

Collective resonance fluorescence of extended systems of rare earth elements in the resonator standing wave in glass materials

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The collective resonance fluorescence of extended systems of rare earth elements in the standing wave of micro-spherical resonator made from doped glass material is discussed. The influence of regular arrangement of the *Dr* or *Sm* elements in interaction with the standing wave of the micro cavity on the fluorescence intensity is analyzed. The collective interference phenomena in the farther field detection region of fiber optics output field are studied. These effects depend on the exchange integrals between two level radiators through vacuum fluorescence field. The fluorescence spectrum as function of the position of two doped radiators in the standing wave resonator is obtained.

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1. Introduction

The processes of collective resonance fluorescence of extended systems of atoms, the formation of two photons as wave packets and precise measurements of atomic data are of great interest to throughout many fields of science [1-5]. The latest studies in these fields result in a deeper understanding of the main principles of quantum mechanics of extended systems. Lifetime measurements are of particular importance to the interpretation of measurements of atomic parity nonconservation, tests of QED and atomic structure theory [7].

In the last time the precision measurements of the excited state lifetime of the $5p^2 P_{1/2}$ and $5p^2 P_{3/2}$ levels of a single and two trapped Cd^+ ions are reported [8-9]. Combining ions trap and ultra-fast laser technologies, the ions are excited with pico-second laser pulses from a mode-locked laser and the distribution of arrival times of spontaneously emitted photons are recorded.

The same effects can take place in doped materials (glasses or crystals) with rare earth elements *Dr* or *Sm*. In this case an interesting behavior is obtained in the studying of resonance fluorescence of natural absorption or emission spectral lines in such materials. Some collective behavior of rare earth impurities already was observed in doped materials, see for example [10].

The aim of this paper is to study the collective phenomena in the presence of dressed laser field in resonance with one active line of rare earth impurities. As the impurities are situated in the crystalline matrix an interesting effect can appear in the standing wave formed

in the process of reflection of laser light by the mirrors of native material. The collective resonance fluorescence of extended systems of rare earth elements in the standing wave of micro-spherical resonator made from doped glass material is discussed. The influence of regular arrangement of the *Dr* or *Sm* elements in interaction with the standing wave of the micro cavity on the fluorescence intensity is analyzed. The collective interference phenomena in the farther field detection region of fiber optics output field are studied. These effects depend on the exchange integrals between two level radiators through vacuum fluorescence field. The interference of fluorescence field as function of the position of two doped radiators in the standing wave resonator is obtained. A system of equations that describes this cooperative effect is obtained.

2. The interaction of rare earth elements with electromagnetic field vacuum

Let us consider the system of radiators in interaction with coherent field. Considering modern experiments with cold atomic systems [8-9, 11-12, see Fig.1] we can observe two possibilities of interaction of atomic systems with strong coherent field. The first possibility corresponds to the situation for which the atoms are situated in the travel wave field (see Fig. 2). This resonance fluorescence was studied in the paper [6]. In the experiments with cold atomic systems the great interest represent the situation, when the atoms are situated in the standing wave field, see Fig. 3.

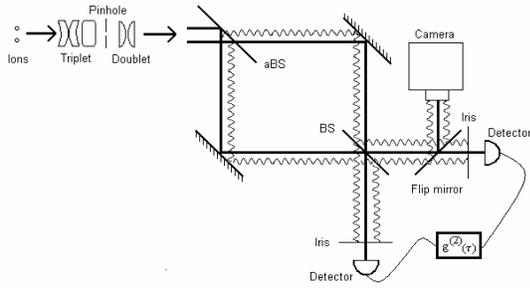


Fig. 1. The correlation detection scheme proposed in the paper [3].

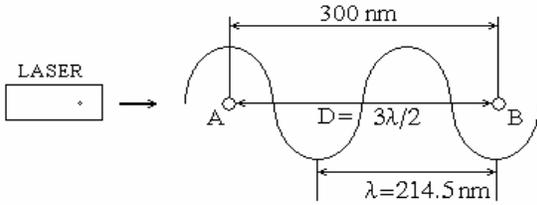


Fig. 2. The scheme of interaction of the atoms situated at the distance \$D\$ in the laser field.

