

# Computing topological indices of certain networks

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A topological index is actually designed by transforming a chemical structure into a number. These topological indices associate certain physico-chemical properties like boiling point, stability, strain energy etc of chemical compounds. Graph theory has found a considerable use in this area of research. The topological properties of certain networks are studied recently. In this paper, we extend this study to interconnection networks and derive analytical closed results of general Randić  $c'$  index  $R_\alpha(G)$  for different values of  $\alpha$  for octagonal network, toroidal polyhex and generalized prism. We also compute first Zagreb,  $ABC$ , and  $GA$  indices for these important classes of networks.

(Received August 31, 2015; accepted September 29, 2016)

**Keywords:** General Randić  $c'$  index, Atom-bond connectivity (ABC) index, Geometric-arithmetic (GA) index, octagonal network  $O_n^m$ , Toroidal polyhex  $H_m^n$  and generalized prism  $P_n^m$ .

## 1. Introduction and basic definitions

*Cheminformatics* is new subject which is a combination of chemistry, mathematics and information science. It studies Quantitative structure-activity (QSAR) and structure-property (QSPR) relationships that are used to predict the biological activities and properties of chemical compounds. In the QSAR /QSPR study, physico-chemical properties and topological indices such as Wiener index, Szeged index, Randić  $c'$  index, Zagreb index and  $ABC$  index are used to predict bioactivity of the chemical compounds.

A *topological index* is a function “*Top*” from “ $\Sigma$ ” to the set of real numbers, where “ $\Sigma$ ” is the set of finite simple graphs with property that  $Top(G) = Top(H)$  if both  $G$  and  $H$  are isomorphic. There is a lot of research which has been done on topological indices of different graph families so far, and is of much importance due to their chemical significance. A topological index is actually a numeric quantity associated with chemical constitution purporting for correlation of chemical structure with many physico-chemical properties, chemical reactivity or you can say that biological activity. Actually topological indices are designed on the ground of transformation of a molecular graph into a number which characterize the topology of that graph.

*Multiprocessor interconnection networks* are often required to connect thousands of homogeneously replicated processor-memory pairs, each of which is called a processing node. Instead of using a shared memory, all synchronization and communication between processing nodes for program execution is often done via message passing. Design and use of multiprocessor interconnection

networks have recently drawn considerable attention due to the availability of inexpensive, powerful microprocessors and memory chips [8]. The octagonal, toroidal polyhex and generalized prism networks have been recognized as versatile interconnection networks for massively parallel computing. This is mainly due to the fact that these families of networks have topologies which reflect the communication pattern of a wide variety of natural problems. Toroidal polyhex networks have recently received a lot of attention for their better scalability to larger networks, as opposed to more complex networks such as hypercubes [10].

A graph  $G(V,E)$  with vertex set  $V$  and edge set  $E$  is connected, if there exist a connection between any pair of vertices in  $G$ . A *network* is simply a connected graph having no multiple edges and loops. A *chemical graph* is a graph whose vertices denote atoms and edges denote bonds between that atoms of any underlying chemical structure. The *degree* of a vertex is the number of vertices which are connected to that fixed vertex by the edges. In a chemical graph the degree of any vertex is atmost 4. The *distance* between two vertices  $u$  and  $v$  is denoted as  $d(u,v)=d_G(u,v)$  and is the length of shortest path between  $u$  and  $v$  in graph  $G$ . The length of shortest path between  $u$  and  $v$  is also called  $u-v$  *geodesic*. The longest path between any two vertices is called  $u-v$  *detour*.

In this article,  $G$  is considered to be network with vertex set  $V(G)$  and edge set  $E(G)$ ,  $d_u$  is the degree of vertex  $u \in V(G)$ . The concept of topological index came from work done by Harold Wiener in 1947 while

he was working on boiling point of paraffin. He named this index as *path number*. Later on, path number was renamed as *Wiener index* [30] and then theory of topological index started.

**Definition.1.** Let  $G$  be a graph. Then the Wiener index of  $G$  is defined as

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u,v)$$

where  $(u,v)$  is any ordered pair of vertices in  $G$  and  $d(u,v)$  is  $u-v$  geodesic. The very first and oldest degree based topological index is *Randi c'* index [28] denoted by  $R_{\frac{1}{2}}(G)$  and introduced by Milan Randić in 1975

**Definition.2.** The Randić  $c'$  index of graph  $G$  is defined as

$$R_{\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}$$

The general Randić index was proposed by Bollobás and Erdős [4] and Amic et al. [1] independently, in 1998. Then it has been extensively studied by both mathematicians and theoretical chemists [19]. Many important mathematical properties have been established [5].

