

# Compressed sensing approach for UWB-OFDM SAR imaging using greedy algorithms

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Compressed sensing (CS) approach has received significant interest in remote sensing applications recently. Only few attempts however have been reported on CS based UWB-OFDM SAR imaging. This paper presents a novel scheme for high-resolution SAR based on UWB-OFDM system using CS techniques. Enhanced resolution is obtained using UWB-OFDM waveforms and the computational burden is reduced using the CS methods. Sparse imaging methods are developed to deal with UWB-OFDM SAR using greedy algorithms including orthogonal matching pursuit (OMP), regularized OMP (ROMP) and compressive sampling matching pursuit (CoSaMP). Performance of these algorithms is analyzed and compared to conventional imaging techniques based on simulated results.

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**Keywords:** Compressed Sensing (CS); Synthetic Aperture Radar (SAR); Ultra-wideband (UWB); Orthogonal Frequency Division Multiplexing (OFDM); Orthogonal Matching Pursuit (OMP), Regularized OMP (ROMP); Compressive Sampling Matching Pursuit (CoSaMP)

## 1. Introduction

Synthetic aperture radar (SAR) provides an effective technique for high-resolution imaging. In SAR pulses are transmitted at spaced intervals called pulse repetition interval (PRI), and the reflections at each PRI are processed to reconstruct SAR image of the terrain [1]. Range resolution of SAR images can highly be increased by exploiting ultra-wideband UWB waveforms that exceeds 500 MHz in bandwidth as radar pulse [2]. In addition to enhancing resolution, UWB techniques provide good capacity of penetration. Another approach for enhancing SAR imaging depends on exploiting OFDM techniques. An OFDM signal includes several orthogonal sub-carriers transmitted over a single transmission path. Each subcarrier occupies a small part of the total signal bandwidth [3]. OFDM can be used to implement SAR imaging in hostile environment. OFDM is used to increase the swath of SAR imaging and to obtain signal diversity to enhance signal to noise ratio. Feasibility study of OFDM based SAR has been investigated recently [4-9].

UWB-OFDM SAR thus has potential of enhancing various SAR applications. Large amount of echo samples, however, must be collected and processed, which leads to an excessive burden on the conventional matched-filter (MF) based SAR imaging processor. Compressed sensing technique is adopted to reconstruct the original signal using limited measurements beyond the Nyquist sampling constraints. CS thus reduces the computational complexity and improves reconstruction results as compared to traditional MF based SAR imaging systems. Moreover, super-resolution can be achieved by employing the sparsity of the target based on CS theory.

CS techniques have been frequently used in many applications due to its compressed sampling and exact reconstruction capability. In radar applications, CS reduces the computational complexity and enhances the resolution of the radar system. Compressive radar imaging is addressed in [10]. CS based random frequency SAR imaging is introduced in [11]. An imaging algorithm for high-resolution space-borne SAR using CS on azimuth displacement phase center antenna (DPCA) is described in [12]. This article opens new prospects for UWB-OFDM SAR waveforms by focusing on the OFDM signal, illustrating how it fits in a natural way with CS based greedy algorithms as processing tools.

The structure of the paper is as follows: UWB-OFDM pulse-shaping is described in section 2. Compressive UWB-OFDM SAR imaging and analysis of greedy algorithms are presented in section 3. Comparison of greedy algorithms based on OFDM signal is analyzed in section 4. UWB-OFDM SAR imaging results based on MF and CS method presented in section 5. Conclusions are given in section 6.

## 2. UWB-OFDM signal generation

UWB-OFDM signal is generated according to the scheme of randomly populating the digital frequency domain vector as:

$$\Psi_{\omega} = [\Pi_{ns} \Pi_0 \Pi_{ps}] \quad (1)$$

where,  $\Pi_{ps}$  and  $\Pi_{ns}$  represent the positive and negative sub-carriers respectively whereas  $\Pi_0$  represents the baseband DC value. Inverse Fast Fourier Transform (IFFT) is applied to  $\Psi_{\omega}$  to get the discrete time domain OFDM signal as:

$$\Psi_{tx}(t) = F^{-1}[\Psi_{\omega}] \quad (2)$$

In the analysis of this paper, UWB-OFDM waveforms are generated using the parameters as follows: number of sub-carriers = 256, sampling period,  $\Delta t_s = 1$  ns results in baseband bandwidth,  $B_0 = 1/2\Delta t_s = 500$  MHz.