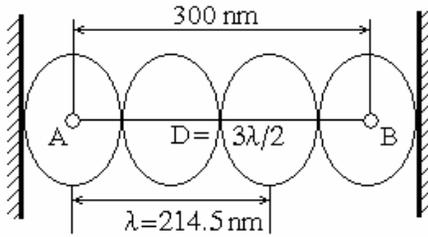


Fig. 3. The interaction of the rare earth elements situated at the distance \$3\lambda/2\$ in the resonator standing wave.

The process of studying the interaction of the system of \$N\$-two level hydrogen-like atoms with dressed coherent field in extended system proposed in the paper [6] can be generalized for this case too. Indeed, the Hamiltonian

$$H = \sum_{j=1}^N \hbar(\omega_0 - \omega) R_{j-} + \sum_k \hbar(\omega_k - \omega) a_k^+ a_k + i \frac{\hbar}{2} \sum_{j=1}^N \left\{ R_j^+ \tilde{\Omega}_j \exp[i(\mathbf{k}_0, \mathbf{r}_j)] - h.c. \right\} + i \frac{\omega_0 d}{c} \sum_{k,j=1}^N g_k \left\{ a_k R_j^+ \exp[i(\mathbf{k}, \mathbf{r}_j)] - h.c. \right\} \quad (1)$$

contains the two interaction parts with electromagnetic field. The first part

$$H_{i1} = i \frac{\hbar}{2} \sum_{j=1}^N \left\{ R_j^+ \tilde{\Omega}_j \exp[i(\mathbf{k}_0, \mathbf{r}_j)] - h.c. \right\} \quad (2)$$

describe the resonance interaction of atomic ensemble with classical coherent field in which Rabi frequency \$\tilde{\Omega}_j\$

can be represent in the following form:

$$\tilde{\Omega}_j = (\mathbf{d}, \mathbf{E}_0) [1 + \lambda \exp(-2i\mathbf{k}_0, \mathbf{r}_j)] / \hbar, \quad (3)$$

which contains the situation when the atomic system is placed in standing wave (\$\lambda = 1\$) and travel wave (\$\lambda = 0\$). In the case when we have travel coherent field this frequency doesn't depend on the position of the atom. The second interaction term

$$H_{i2} = i \frac{\omega_0 d}{c} \sum_k \sum_{j=1}^N g_k \left\{ a_k R_j^+ \exp[i(\mathbf{k}, \mathbf{r}_j)] - h.c. \right\} \quad (4)$$

describes the interaction of extended atomic system with electromagnetic field vacuum. The first and second terms in the Hamiltonian (1) describe the free atomic and electromagnetic parts of the Hamiltonian in the non-inertial system that is rotated with laser frequency \$\mathcal{Y}_0\$. In the Hamiltonian representation we have: \$\mathcal{Y}_0\$ is the frequency of the transition, \$R_j^{\pm}\$ (\$R_j^{\mp}\$) are the operators of creation (annihilation) of the excited state of \$j\$ atom, \$a_k^+\$ (\$a_k\$) are the radiation field creation (annihilation) operators with impulse \$k\$, energy \$\omega_k\$ and polarization \$\gamma\$ (\$\gamma = 1, 2\$), \$k_0\$ is the wave vector of external laser field, \$g_k = \sqrt{2\pi c^2 \hbar / V \omega_k}\$, \$V\$ is the quantization volume, \$\mathbf{n}_d = \mathbf{d} / d\$. These operators satisfy the commutation relations \$[R_i^+, R_j^-] = 2R_{zj} \delta_{ij}\$, \$[R_i^{\pm}, R_{zj}] = \mp R_i^{\pm} \delta_{ij}\$, \$[a_k, a_{k'}^+] = \delta_{kk'}\$, \$[a_k, a_{k'}] = [a_k^+, a_{k'}^+] = 0\$. Here \$\mathbf{E}_0\$ is the amplitude of laser field; \$d\$ is the dipole moment of the transition.