The general Randić index  $R_\alpha(G)$  is the sum of  $(d_u d_v)^\alpha$  over all edges  $e = uv \in E(G)$  defined as

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha$$

Obviously  $R_{\frac{1}{2}}(G)$  is the particular case of  $R_\alpha(G)$

when  $\alpha = -\frac{1}{2}$ .

An important topological index introduced about forty years ago by Ivan Gutman and Trinajstić is the *Zagreb index* or more precisely first zagreb index denoted by  $M_1(G)$  and was defined as the sum of degrees of end vertices of all edges of  $G$ .

**Definition.3.** Consider a graph  $G$ , then first zagreb index is defined as

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$$

**Definition.4.** Consider a graph  $G$ , then second zagreb index is defined as

$$M_2(G) = \sum_{uv \in E(G)} (d_u \times d_v)$$

One of the well-known degree based topological index is *atom-bond connectivity (ABC)* index introduced by Estrada et al. in [13].

**Definition.5.** For a graph  $G$ , the *ABC* index is defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

Another well-known connectivity topological descriptor is *geometric-arithmetic (GA)* index which was introduced by Vukičević [29].

**Definition.6.** Consider a graph  $G$ , then its *GA* index is defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)}$$

**Definition.7.** A *cartesian product*  $G_1 W G_2$  of two graphs  $G_1$  and  $G_2$  is the graph with the vertex set  $V(G_1) \times V(G_2)$  and the vertex  $(a,b)$  is adjacent to the vertex  $(c,d)$  if and only if  $a=c$  and  $b$  is adjacent to  $d$  or  $b=d$  and  $a$  is adjacent to  $c$ .

## 2. Main Results

In this paper, we study the general Randić, First Zagreb, *ABC* and *GA* indices and give closed formulae of these indices for octagonal network, toroidal polyhex and generalized prism. For further study of topological indices of various graph families see, [15-18,20,27].

### 2.1. Results for octagonal networks

For  $n, m \geq 2$  we denote octagonal network by  $O_n^m$ , the planar map labeled as in Figure (1) with  $m$  rows and  $n$  columns of octagons. The symbols  $V(O_n^m)$  and  $E(O_n^m)$  will denote the vertex set and the edge set of  $O_n^m$ , respectively.

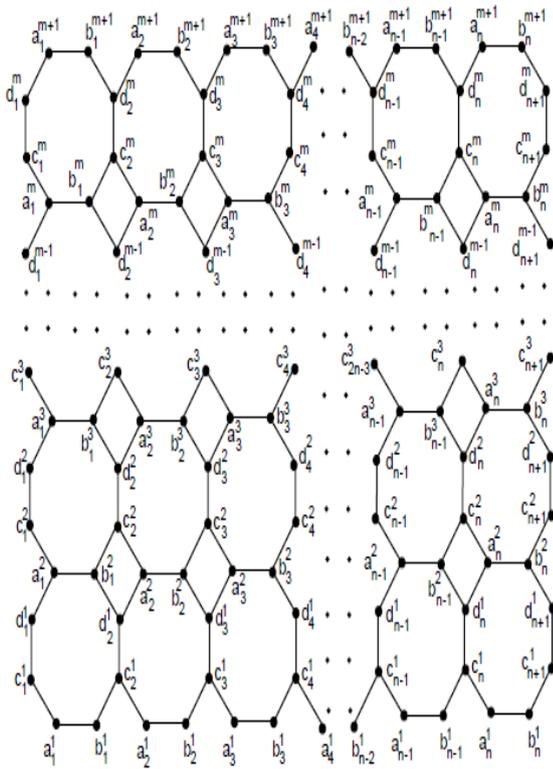


Fig 1. The Octagonal network  $O_n^m$

$$V(O_n^m) = \{x_i^j; 1 \leq i \leq 2n-1, i \text{ odd and } 1 \leq j \leq 3m+1\} \\ \cup \{x_i^{3j-2}; 1 \leq i \leq 2n, i \text{ even and } 1 \leq j \leq m+1\} \\ \cup \{x_{2n}^{3j-1}, x_{2n}^{3j}; 1 \leq j \leq m\}$$