### 3. Compressive SAR Imaging based on UWB-OFDM

In some special applications (such as ocean ship monitoring, aircraft detecting and tumor detection in human head), the number of dominant scatterers is much smaller than the number of overall samples. In these cases, SAR echoes are considered as sparse signal. Thus, sparse reconstruction can be used in such applications. High-resolution imaging with SAR can be achieved by using UWB-OFDM waveforms. However, due to higher sampling speed of UWB scale, it leads to extremely high data rate on SAR imaging processor. Sparse signal recovery using CS theory is used, by which only a small amount of radar echoes are used for SAR imaging. The raw echo signal can be processed for SAR imaging with high probability by using greedy algorithms such as OMP, ROMP and CoSaMP. Thus, sample size of SAR echoes can be reduced considerably by CS method.

#### A. Orthogonal Matching Pursuit (OMP)

OMP is an iterative greedy algorithm that selects the column which is most correlated with the current residuals at each step [13-15]. For a given measurement matrix  $\Psi \in \mathbb{R}^{m \times n}$  ( $n > m$ ), the CS recovery algorithm generates an estimate of  $K$ -sparse vector  $x \in \mathbb{R}^n$  from a set of linear measurements given as

$$y = \Psi x + \varepsilon \quad (3)$$

where,  $y$  is the measurement vector,  $\Psi$  denotes the measurement matrix,  $x$  is the original signal to be recovered and  $\varepsilon$  denotes the amount of noise. Due to the prior information of signal sparsity,  $x$  can be perfectly recovered using efficient recovery algorithm. Among many recovery algorithms in the literature, greedy methods receive significant attention for practical benefits. OMP algorithm has received significant interest because of its simplicity and efficient recovery performance. It has also been shown that OMP is reliable for reconstructing both sparse and near-sparse signals [16]. OMP estimates the sparse signal as

$$\hat{x} = \arg \min_x \|y - \Psi x\|_2 \quad (4)$$

A widely used framework for OMP based sparse signal recovery is the *Mutual Incoherence Property* (MIP) and is defined by

$$\mu = \max_{i \neq j} |\langle \Psi_i, \Psi_j \rangle| \quad (5)$$

where,  $\Psi_i$  and  $\Psi_j$  denote the  $i^{\text{th}}$  and  $j^{\text{th}}$  column of  $\Psi$  respectively.

#### B. Regularized Orthogonal Matching Pursuit (ROMP)

ROMP provides the strong guarantees of the optimization method and faster processing. ROMP is a greedy algorithm, but correctly recovers any sparse signal using any measurement matrix that satisfies the Restricted Isometry Property (RIP).

Similar to OMP, the observation vector,  $y = \Psi * \Psi x$  is used as a good local approximation to the sparse signal  $x$ . Since the RIP guarantees that every  $s$  columns of  $\Psi$  are close to an orthonormal system, it doesn't not choose just one coordinate as in OMP, but up to  $s$  coordinates at each iteration using the observation vector. A *regularization* step is included which will guarantee that each coordinate selected contains an even share of the information about the signal. This allows us to translate captured energy of the signal into captured support of the signal.

#### C. Compressive Sampling Matching Pursuit (CoSaMP)

ROMP bridges a critical gap between the major approaches in compressed sensing. It provides the speed of the greedy approach and the strong guarantees of the convex optimization approach. However, the requirements imposed by ROMP on the RIP were slightly stronger than the convex optimization. ROMP provides weakened error bounds in case of noisy signals and measurements. These issues were resolved by Compressive Sampling Matching Pursuit (CoSaMP). It provides both uniform guarantees as well as fast runtime, while improving upon the error bounds and restricted isometry requirements of ROMP. Unlike some other greedy algorithms, CoSaMP selects many components at each iteration.

The advantage of using OFDM signal in greedy algorithm based SAR imaging is that it ensures the Mutual Incoherence Property (MIP) to be small enough because the SAR system transmits a unique pulse at each PRI by using random sub-carrier composition. Moreover, it ensures lower cross-correlation among the pulses. The procedure for CS based SAR image reconstruction used in this article is as follows:

- Generate SAR raw echoes considering point targets at each cell of the target area.
- Select randomly a small amount of raw echoes to create measurement matrix ( $\Psi$ ).
- Create sparse signal ( $x$ ) and compute observation vector ( $y$ ).
- Recover the signal using chosen algorithm ( $\hat{x}$ ).
- 2D SAR imaging of sparse targets.

### 4. Comparison of Greedy Algorithms based on UWB-OFDM Signal

The relationship between the sparsity and the number of measurements is investigated in this section. In the simulation, for each trial we generate binary signals, as well as an independent Gaussian measurement matrix, considering different sparsity levels and various number of

measurements. Then, we apply OMP on the measurements of that signal and count the number of times the signal is recovered correctly out of 250 trials. Figure 1 shows the relationship between the number of measurements and the sparsity level to guarantee that correct recovery occurs for 99% of the time.