Introducing the dependence of Rabi frequency in the standing wave amplitude and the specially phase \$(\mathbf{k}_0, \mathbf{r}_j)\$, one can generalized the proposed method of diagonalization of the first two terms in the Hamiltonian (1) in according with paper [6]

$$U(\theta) = \exp \left[i \sum_{j=1}^N R_j^+ \theta_j \exp[i(\mathbf{k}_0, \mathbf{r}_j)] + h.c. \right], \quad (5)$$

where \$\theta_j = \frac{1}{2} \arctg[\tilde{\Omega}_j / (\omega_0 - \omega)]\$. Indeed, doing the diagonalization of free atomic parts of the Hamiltonian (1), one can obtain the dressed effective Hamiltonian of the system atoms and radiation field

$$\tilde{H} = \tilde{H}_0 + \tilde{H}_{i1} + \tilde{H}_{i2} + \tilde{H}_{i3}, \quad (6a)$$

where

$$\tilde{H}_0 = \sum_k \hbar(\omega_k - \omega) a_k^+ a_k + \hbar \sum_{j=1}^N \Omega_j D_{zj}, \quad (6b)$$

is free dressed part of the system and

$$\tilde{H}_{i1} = i \frac{\omega_0 d}{2c} \sum_k \sum_{j=1}^N g_k \{ a_k D_j^+(t) \Delta_{1j} \exp[i(\mathbf{k} - (1-\lambda)\mathbf{k}_0, \mathbf{r}_j)] - h.c. \} \tag{6c}$$

$$\tilde{H}_{i2} = i \frac{\omega_0 d}{2c} \sum_k \sum_{j=1}^N g_k \{ a_k D_j^-(t) \Delta_{2j} \exp[i(\mathbf{k} - (1-\lambda)\mathbf{k}_0, \mathbf{r}_j)] - h.c. \} \tag{6d}$$

$$\tilde{H}_{i3} = \frac{\omega_0 d}{c} \sum_k \sum_{j=1}^N g_k D_{zj}(t) \{ a_k \Delta_{3j} \exp[i(\mathbf{k} - (1-\lambda)\mathbf{k}_0, \mathbf{r}_j)] - h.c. \} \tag{6f}$$

is the interaction Hamiltonian in rotation wave approximation relatively Rabi frequency. The new dressed atomic operators D_j^{\square} , D_j^{\times} , D_{zj} are expressed through the old operators in the following way

$$\begin{aligned} D_j^+ &= \frac{\Delta_{2j}}{2} R_j^- e^{-i(1-\lambda)(\mathbf{k}_0, \mathbf{r}_j)} + \frac{\Delta_{1j}}{2} R_j^+ e^{i(1-\lambda)(\mathbf{k}_0, \mathbf{r}_j)} + i\Delta_{3j} R_{zj}, \\ D_j^- &= \frac{\Delta_{2j}}{2} R_j^+ e^{i(1-\lambda)(\mathbf{k}_0, \mathbf{r}_j)} + \frac{\Delta_{1j}}{2} R_j^- e^{-i(1-\lambda)(\mathbf{k}_0, \mathbf{r}_j)} - i\Delta_{3j} R_{zj}, \\ D_{zj} &= \frac{\delta}{\Omega_j} R_{zj} - i \frac{\Delta_{3j}}{2} \left\{ R_j^- e^{-i(1-\lambda)(\mathbf{k}_0, \mathbf{r}_j)} - h.c. \right\} \end{aligned} \tag{7}$$

Here we take into account both situations: travel wave case, $\lambda = 0$, for which the value of Rabi frequency $\Omega_j = \sqrt{\delta^2 + \Omega_0^2}$ doesn't depend on the position of the atoms, but the operators contain the faze position factors $\exp(\pm i k_0 r_j)$ and the standing wave, $\lambda = 1$, for which the counter propagate fazes of the waves in the resonator is contained in the Rabi frequency $\Omega_j = \sqrt{\delta^2 + 4\Omega_0^2 \cos^2(\mathbf{k}_0, \mathbf{r}_{0j})}$, $\delta = \omega_0 - \omega$ is the resonance detuning, $A_{1j} = 1 + (\delta / \Omega_j)$,