$$E(O_n^m) = \{x_i^j x_{i+1}^{j+1}; 1 \leq i \leq 2n-1, i \text{ odd and } 1 \leq j \leq 3m\} \\ \cup \{x_i^{3j-2} x_{i+1}^{3j-2}; 1 \leq i \leq 2n-1, i \text{ odd and } 1 \leq j \leq m+1\} \\ \cup \{x_i^{3j-2} x_{i+1}^{3j-1}; 1 \leq i \leq 2n-2, i \text{ even and } 1 \leq j \leq m\} \\ \cup \{x_i^{3j} x_{i-1}^{3j+1}; 3 \leq i \leq 2n-1, i \text{ odd and } 1 \leq j \leq m\} \\ \cup \{x_{2n}^j x_{2n}^{j+1}; 1 \leq j \leq 3m\}$$

The number of vertices and number of edges in an Octagonal network are  $(4m+2)n+2m$  and  $(6m+1)n+m$  respectively. We compute general Randi  $c'$  index  $R_\alpha(G)$  with  $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$  in the following theorem of Octagonal network.

**Theorem.1.** Consider the Octagonal network  $O_n^m$ , then its general Randi  $c'$  index is equal to

$$R_\alpha(O_n^m) = \begin{cases} 54mn - 13(m+n) + 4, & \alpha = 1; \\ 18mn - 11(m+n) + 20 + \sqrt{6(4n+4m+8)}, & \alpha = \frac{1}{2}; \\ \frac{12mn + 11(m+n) + 2}{18}, & \alpha = -1; \\ \frac{6mn - 2(m+n) + 9}{3} + \frac{4n + 4m + 8}{\sqrt{6}}, & \alpha = -\frac{1}{2}. \end{cases}$$

**Proof.** Let  $G$  be an Octagonal network  $O_n^m$ . The number of vertices and edges in  $O_n^m$  are  $(4m+2)n+2m$  and  $(6m+1)n+m$  respectively. There are three types of edges in  $O_n^m$  based on degrees of end vertices of each edge. Table 1 shows such an edge partition of  $O_n^m$ .

Table 1. Edge partition of Octagonal network  $O_n^m$

$(d_u, d_v)$ where $uv \in E(G)$	(2,2)	(2,3)	(3,3)
Number of edges	$2n+2m$ $+4$	$4n+4m$ $-8$	$6mn-5n$ $-5m+4$

For  $\alpha = 1$ , we apply the formula of  $R_\alpha(G)$

$$R_1(G) = \sum_{uv \in E(G)} (d_u \times d_v)$$

By using edge partition given in Table 1, we get

$$R_1(G) = (2n+2m+4)(2 \times 2) + (4n+4m-8)(2 \times 3) \\ + (6mn-5n-5m+4)(3 \times 3) \\ \Rightarrow R_1(G) = 54mn - 13(m+n) + 4$$

For  $\alpha = \frac{1}{2}$ , we apply the formula of  $R_\alpha(G)$

$$R_{\frac{1}{2}}(G) = \sum_{uv \in E(G)} \sqrt{(d_u \times d_v)}$$

By using edge partition given in Table 2, we get

$$R_{\frac{1}{2}}(G) = (2n + 2m + 4)\sqrt{(2 \times 2)} + (4n + 4m - 8)\sqrt{(2 \times 3)} + (6mn - 5n - 5m + 4)\sqrt{(3 \times 3)}$$

$$\Rightarrow R_{\frac{1}{2}}(G) = 18mn - 11(m + n) + 20 + \sqrt{6}(4n + 4m + 8)$$

For  $\alpha = -1$ , we apply the formula of  $R_{\alpha}(G)$ .

