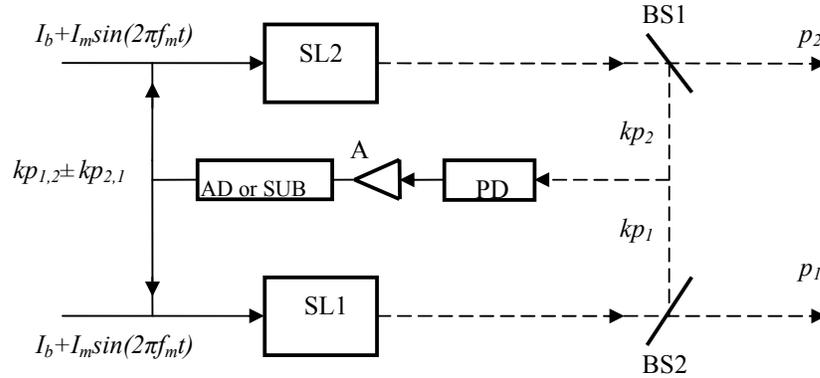

Fig. 1 Schematic diagram for coupled lasers. SL:semiconductor laser, PD:photodetector, A:amplifier,   BS:beamsplitter, AD:Adder, SUB:Substracter.

In such a scheme, the rate equations representing semiconductor lasers with optoelectronic feedback and bidirectional couplings are expressed as [18-20]:

$$\frac{dp_{1,2}}{dt} = \frac{1}{\tau_p}[\frac{N_{1,2} - \delta}{1 - \delta}(1 - \varepsilon p_{1,2})p_{1,2} - p_{1,2} + \beta N_{1,2}] \quad (1)$$

$$\frac{dN_{1,2}}{dt} = \frac{1}{\tau_e}(\frac{I}{I_{th}} - N_{1,2} - \frac{N_{1,2} - \delta}{1 - \delta}p_{1,2}) \quad (2)$$

$$I(t) = I_b + I_m \sin(2\pi f_m t) \quad (3)$$

where $N_{1,2}$ and $p_{1,2}$ are the carrier and photon densities of SL1 and SL2 ,respectively.τ_e and τ_p are the electron and photon lifetimes, respectively. $\delta = n_0/n_{th}$ and $\varepsilon = \varepsilon_{NL}S_0$ are dimensionless parameters where n_0 is the carrier density required for transparency,$n_{th} = (\tau_e I_{th}/eV)$ is the threshold carrier density, ε_{NL} is the factor governing the nonlinear gain reduction occurring with an increase in S, $S_0 = \Gamma(\tau_p/\tau_e)nth, I_{th}$ is the threshold current.e is the electron charge, V is the active volume andΓ is the confinement factor. I, I_b and I_m denote the driving current, the bias current and the amplitude of the modulation current, respectively. f_m is the modulation frequency and β is the spontaneous emission factor.

The difference coupling and sum-coupling are realized by adding a current proportional to the output power difference/sum of the first laser and the second. The rate equations are described as:

$$I(t) = I_b + I_m \sin(2\pi f_m t) + k(p_{1,2} \pm p_{2,1}) \quad (4)$$

where k is the coupling strength. p_1 and p_2 is the photon densities of SL1 and SL2,respectively.

3. Results and discussion

The rate equations (1)-(4) can be numerically solved by the fourth-order Runge-Kutta method. During the calculations,   the   used   data are[19]: $\tau_p = 6ps$, $\tau_c = 3ns$, $I_{th} = 26mA$,