$A_{2j} = 1 - (\delta / \Omega_j)$, $A_{3j} = \tilde{\Omega}_j e^{i\lambda(\mathbf{k}_0, \mathbf{r}_j)} / \Omega_j$, D_j^{\square} , D_j^{\times} and D_{zj} are new quasi-spin operators connected with the processes of transition between quasi-levels of split state of j -th atom in the strong laser field. The Hamiltonians \tilde{H}_{i1} and \tilde{H}_{i2} describe the processes of collective generation of light with frequencies $\omega + \Omega_j$ and $\omega - \Omega_j$ in the external strong laser field. The light with frequencies $\omega + \Omega_j$, $\omega - \Omega_j$ is generated in the process of transition from quasi-level $\hbar\Omega/2$ to quasi-level $-\hbar\Omega/2$, and the light with frequency, ω is scattered by the atom without the transitions between new quasi-level (see Fig. 4).

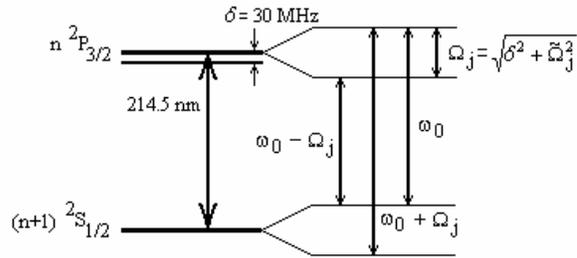


Fig. 4. The relevant energy levels of rare earth elements.

The Heisenberg equation for the mean value of atomic operator, $O(t)$, operator for the atoms A and B can be written

$$\begin{aligned} \frac{d\langle O(t) \rangle}{dt} &= \sum_{j=A,B} i\Omega_j \langle [D_{zj}(t), O(t)] \rangle \\ &+ \frac{i}{\hbar} \sum_{j=A,B} \sum_k g_k \left[\langle [\varphi_{21}^j(k) (\Delta_{1j} D_j^+(t)/2 + \Delta_{2j} D_j^-(t)/2 + \Delta_{zj} D_{zj}(t)) \right. \\ &\quad \left. \exp[-i(k_0, r_{0j})], O(t) \rangle a_k \right] + \langle a_k^+ [\varphi_{12}^j(k) (\Delta_{1j} D_j^-(t)/2 \\ &\quad \left. + \Delta_{2j} D_j^+(t)/2 + \Delta_{zj} D_{zj}(t)) \exp[-i(k_0, r_{0j})], O(t) \rangle \right] \end{aligned} \tag{8}$$

where $\varphi_{21}^j(k) = i \frac{\omega_0 d}{c} \exp[i(\mathbf{k}_0, \mathbf{r}_j)]$. Here the averages $\langle \dots \rangle$ take place at the initial state of the system. Using the solution of the Heisenberg equation for the electromagnetic field operators

$$\begin{aligned} a_k^+(t) &= a_k^+(0) \exp[i(\omega_k - \omega)t] + \sum_{l=A,B} \sum_k \frac{ig_k}{\hbar} \int_0^t dt' \exp[i(\omega_k - \omega)t'] \left[\varphi_{21}^j(k) \left(\frac{\Delta_{1l}}{2} D_l^+(t-\tau) \right. \right. \\ &\quad \left. \left. + \frac{\Delta_{2l}}{2} D_l^-(t-\tau) + \Delta_{zj} D_{zj}(t-\tau) \right) \exp[-i(k_0, r_{0j})] \right], \end{aligned} \tag{9}$$

$$a_k(t) = [a_k^+(t)]^+,$$

and substituting these expressions into (8), we can eliminate the free parts of Exp. (8). We take into account the Born-Marcoff approximation, and use the properties of electromagnetic field operators $a_k(0)|0\rangle = 0$ and $\langle 0|a_k^+(0) = 0$. Averaging over the initial state, one can obtain the following equation for atomic operator, $O(t)$.