$$R_{-1}(G) = \sum_{uv \in E(G)} \frac{1}{(d_u \times d_v)}$$

$$R_{-1}(G) = (2n + 2m + 4)\frac{1}{(2 \times 2)} + (4n + 4m - 8)\frac{1}{(2 \times 3)} + (6mn - 5n - 5m + 4)\frac{1}{(3 \times 3)}$$

$$\Rightarrow R_{-1}(G) = \frac{12mn + 11(m + n) + 2}{18}$$

For  $\alpha = -\frac{1}{2}$ , we apply the formula of  $R_{\alpha}(G)$

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u \times d_v)}}$$

$$R_{-\frac{1}{2}}(G) = (2n + 2m + 4)\frac{1}{\sqrt{(2 \times 2)}} + (4n + 4m - 8)\frac{1}{\sqrt{(2 \times 3)}} + (6mn - 5n - 5m + 4)\frac{1}{\sqrt{(3 \times 3)}}$$

$$\Rightarrow R_{-\frac{1}{2}}(G) = \frac{6mn - 2(m + n) + 9}{3} + \frac{4n + 4m + 8}{\sqrt{6}}$$

In the following theorem, we compute first Zagreb index of Octagonal networks  $O_n^m$ .

**Theorem.2.** For an Octagonal network  $O_n^m$ , the first Zagreb index is equal to

$$M_1(O_n^m) = 36mn - 2n - 2m$$

**Proof.** Let  $G$  be an Octagonal network  $O_n^m$ . By using edge partition from Table 2, we easily prove it. We know

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$$

$$M_1(G) = (2n + 2m + 4)(2 + 2) + (4n + 4m - 8)(2 + 3) + (6mn - 5n - 5m + 4)(3 + 3)$$

By doing some calculation, we get

$$\Rightarrow M_1(G) = 36mn - 2n - 2m$$

Now we exhibit  $ABC$  index of Octagonal network  $O_n^m$  in the following theorem.

**Theorem.3.** For an Octagonal network  $O_n^m$ , the

$ABC$  index is equal to

$$ABC(O_n^m) = \sqrt{2}(3n + 3m - 2) + \frac{12mn - 10n - 10m + 8}{3}$$

**Proof.** Let  $G$  be Octagonal network  $O_n^m$ . By using edge partition given in Table 2, we easily prove it. We know

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$$ABC(G) = (2n + 2m + 4)\sqrt{\frac{2 + 2 - 2}{2 \times 2}} + (4n + 4m - 8)\sqrt{\frac{2 + 3 - 2}{2 \times 3}} + (6mn - 5n - 5m + 4)\sqrt{\frac{3 + 3 - 2}{3 \times 3}}$$

By doing some calculation, we get

$$\Rightarrow ABC(G) = \sqrt{2}(3n + 3m - 2) + \frac{12mn - 10n - 10m + 8}{3}$$

In the following theorem, we compute  $GA$  index of Octagonal network  $O_n^m$ .

**Theorem.4.** Consider an Octagonal network  $O_n^m$ , then its  $GA$  index is equal to

$$GA(O_n^m) = 6mn - 3(n + m) + 8 + \frac{8\sqrt{6}(n + m - 2)}{5}$$

**Proof.** Let  $G$  be an Octagonal network  $O_n^m$ . By using edge partition given in Table 2, we easily prove it. We know

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)}$$

$$GA(G) = (2n + 2m + 4)\left(\frac{2\sqrt{2 \times 2}}{2 + 2}\right) + (4n + 4m - 8)\left(\frac{2\sqrt{2 \times 3}}{2 + 3}\right) + (6mn - 5n - 5m + 4)\left(\frac{2\sqrt{3 \times 3}}{3 + 3}\right)$$

By doing some calculation, we get

$$\Rightarrow GA(G) = 6mn - 3(n + m) + 8 + \frac{8\sqrt{6}(n + m - 2)}{5}$$

### 2.2. Results for toroidal polyhex

The discovery of the fullerene molecules has stimulated much interests in other possibilities for carbons. Many properties of fullerenes can be studied using mathematical tools such as graph theory and group theory.

A fullerene can be represented by a trivalent graph on a closed surface with pentagonal and hexagonal faces, such that its vertices are carbon atoms of the molecule. Two vertices are adjacent if there is a bond between corresponding atoms. Deza et al. [12] considered fullerene's extension to other closed surfaces and showed that only four surfaces are possible, namely sphere, torus, Klein bottle and projective (elliptic) plane. The usual spherical fullerenes have 12 pentagons and elliptic fullerenes have 6 pentagons. The toroidal and Klein bottle's fullerenes contain no pentagons, see [12,24].