$f_m = 0.8Ghz$, $I_b = 1.5I_{th}$, $I_m = 0.3I_{th}$, $\delta = 0.692$, $\varepsilon = 1 \times 10^{-4}$, $\beta = 5 \times 10^{-5}$ .Figure 2 plots the time series, phase and correlation diagram of the output of the two lasers for various values of the coupling constant k in the sum-coupled and difference coupled. It can be clear in figure 2(a) that, when k is small(k=0.05e-3),the output dynamics is chaotic from the time series and phase diagram; As the increase of k, the time series show a double peak structure, and the orbit in the phase diagram changes apparently, which shows a four cycle at k=3.5e-3;For k=4e-3,the time series and phase diagram both show a two-cycle. Compared with the previous results, Figure 2(b) presents time series, phase and correlation diagram of the output of the two lasers for the same coupling constant k in the difference-coupled. From these diagrams, when k is 0.05e-3, the phase portrait shows chaotic. With the increase of k,the orbit remains unchanged at k =3.5e-3.As k is further increased, the time series, phase and correlation diagram still show chaotic for k=4e-3,even for k=11.2e-3. In Fig. 2, the correlation diagrams of the photonic intensity of two lasers realized by difference coupled show that, the high-quality chaos synchronization between the two lasers can be realized. From two schemes, every dynamics states can be controlled much easier, and the sum-coupled is much easier than the difference coupled; On the other hand, the stability in the difference coupled is better than the sum-coupled, and the difference coupled system can be adjusted and controlled in a large parameter range.
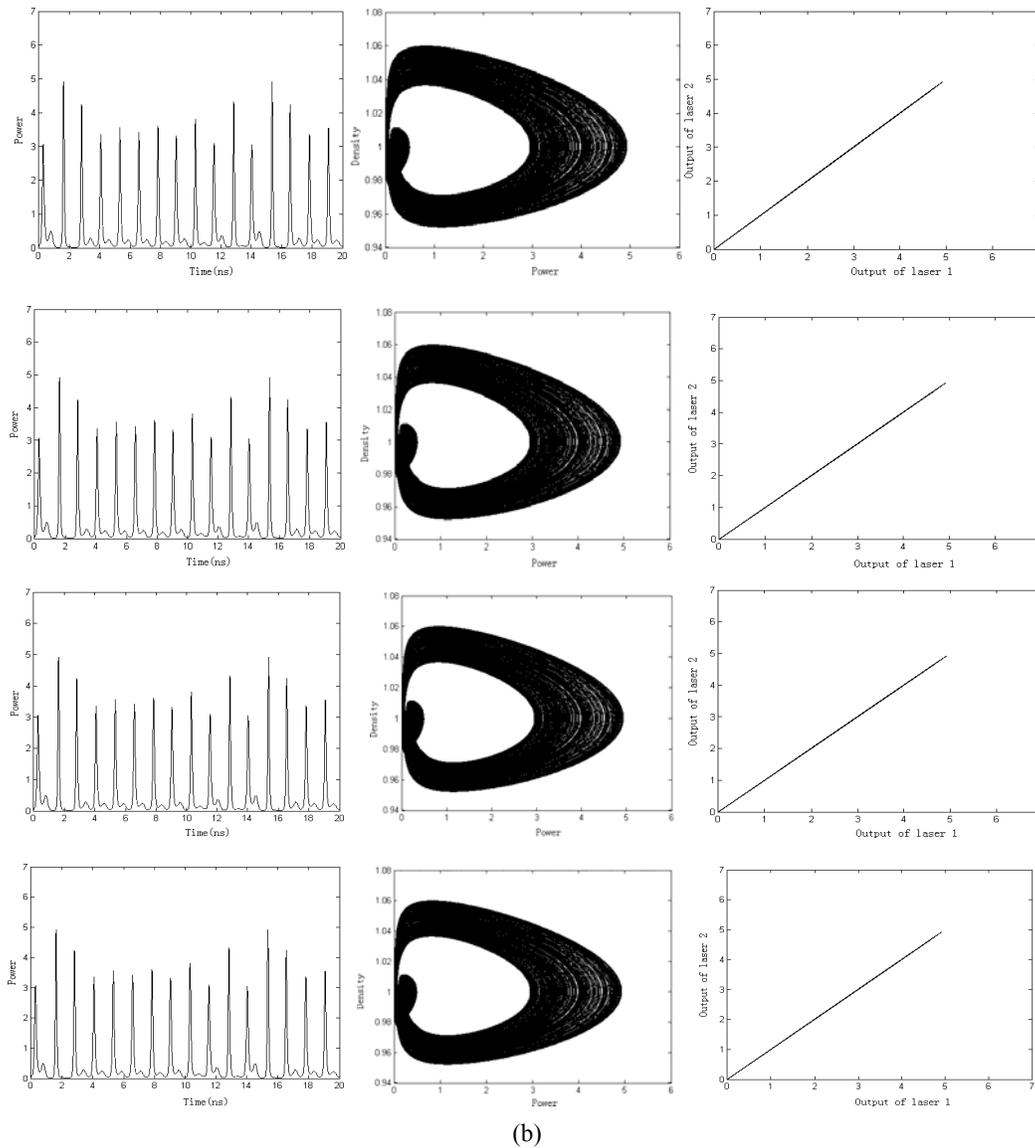
(a)