$$\begin{aligned} \frac{d\langle O(t) \rangle}{dt} &= \sum_{j=A,B} i\Omega_j \langle [D_{zj}(t), O(t)] \rangle - \sum_{j,l=A,B} \sum_k \frac{g_k^2}{\hbar^2} \varphi_{21}^j(k) \varphi_{12}^l(k) \exp[-i(k_0, r_{0j})] \\ &\times \left[\frac{\Delta_{1j} \Delta_{1l}}{4} \left(\frac{iP}{\omega_k - (\omega + \Omega_j)} + \pi\delta[\omega_k - (\omega + \Omega_j)] \right) \langle [D_j^+(t), O(t)] D_l^-(t) \rangle \right. \\ &+ \frac{\Delta_{2j} \Delta_{2l}}{4} \left(\frac{iP}{\omega_k - (\omega - \Omega_j)} + \pi\delta[\omega_k - (\omega - \Omega_j)] \right) \langle [D_j^-(t), O(t)] D_l^+(t) \rangle \\ &\left. + \Delta_{3j} \Delta_{3l} \left(\frac{iP}{\omega_k - \omega} + \pi\delta[\omega_k - \omega] \right) \langle [D_{zj}(t), O(t)] D_{zl}(t) \rangle \right] + h.c. \end{aligned} \tag{10}$$

From generalized Eq. (10) we can obtain the equation for inversion $d\langle D_{zA}(t-\tau) \rangle / dt$ of the j atom.

$$\begin{aligned} \frac{d\langle D_{zA}(t) \rangle}{dt} = & -\frac{\pi}{2} \sum_k \frac{g_k^2}{\hbar^2} \varphi_{21}^A(k) \varphi_{12}^{+A}(k) [-\Delta_{1A}^2 \delta[\omega_k - (\omega + \Omega_A)] \langle D_{zA}(t) \rangle + 1/2] \\ & + \Delta_{2A}^2 \delta[\omega_k - (\omega - \Omega_A)] [1/2 - \langle D_{zA}(t) \rangle] \\ & - \pi \sum_k \frac{g_k^2}{\hbar^2} \varphi_{21}^A(k) \varphi_{12}^{+B}(k) [-\Delta_{1A} \Delta_{1B} \delta[\omega_k - (\omega + \Omega_B)] \\ & + \Delta_{2A} \Delta_{2B} \delta[\omega_k - (\omega - \Omega_B)]] \langle D_A^+(t) D_B^-(t) \rangle \end{aligned} \quad (11a)$$

In the similar way the expressions for correlators $\langle D_A^+(t) D_B^-(t) \rangle$, $\langle D_{zA}(t) D_{zB}(t) \rangle$ can be obtained from Eq. (10) where $j, l = A, B$, $j \neq l$.

$$\begin{aligned} \frac{d\langle D_A^+(t) D_B^-(t) \rangle}{dt} = & -\sum_k \frac{g_k^2}{\hbar^2} [\pi \varphi_{21}^A(k) \varphi_{12}^{+B}(k) \{ \Delta_{1A} \Delta_{1B} \delta[\omega_k - (\omega + \Omega_A)] \\ & + \Delta_{2A} \Delta_{2B} \delta[\omega_k - (\omega - \Omega_B)] \} \langle D_{zA}(t) D_{zB}(t) \rangle \\ & + \left[\frac{\Delta_{1A} \Delta_{1B}}{2} \delta[\omega_k - (\omega + \Omega_A)] - \frac{\Delta_{2A} \Delta_{2B}}{2} \delta[\omega_k - (\omega - \Omega_B)] \right] \langle D_{zA}(t) \rangle \\ & + 2\Delta_{3A} \Delta_{3B} \delta[\omega_k - \omega] \langle D_A^+(t) D_B^-(t) \rangle - 2\pi \varphi_{21}^A(k) \varphi_{12}^{+A}(k) \left[\frac{\Delta_{2A}^2}{2} \delta[\omega_k - (\omega - \Omega_A)] \right. \\ & \left. + \frac{\Delta_{1B}^2}{2} \delta[\omega_k - (\omega + \Omega_B)] + \left(\frac{\Delta_{3A}^2}{2} + \frac{\Delta_{3B}^2}{2} \right) \delta[\omega_k - \omega] \right] \langle D_A^+(t) D_B^-(t) \rangle] \end{aligned} \quad (11b)$$