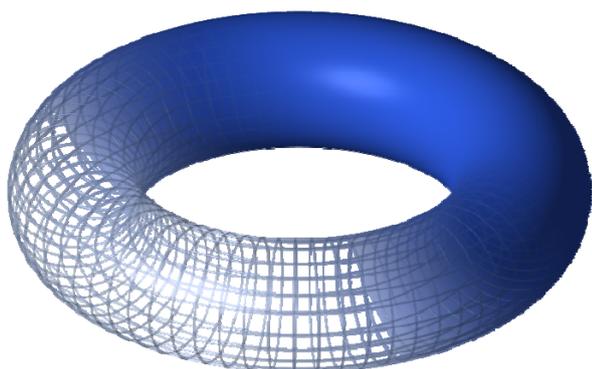


Fig. 2. 3D Polyhex Torus

A toroidal fullerene (toroidal polyhex), obtained from 3D Polyhex Torus Fig. 2, is a cubic bipartite graph embedded on the torus such that each face is a hexagon. Note that the torus is a closed surface that can carry graph of toroidal polyhex such that all its vertices have degree 3 and all faces of the embedding are hexagons.

Some features of toroidal polyhexes with chemical relevance were discussed in [22,23]. For example, a systematic coding and classification scheme were given for the enumeration of isomers of toroidal polyhexes, the calculation of the spectrum and the count for spanning trees. The optical and vibrational properties of toroidal carbon nanotubes can be found in [3]. There have appeared a few works in the enumeration of perfect matchings of toroidal polyhexes by applying various techniques, such as transfer-matrix and permanent of the adjacency matrix [6]. Ye et al. [30] have studied a k-resonance of toroidal polyhexes and Kang [21] classified all possible structures of fullerene Cayley graphs.

Let  $L$  be a regular hexagonal lattice and  $P_m^n$  be an  $m \times n$  quadrilateral section (with  $m$  hexagons on the top and bottom sides and  $n$  hexagons on the lateral sides,  $n$  is even) cut from the regular hexagonal lattice  $L$ . First identify two lateral sides of  $P_m^n$  to form a cylinder, and finally identify the top and bottom sides of  $P_m^n$  at their corresponding points, see Figure 2. From this we get a toroidal polyhex  $H_m^n$  with  $mn$  hexagons.

Let  $V(H_m^n) = \{u_j^i, v_j^i : 0 \leq i \leq n-1, 0 \leq j \leq m-1\}$  be the vertex set. The set of edges of  $H_m^n$  we split into mutually disjoint subsets such that for  $i$  even,  $0 \leq i \leq n-2$

$$A_i = \{u_j^i v_j^i : 0 \leq j \leq m-1\}$$

and

$$A'_i = \{u_j^i u_{j+1}^i : 0 \leq j \leq m-1\}$$

for  $i$  odd,  $1 \leq i \leq n-1$ ,

$$B_i = \{v_j^i v_j^i : 0 \leq j \leq m-1\}$$

and

$$B'_i = \{u_j^i v_{j+1}^i : 0 \leq j \leq m-1\}$$

, and for  $0 \leq i \leq n-1$

$$C_i = \{v_j^i u_{j+1}^{i+1} : 0 \leq j \leq m-1\}$$

, where  $i$  is taken modulo  $n$  and  $j$  is taken modulo  $m$ .

Thus

$$E(H_m^n) = \bigcup_{i=0}^{n-1} (A_{2i} \cup A'_{2i} \cup B_{2i+1} \cup B'_{2i+1}) \cup \bigcup_{i=0}^{n-1} C_i$$

We easily know that  $H_m^n$  has  $2mn$  vertices and  $3mn$  edges.