(b)
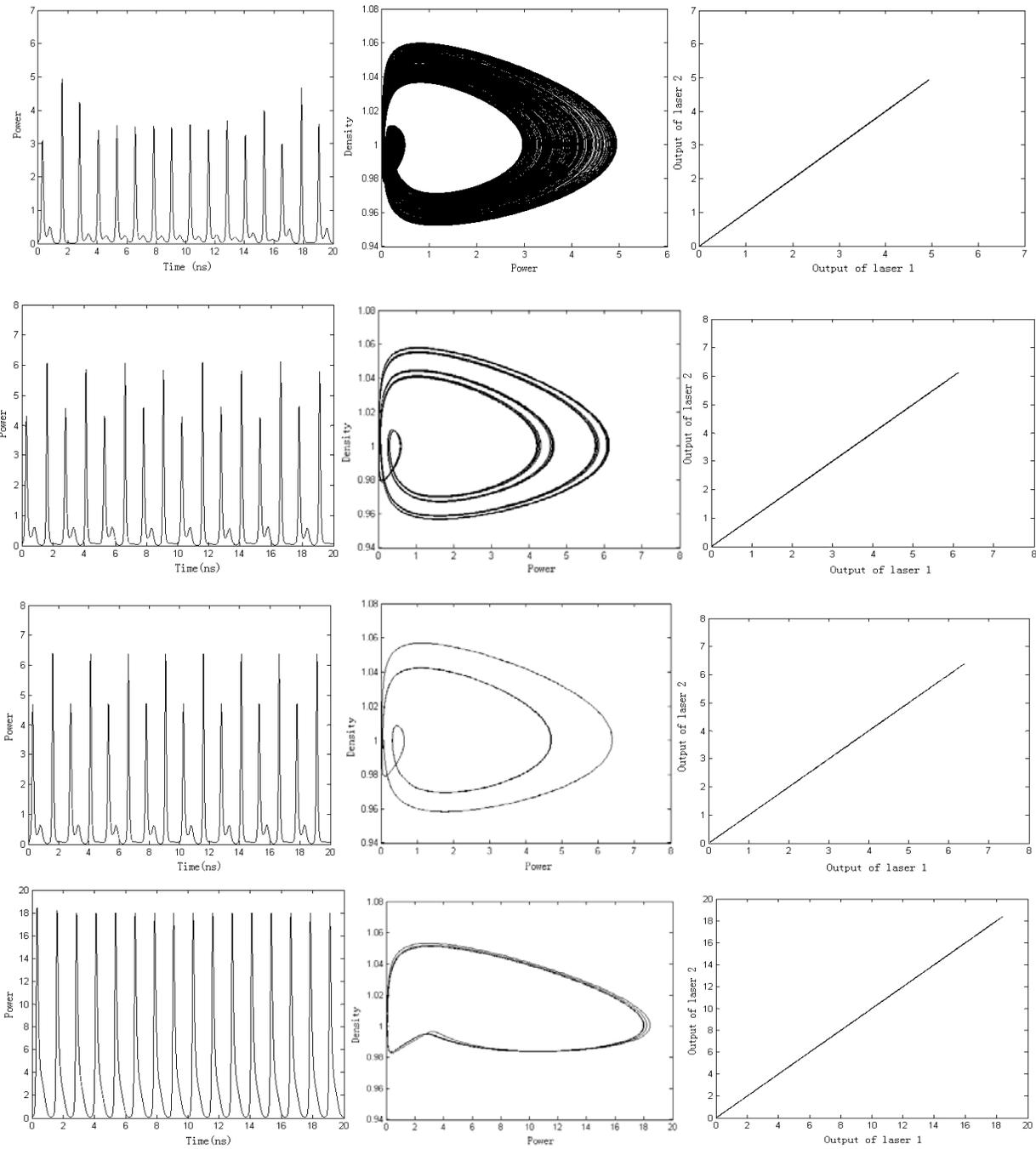
Fig. 2. The time series, phase and correlation diagram of sum coupled (a) and difference coupled (b) (k=0.05e-3,

3.5e-3,4e-3,11.2e-3).

In order to achieve chaos and chaos synchronization, the parameters of these two lasers are very important. Therefore, it is necessary to investigate the effects of parameter on the chaos and chaotic synchronization of these lasers in the two schemes, we concentrate on the dynamics for the different k and ε,the term governing nonlinear gain. Figure 3(a) shows the bifurcation diagram of the output dynamics versus k at ε=0.0001 in sum-coupling. For coupling values between 0.05e-3 and 12e-3, the bifurcation diagram shows the system undergoes chaotic and attains steady state(one cycle) through a sequence of reverse period doubling. It makes clear that, when the feedback fraction is at a low value of k=0.05e-3, even less than 2e-3, the bifurcation diagram shows chaotic. Meanwhile the bifurcation diagram namely