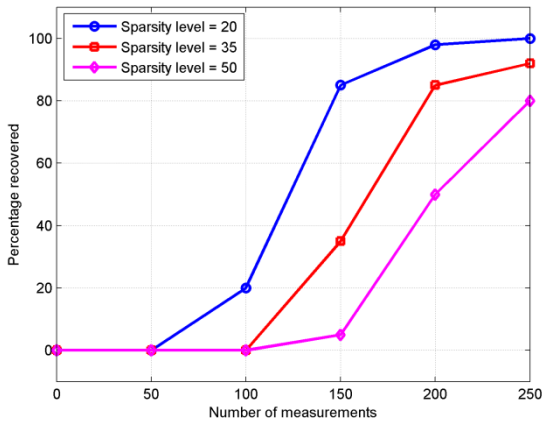


Fig. 1. Sparse signal recovery using OMP as a function of the number of measurements for different sparsity levels.

Figure 2 depicts the number of iterations executed by CoSaMP under the same setting described above for sparse signals. The number of iterations in each scenario is averaged over the 250 trials, and the results are plotted against the sparsity of the signal.

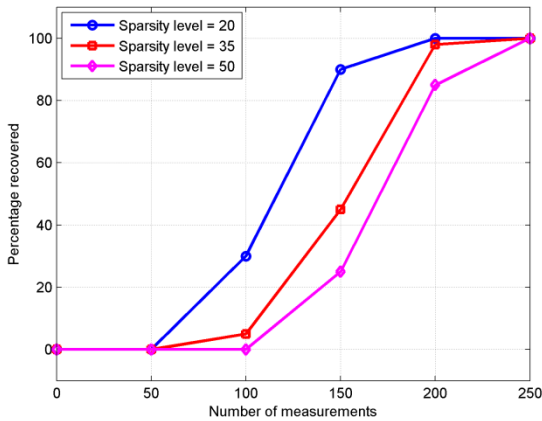


Fig. 2. Sparse signals recovery using CoSaMP as a function of the number of measurements for different sparsity levels.

Fig. 3 depicts the percentage of sparse signals reconstructed exactly using ROMP. The horizontal axis represents the number of measurements and the vertical axis represents the exact recovery percentage.

The plot demonstrates that often far less iteration is actually needed in some cases. It is observed that the ROMP provides 100% recovery with minimum number of iterations among the three algorithms.

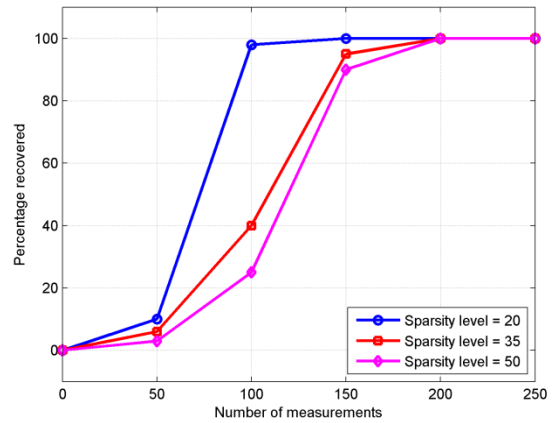


Fig. 3. Sparse signal recovery using ROMP as a function of the number of measurements for different sparsity levels.

### 5. Compressed Sensing based SAR imaging

The scenario involves the UWB-OFDM SAR imaging of target profile with three targets. The aim is to verify the imaging capability of the orthogonal waveforms as SAR transmitted pulse. ROMP is used as it provides the minimum number of iterations to achieve 100% recovery. Let us consider the imaging of a space with three-target profile as shown in Fig. 4.

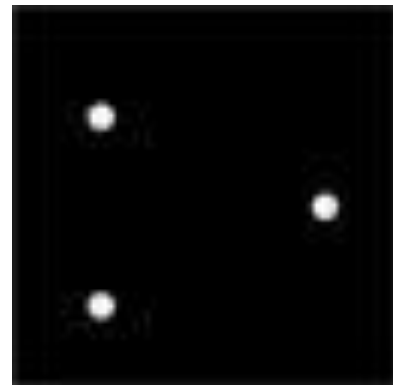
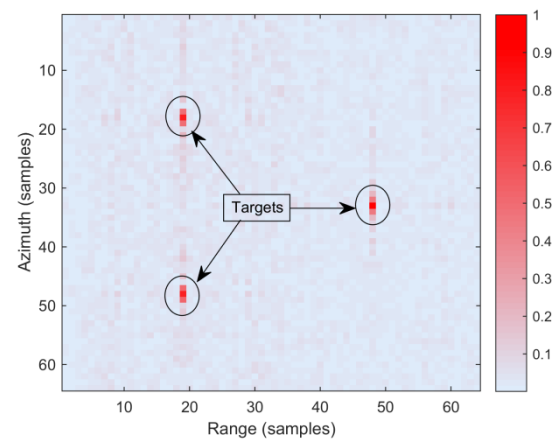
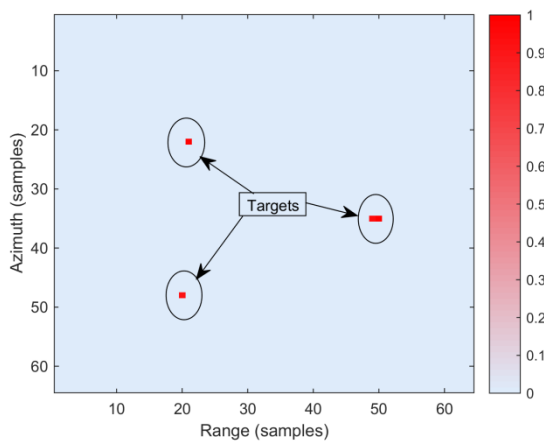