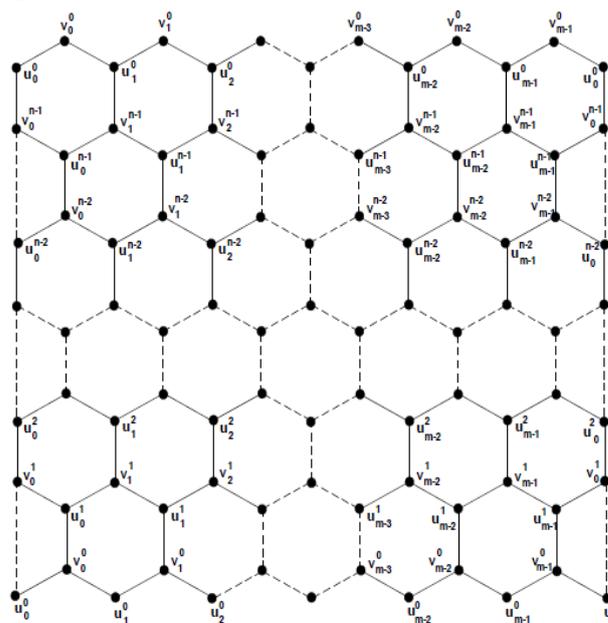


Fig. 3. Quadrilateral section  $P_m^n$  cuts from the regular hexagonal lattice

Baca et.al [2], compute atom-bond connectivity index and geometric–arithmetic index of toroidal polyhex  $H_m^n$ . Now we compute its general Randic index  $R_\alpha(G)$  with  $\alpha = 1, -1, \frac{1}{2}, -\frac{1}{2}$  in the following theorem.

**Theorem. 5.** Consider the toroidal polyhex  $H_m^n$ , then its general Randic index is equal to

$$R_\alpha(H_m^n) = \begin{cases} 27mn, & \alpha = 1; \\ 9mn, & \alpha = \frac{1}{2}; \\ \frac{mn}{3}, & \alpha = -1; \\ mn, & \alpha = -\frac{1}{2}. \end{cases}$$

**Proof.** Let  $G$  be the toroidal polyhex  $H_m^n$  with defining parameters as  $m$  and  $n$ . The number of vertices and edges in toroidal polyhex  $H_m^n$  are  $2mn$  and  $3mn$  respectively. There are only one types of edges in toroidal polyhex  $H_m^n$  based on degrees of end vertices of each edge. Table 2 shows such an edge partition of toroidal polyhex  $H_m^n$ .

Table 2. Edge partition of toroidal polyhex  $H_m^n$ .

$(d_u, d_v)$ where $uv \in E(G)$	Number of edges
(3,3)	$3mn$

For  $\alpha = 1$ , we apply the formula of  $R_\alpha(G)$

$$R_1(G) = \sum_{uv \in E(G)} (d_u \times d_v)$$

By using edge partition given in Table 2, we get

$$R_1(G) = 3mn(3 \times 3) = 27mn$$

For  $\alpha = \frac{1}{2}$ , we apply the formula of  $R_\alpha(G)$

$$R_{\frac{1}{2}}(G) = \sum_{uv \in E(G)} \sqrt{d_u \times d_v}$$

By using edge partition given in Table 3, we get

$$R_{\frac{1}{2}}(G) = 3mn\sqrt{3 \times 3} = 9mn$$

For  $\alpha = -1$ , we apply the formula of  $R_\alpha(G)$

$$R_{-1}(G) = \sum_{uv \in E(G)} \frac{1}{d_u \times d_v} = 3mn \times \frac{1}{3 \times 3} = \frac{mn}{3}$$

For  $\alpha = -\frac{1}{2}$ , we apply the formula of  $R_\alpha(G)$

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u \times d_v}} = 3mn \times \frac{1}{\sqrt{3 \times 3}} = mn$$

In the following theorem, we compute first Zagreb index of toroidal polyhex  $H_m^n$ .

**Theorem.6.** For toroidal polyhex  $H_m^n$ , the first Zagreb index is equal to

$$M_1(H_m^n) = 18mn$$

**Proof.** Let  $G$  be the toroidal polyhex  $H_m^n$ . By using edge partition from Table 3, we get the result. We know

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v) = 3mn(3 + 3) = 18mn$$

### 2.3. Result for generalized prism

The *generalized prism*  $P_n^m$  can be defined as the Cartesian product  $C_n WP_m$  of a cycle on  $n$  vertices with a path on  $m$  vertices. If we consider a cycle  $C_n$  with

$$V(C_n) = \{x_i : 1 \leq i \leq n\},$$

$$E(C_n) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_n x_1\}$$

and a path  $P_m$  with  $V(P_m) = \{y_j : 1 \leq j \leq m\}$ ,

$$E(P_m) = \{y_j y_{j+1} : 1 \leq j \leq m-1\},$$

then

$$V(P_n^m) = V(C_n WP_m) = \{(x_i, y_j) : 1 \leq i \leq n, 1 \leq j \leq m\}$$

$$E(P_n^m) = E(C_n WP_m) = \{(x_i, y_j)(x_{i+1}, y_j) :$$

$$\text{for } 1 \leq i \leq n-1, 1 \leq j \leq m\}$$

$$\cup \{(x_n, y_j)(x_1, y_j) : 1 \leq j \leq m\}$$

$$\cup \{(x_i, y_j)(x_i, y_{j+1}) : 1 \leq i \leq n, 1 \leq j \leq m-1\}$$

is the vertex set of the graph  $P_n^m$  and is the edge set of  $P_n^m$  respectively.