$$\begin{aligned} \frac{d\langle D_{zA}(t) D_{zB}(t) \rangle}{dt} = & 2\pi \sum_k \frac{g_k^2}{\hbar^2} \varphi_{21}^A(k) \varphi_{12}^{+A}(k) \left[\left(\frac{\Delta_{1A}^2}{2} \delta[\omega_k - (\omega + \Omega_A)] \right. \right. \\ & \left. \left. + \frac{\Delta_{1B}^2}{2} \delta[\omega_k - (\omega + \Omega_B)] + \frac{\Delta_{2A}^2}{2} \delta[\omega_k - (\omega - \Omega_A)] + \frac{\Delta_{2B}^2}{2} \delta[\omega_k - (\omega - \Omega_B)] \right) \langle D_{zA}(t) D_{zB}(t) \rangle \right. \\ & \left. + \left(\frac{\Delta_{1A}^2}{2} \delta[\omega_k - (\omega + \Omega_A)] + \frac{\Delta_{1B}^2}{2} \delta[\omega_k - (\omega + \Omega_B)] - \frac{\Delta_{2A}^2}{2} \delta[\omega_k - (\omega - \Omega_A)] - \frac{\Delta_{2B}^2}{2} \delta[\omega_k - (\omega - \Omega_B)] \right) \langle D_{zA}(t) \rangle \right] \\ & - \pi \sum_k \frac{g_k^2}{\hbar^2} \varphi_{21}^A(k) \varphi_{12}^{+B}(k) \left[\frac{\Delta_{1A} \Delta_{1B}}{2} (\delta[\omega_k - (\omega + \Omega_A)] + \delta[\omega_k - (\omega + \Omega_B)]) + \right. \\ & \left. + \frac{\Delta_{2A} \Delta_{2B}}{2} (\delta[\omega_k - (\omega - \Omega_A)] + \delta[\omega_k - (\omega - \Omega_B)]) \right] \langle D_A^+(t) D_B^-(t) \rangle \end{aligned} \quad (11c)$$

Using the notation $X(t) = \langle D_{zj}(t) \rangle$, $Y(t) = \langle D_A^+(t) D_B^-(t) \rangle$, $Z(t) = \langle D_{zA}(t) D_{zB}(t) \rangle$, and integrating the right-hand sides of Eqs. (11) over \mathbf{k} , λ , one can obtain the following system of equations:

$$\frac{dX(t)}{dt} = \frac{\Delta_{1A}^2}{4} J_{AA}^a [X(t) + 1/2] - \frac{\Delta_{2A}^2}{4} J_{AA}^s [1/2 - X(t)] + (\Delta_{1A} \Delta_{1B} J_{AB}^a - \Delta_{2A} \Delta_{2B} J_{AB}^s) Y(t) \quad (12a)$$

$$\begin{aligned} \frac{dY(t)}{dt} = & \left(\frac{\Delta_{2A}^2}{4} J_{AA}^s + \frac{\Delta_{1B}^2}{4} J_{AA}^a + \left(\frac{\Delta_{3A}^2}{2} + \frac{\Delta_{3B}^2}{2} \right) J_{AA}^e \right) Y(t) - (\Delta_{1A} \Delta_{1B} J_{AB}^a + \Delta_{2A} \Delta_{2B} J_{AB}^s) Z(t) \\ & + \left(\frac{\Delta_{2A} \Delta_{2B}}{2} J_{AB}^s - \frac{\Delta_{1A} \Delta_{1B}}{2} J_{AB}^a \right) X(t) - 2\Delta_{3A} \Delta_{3B} J_{AB}^e Y(t) \end{aligned} \quad (12b)$$

$$\begin{aligned} \frac{dZ(t)}{dt} = & \left(\frac{\Delta_{1A}^2}{4} J_{AA}^a + \frac{\Delta_{1B}^2}{4} J_{AA}^s + \frac{\Delta_{2A}^2}{4} J_{AA}^s + \frac{\Delta_{2B}^2}{4} J_{AA}^a \right) Z(t) \\ & + \left(\frac{\Delta_{1A}^2}{4} J_{AA}^a + \frac{\Delta_{1B}^2}{4} J_{AA}^s - \frac{\Delta_{2A}^2}{2^3} J_{AA}^s - \frac{\Delta_{2B}^2}{2^3} J_{AA}^a \right) X(t) \\ & - \left(\frac{\Delta_{1A} \Delta_{1B}}{4} (J_{AB}^a + J_{AB}^s) + \frac{\Delta_{2A} \Delta_{2B}}{2} (J_{AB}^s + J_{AB}^a) \right) Y(t). \end{aligned} \quad (12c)$$