Fig.4. Space with three-target profile

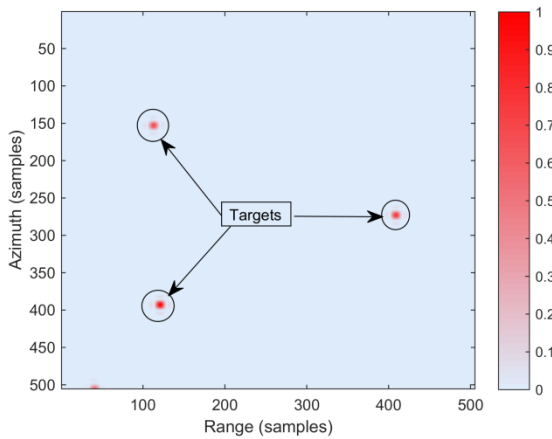
Stripmap SAR imaging topology is used for echo generation based on the proposed UWB-OFDM waveform as transmitted pulse while greedy algorithm is used for image reconstruction. The imaging results based on greedy algorithms are shown in Fig. 5. Figure 5(a) shows the reconstructed image using MF based RDA while Figure 5(b) shows the imaging results using ROMP based compressed sensing techniques. The CS based result shows that the position and scattering coefficients are clearly reconstructed without ambiguities as compared to MF based RDA. Figure 5(c) shows the image reconstruction using ROMP based CS approach with linear interpolation to achieve the actual size and shape of the targets.



(a)



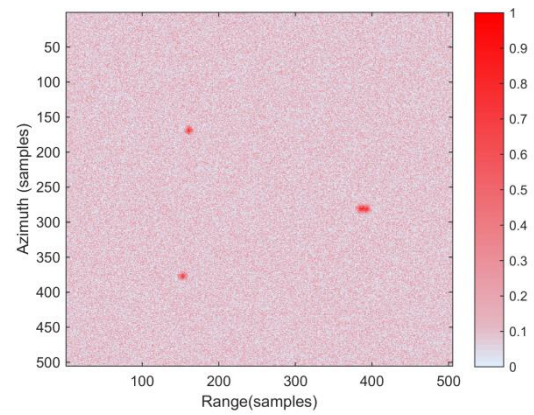
(b)



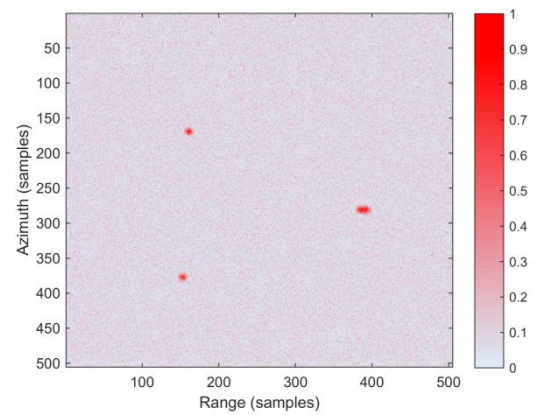
(c)

Fig. 5. Reconstructed image with three targets (a) using MF based RDA (b) using ROMP based CS technique. (c) using ROMP based CS with interpolation.

The performance of the CS method in the noisy environments depends on the number of samples used in the measurements. Figure 6 shows the effects of amount of echo samples used in CS method with  $SNR = 10$  dB. It is observed that the scattering centers are reconstructed with higher amplitudes when larger number of echo samples (50%) is used.



(a)



(b)

Fig. 6. CS based imaging with  $SNR = 10$  dB. using 25% echo samples (a) and 50% of echo samples (b).

## 6. Conclusions

UWB-OFDM SAR imaging has been investigated with different CS techniques. Various greedy algorithms are used to reduce the processing complexity of raw SAR data. It is shown that the proposed UWB-OFDM SAR indeed provides a potential solution to high-resolution remote sensing. Simulated results has been demonstrated and analyzed to show the effectiveness of the proposed method.

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