The cartesian product of cycle and path is depicted on Fig. 4.

The generalized prism  $P_n^m$  has been studied extensively in recent years. Kuo et al. [25] and Chiang et al. [9] studied distance-two labelings of  $P_n^m$ . Deming et al. [11] gave complete characterization of the Cartesian product of cycles and paths for their incidence chromatic numbers. Gravier et al. [14] showed the link between the existence of perfect Lee codes and minimum dominating sets of  $P_n^m$ . Lai et al. [26] determined the edge addition number for the Cartesian product of a cycle with a path. Chang et al. [7] established upper bounds and lower bounds for global defensive alliance number of  $P_n^m$

Now we compute these topological indices for generalized prism  $P_n^m$

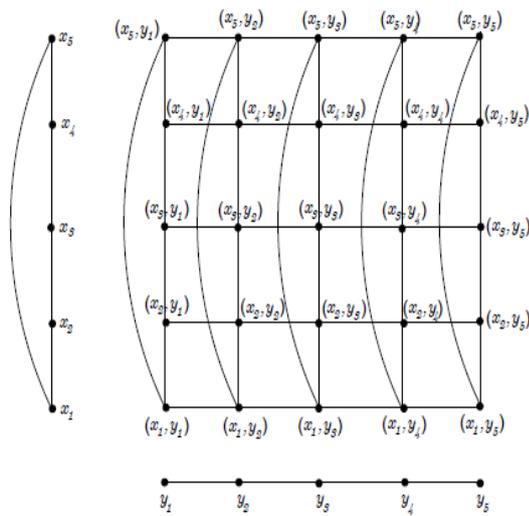


Fig 4. Cartesian product of cycle and path.

**Theorem.7.** Consider the generalized prism  $P_n^m$ , then its general Randić  $C'$  index is equal to

$$R_\alpha(P_n^m) = \begin{cases} 32mn - 38n, & \alpha = 1; \\ 8mn - 14n + 4\sqrt{3}n, & \alpha = \frac{1}{2}; \\ \frac{18mn + 11n}{144}, & \alpha = -1; \\ \frac{6mn - 7n}{12} + \frac{n}{\sqrt{3}}, & \alpha = -\frac{1}{2}. \end{cases}$$

**Proof.** Let  $G$  be generalized prism  $P_n^m$  with defining parameters  $m$  and  $n$ . The number of vertices and edges in generalized prism  $P_n^m$  are  $nm$  and  $n(2m - 1)$  respectively. There are three types of edges in  $P_n^m$  based on degrees of end vertices of each edge. Table 3 shows such a edge partition of  $P_n^m$ .

Table 3. Edge partition of  $P_n^m$ .

$(d_u, d_v)$ $uv \in E(G)$	where	Number of edges
(3, 3)		$2n$
(3, 4)		$2n$
(4, 4)		$2mn - 5n$

For  $\alpha = 1$ , we apply the formula of  $R_\alpha(G)$

$$R_1(G) = \sum_{uv \in E(G)} (d_u \times d_v)$$

By using edge partition given in Table 4, we get  
 $R_1(G) = 2n(3 \times 3) + 2n(3 \times 4) + (2mn - 5n)(4 \times 4)$   
 $\Rightarrow R_1(G) = 32mn - 38n$