the output of the two lasers, undergoes a four-cycle, a reverse period doubling, and a one-cycle with further increase of k.The bifurcation diagrams for the different ε for four values of coupling constant k(=0.05e-3,2.5e-3, 3.3e-3,3.5e-3,4e-3) in sum-coupling are shown as Figure 3(b)-(f).For ε values from 0 to 0.04,the bifurcation diagrams shows that, the system shows chaotic for low values of ε≦0.0002,the results agree with those shown as Figure (a) in sum-coupling, and experiencing the multi-period, period-doubling in a increase of ε,finally steady state(one cycle) for high values for k=0.05e-3;As the further increase of k(=2.5e-3),the chaotic regions get small, and the systems show the multi-period,period-doubling,even the steady state with a low values of ε;For k(=3.3e-3),the chaos disappears

gradually, then the multi-period get smaller, the period-doubling and steady state(one-cycle) can adjust in a large scale;Obviousely the period-doubling and steady state with the different values of ε are shown as Figure 3(e) ;In Figure 3(f),as ε is increased further, the bifurcation diagram shows the single power,namly the system remains the steady state. This can imply that, the sum-coupled system reaches a good synchronization faster and has a larger range of good synchronization than the difference-coupled system; these results show that there are different ranges of coupling strengths and gain nonlinearity, which can provide us with different dynamical behaviors. Therefore, by choosing appropriate coupling and gain nonlinearity values, we can use the method of optoelectronic feedback and bidirectional couplings for chaos and chaotic synchronization, suppression of chaos.
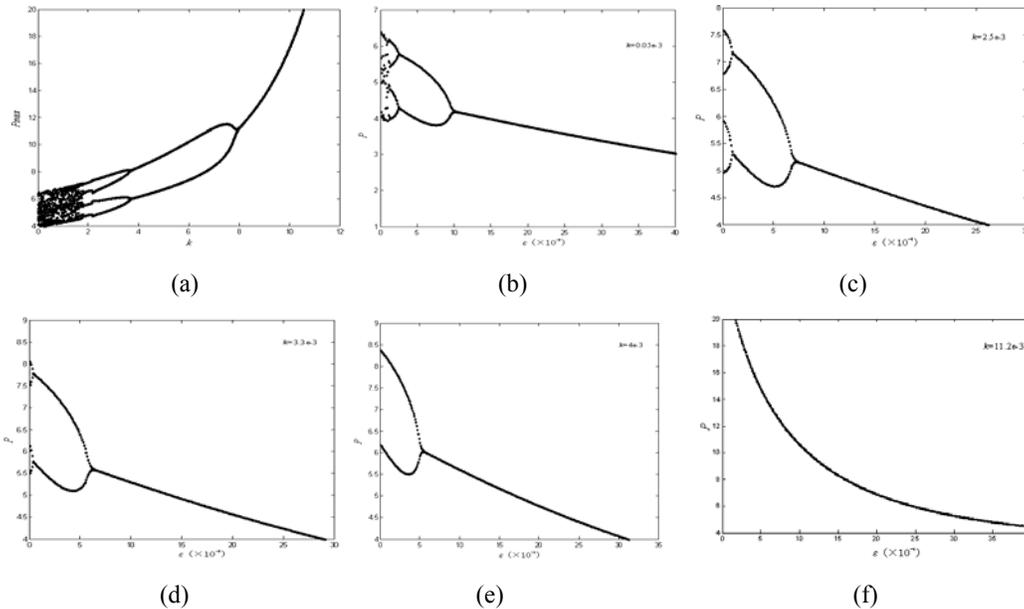

| (a) | (b) | (c) |
| (d) | (e) | (f) |

Fig. 3. Bifurcation diagrams of sum-coupled (a)ε=0.0001, (b) k=0.05e-3, (c)k=2.5e-3, (e) k=3.5e-3, (e) k=4e-3, (f)k=11.2e-3

4. Conclusions

In summary, two schemes of difference coupling and sum-coupling in which the semiconductor lasers subject to optoelectronic feedback for chaos and chaotic synchronization have been investigated. The results show that, chaos can be suppressed easily, two lasers have synchronization between the two outputs from laser system with the optoelectronic feedback and difference coupling or sum-coupling, and good synchronization quality and robustness has been observed in the configuration of two schemes. Our observations indicate that coupling constant k and nonlinear gain ε of SL may play an important role during the process of chaos and chaotic synchronization. We hope this work would offer a physical method to suppress and achieve chaos and chaotic synchronization for the SL based chaos cryptosystems. Also, this work may offer a useful insight to the nonlinear dynamics of SL system.

Acknowledgements

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