Here we have noted

$$J_{AA}^a = 2\pi \sum_k \frac{g_k}{\hbar^2} \varphi_{21}^A(k) \varphi_{12}^{+A}(k) \delta[\omega_k - \omega_{\alpha, \beta}] = \frac{4}{3} \frac{\omega_0^2 \omega_{\alpha} d^2}{\hbar c^3} \quad (13)$$

$$\begin{aligned} J_{AB}^a = & \pi \sum_k \frac{g_k}{\hbar^2} \varphi_{21}^A(k) \varphi_{12}^{+B}(k) \delta[\omega_k - \omega_{\alpha}] \\ = & \frac{\omega_0^2 \omega_{\alpha} d^2}{\hbar c^3} \left\{ 1 + \cos^2 \xi \frac{\partial^2}{\partial u^2} - \left(1 + \frac{\partial^2}{\partial u^2} \right) \frac{\sin^2 \xi}{2} \right\} \frac{\sin u}{u} \quad u = \omega_{\alpha} D/c \end{aligned} \quad (14)$$

where $\omega_{\alpha} = \{\omega, \omega - \Omega_{A,B}, \omega + \Omega_{A,B}\}$. For $\xi, \beta = s$ we have the fluorescent generation at Stokes frequency $\omega_s = \omega - \Omega_{A,B}$, for $\alpha, \beta = a$ the frequency is $\omega_a = \omega + \Omega_{A,B}$, and for $\alpha, \beta = e$ we have spontaneous scattering effect without changes of frequency. The coefficient J_{AA}^a represents the decay rate at the frequency ω_{α} and the coefficient J_{AB}^a represents the cooperative decay rate between the atoms and describes the mutual influence of the spontaneous emission light by the atoms in the process of resonance fluorescence. The solution of the system of equations (12) describes the collective resonant spontaneous emission of two impurities situated at the distance D in the dressed standing wave (see Fig. 5).

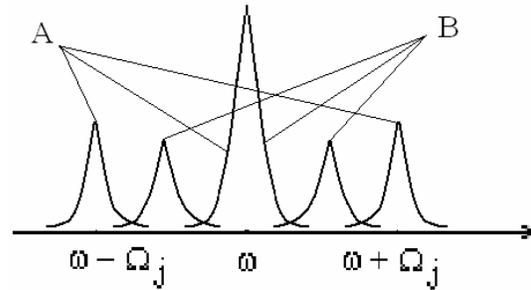


Fig. 5. The spectrum of resonance fluorescence for two atoms situated in the different regions of the standing wave.

3. Conclusions

From the proposed approach it follows that, in collective resonant spontaneous emission of two atoms the sign of collective decay rate (14) strongly depend on the distance between the atoms. For small distance $D \ll \lambda$, $J_{AB}^a = J_{AA}^a$ and for a large distance $D > \lambda$, J_{AB}^a tends to $J_{AA}^a \frac{\sin k_{\alpha} D}{k_{\alpha} D}$. In this paper the cooperative spontaneous emission of two rare earth elements situated at the distance D in the standing wave of the Laser field was studied. The closed system of equations that describes the collective resonant spontaneous emission of two impurities in the

dressed standing wave is obtained. From the symmetrical point of view the mutual influences of the atoms it is studied.

Another interesting effect is the dependence of Rabi frequency on the atomic position in the standing wave. If radiator "A" is situated in oscillating loop, the Rabi frequency $\tilde{\Omega}_j$ takes the maximal value; in the case when radiator "A" is situated in node of wave, the Rabi frequency takes minimal value. That is connected with the behavior of the function $4(\vec{d}, \vec{E}_0)\cos(r_j, k_0)/\hbar$; if $r_j k_0 = n\pi$ the value of $|\tilde{\Omega}_j|$ is maximal; if $r_j k_0 = (n+1/2)\pi$, the Rabi frequency $|\tilde{\Omega}_j|$ takes zero value.

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