For  $\alpha = \frac{1}{2}$ , We apply the formula of  $R_\alpha(G)$

$$R_{\frac{1}{2}}(G) = \sum_{uv \in E(G)} \sqrt{(d_u \times d_v)}$$

By using edge partition given in Table 3, we get

$$R_{\frac{1}{2}}(G) = 2n\sqrt{(3 \times 3)} + 2n\sqrt{(3 \times 4)} + (2mn - 5n)\sqrt{(4 \times 4)}$$

$$\Rightarrow R_{\frac{1}{2}}(G) = 8mn - 14n + 4\sqrt{3}n$$

For  $\alpha = -1$ , we apply the formula of  $R_\alpha(G)$

$$R_{-1}(G) = \sum_{uv \in E(G)} \frac{1}{(d_u \times d_v)}$$

$$R_{-1}(G) = 2n \frac{1}{(3 \times 3)} + 2n \frac{1}{(3 \times 4)} + (2mn - 5n) \frac{1}{(4 \times 4)}$$

$$\Rightarrow R_{-1}(G) = \frac{18mn + 11n}{144}$$

For  $\alpha = -\frac{1}{2}$ , we apply the formula of  $R_\alpha(G)$

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{(d_u \times d_v)}}$$

$$R_{-\frac{1}{2}}(G) = 2n \frac{1}{\sqrt{(3 \times 3)}} + 2n \frac{1}{\sqrt{(3 \times 4)}} + (2mn - 5n) \frac{1}{\sqrt{(4 \times 4)}}$$

$$\Rightarrow R_{-\frac{1}{2}}(G) = \frac{6mn - 7n}{12} + \frac{n}{\sqrt{3}}$$

In the following theorem, we compute first Zagreb index of generalized prism  $P_n^m$ .

**Theorem.8.** For a generalized prism  $P_n^m$ , the first Zagreb index is equal to

$$M_1(P_n^m) = 16mn - 14n$$

**Proof.** Let  $G$  be generalized prism  $P_n^m$ . By using edge partition from Table 4, we easily prove it. We know

$$M_1(G) = \sum_{uv \in E(G)} (d_u + d_v)$$

$$M_1(G) = 2n(3 + 3) + 2n(3 + 4) + (2mn - 5n)(4 + 4)$$

By doing some calculation, we get

$$\Rightarrow M_1(G) = 16mn - 14n$$

**Theorem.9.** For generalized prism  $P_n^m$ , the  $ABC$  index is equal to

$$ABC(P_n^m) = \frac{4n}{3} + \sqrt{\frac{5}{3}}n - \frac{\sqrt{6}(2mn - 5n)}{4}$$

**Proof.** Let  $G$  be a generalized prism  $P_n^m$ . By using edge partition given in Table 4, we easily prove it. We know

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}$$

$$ABC(G) = 2n\sqrt{\frac{3+3-2}{3 \times 3}} + 2n\sqrt{\frac{3+4-2}{3 \times 4}}$$

$$+ (2mn - 5n)\sqrt{\frac{4+4-2}{4 \times 4}}$$

$$\Rightarrow ABC(G) = \frac{4n}{3} + \sqrt{\frac{5}{3}}n - \frac{\sqrt{6}(2mn - 5n)}{4}$$

In the following theorem, we compute  $GA$  index of a generalized prism  $P_n^m$

**Theorem.10.** Consider a generalized prism  $P_n^m$ , then its  $GA$  index is equal to

$$GA(P_n^m) = 2mn - 3n + \frac{8n\sqrt{3}}{7}$$

**Proof.** Let  $G$  be a generalized prism  $P_n^m$ . By using

edge partition given in Table 4, we easily prove it. We know

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{(d_u + d_v)}$$

$$GA(G) = 2n\left(\frac{2\sqrt{3 \times 3}}{3 + 3}\right) + 2n\left(\frac{2\sqrt{3 \times 4}}{3 + 4}\right)$$

$$+ (2mn - 5n)\left(\frac{2\sqrt{4 \times 4}}{4 + 4}\right)$$

$$\Rightarrow GA(G) = 2mn - 3n + \frac{8n\sqrt{3}}{7}$$

### 3. Conclusion and closing remarks

In this paper, certain degree based topological indices, namely general Randić index, atomic-bond connectivity index(ABC), geometric-arithmetic index ( $GA$ ) and first zagreb index for octagonal networks as well as toroidal polyhex and generalized prism, were studied for the first time. To construct and study new architectures has always been an open problem in both network and art/design sciences. In future, we are interested to design some new architectures/networks and then study their topological indices which will be quite helpful to understand their underlying topologies.

### Acknowledgement

We are very grateful to the referee for his valuable corrections and suggestions for the improvement of the first version.

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