

# Dipolar interactions in chains of ferromagnetic nanowires

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In the present paper we present a theoretical model used to estimate the dipolar interaction field in ferromagnetic nanowire chains. Based on this model, the dependence of the dipolar field on the nanowires separation for two different orientations of the magnetization inside the nanowires was evaluated.

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## 1. Introduction

In recent years, the quest for alternative technologies has stimulated an interest in nanometer-scale materials and devices. The nanowires are one of the most attractive materials because of their unique properties that may lead to a variety of applications, such as interconnects for nanoelectronics, magnetic devices, chemical and biological sensors, and biological labels [1-3].

The interest for magnetic nanowires has given rise to several studies on the interesting magnetic properties of these systems [4-7]. Not much is known about the details of the magnetization processes occurring in the arrays of closely packed magnetic nanowires. Even in single nanowires, the unusual domain structures were found [7]. The issue becomes highly complex when the long-range dipolar coupling between the wires is considered [5,6]. The corresponding dipolar field adds to the internal field of a single wire and it is measurable in experiments such as FMR [8,9].

We have calculated the dipolar interactions between the nanowires which are arranged in a 1D chain as function of the nanowire separation ( $s$ ). The nanowires were approximated by rectangular particles and a chain of perfectly parallel and identical ferromagnetic nanowires. Then the dipolar field can be easily calculated by summing the strayfield arising from "charged sheets". The corresponding formalism used here it was described in [10].

## 2. Theoretical model

Numerical calculations were performed in order to determine the interaction field for ferromagnetic nanowires arrays with nanowire length being one hundred larger than the wire diameter.

Starting from Maxwell's equations  $\text{div} \mathbf{B} = \text{div}(\mu_0 \mathbf{H} + \mathbf{J}) = 0$ , we define as the stray field,  $H_d$ , the field generated by the divergence of the magnetization  $\mathbf{J}$ :

$$\text{div} H_d = -\text{div} \left( \frac{\mathbf{J}}{\mu_0} \right) \quad (1)$$

The sources of the magnetization act like positive and negative "magnetic charges" for the stray field. The field can be calculated in the same way as the electrostatic field which arises from the electrical charges. The only difference is that the magnetic charges never appear isolated but they are always balanced by opposite charges. The energy connected to the stray field is [10]:

$$E_d = \frac{1}{2} \mu_0 \int_{\text{allspace}} H_d^2 dV = -\frac{1}{2} \int_{\text{sample}} H_d \mathbf{J} dV \quad (2)$$

A general solution of the stray field problem is given by the potential theory. In this approach the potential of the stray field at position  $\mathbf{r}$  is given by integration over  $\mathbf{r}'$ :

$$\Phi_d(\mathbf{r}) = \frac{J_s}{4 \cdot \pi \cdot \mu_0} \left[ \int \frac{\lambda_v(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dV' + \int \frac{\sigma_s(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dS' \right] \quad (3)$$

Using this expression we can calculate the stray field, resulting in:

$$H_d(\mathbf{r}) = -\text{grad} [\Phi_d(\mathbf{r})] \quad (4)$$

The direct computation of integrals would be time consuming. In order to avoid this situation we must try to analytically perform as many steps as possible. One approach is to replace the integration by a summation over

self energy and interaction energy terms of finite elements. We consider that the finite elements are volume and surface elements in which the magnetic charges are assumed to be constant. If the elements are of the rectangular shape, all integration can be reduced by substitution to multiple integrals of the core function  $F_{000} = 1/r$  [10]. The integrals of increasing order have the following expressions:

$$F_{100} = \int F_{000} \cdot dx = \frac{1}{2} \cdot \ln \left[ \frac{r+x}{r-x} \right] = L_x \quad (5)$$

$$F_{200} = \int F_{100} \cdot dx = x \cdot L_x - r \quad (6)$$

$$F_{110} = \int F_{100} dy = y \cdot L_x + x \cdot L_y - z \cdot \arctan \left( \frac{x \cdot y}{z \cdot r} \right) \quad (7)$$

Considering a rectangular element with the magnetization oriented on x direction assuming that there is no volume charges but only the surface charges, the Eq. 3 is given by:

$$\begin{aligned} \Phi_d(x, y, z) = & M_S \cdot [F_{011}(x-x_2, y-y_2, z-z_2) - F_{011}(x-x_2, y-y_2, z-z_1)] - \\ & - M_S \cdot [F_{011}(x-x_1, y-y_2, z-z_2) - F_{011}(x-x_1, y-y_2, z-z_1)] - \\ & - M_S \cdot [F_{011}(x-x_2, y-y_1, z-z_2) - F_{011}(x-x_2, y-y_1, z-z_1)] + \\ & + M_S \cdot [F_{011}(x-x_1, y-y_1, z-z_2) - F_{011}(x-x_1, y-y_1, z-z_1)] \end{aligned} \quad (8)$$

The dipolar field components are given by:

$$H_x = -\frac{\partial \Phi_d}{\partial x}, \quad H_y = -\frac{\partial \Phi_d}{\partial y}, \quad H_z = -\frac{\partial \Phi_d}{\partial z} \quad (9)$$

### 3. Results and discussion

We have used the method previously described to determine the dipolar interaction field for 1D nanowires chain. Two cases were considered: (i) the orientation of magnetization inside the nanowire is perpendicular to the long nanowire axis; (ii) the orientation is parallel to that axis.

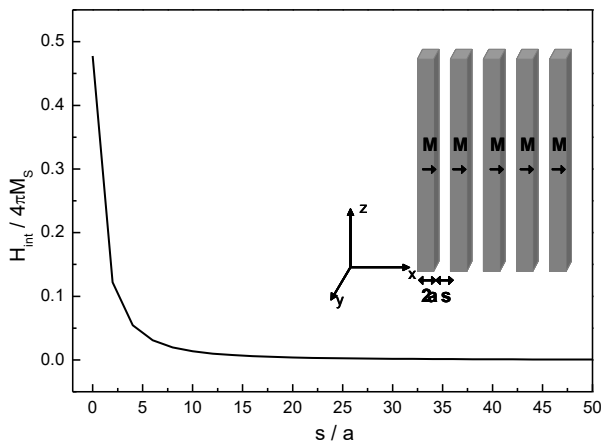


Fig. 1. Calculated normalized dipolar interaction field as a function of  $s/a$  for  $M_S \perp z$ .

For the simplicity of the calculation the nanowires were approximated by parallelepipeds. The interaction field is evaluated in the center of the chain and it is obtained by summing over all charged surfaces of the whole chain. A number of  $10^4$  neighboring wires which guarantees convergence of the dipolar field sum was taken into the consideration. Then the total calculated field is the sum of the interaction field from all other wires and the self demagnetization field:

$$H_{\text{tot}} = H_{\text{int}} + H_{\text{self}} \quad (10)$$

The results for the nanowire chains magnetized in the direction perpendicular to the long nanowire axis  $M_S \perp z$  are shown in Fig.1. The field is normalized to the value  $4\pi M_S$  and the separation ( $s$ ) is scaled to the nanowire radius ( $r$ ). We have used the normalized value in order to be able to apply the results to all types of nanowire material. It was observed that the dipolar interaction field decreases with the increase of the separation between the nanowires.

For the separation  $s=0$ , the interaction field becomes:

$$H_{\text{int}} = H_{\text{tot}} - H_{\text{self}} \quad (11)$$

Considering that the  $H_{\text{self}} = -2\pi M_S$ , a value of  $H_{\text{int}} = 2\pi M_S$  was obtained for the interaction field.

For increasing the separation ( $s > 0$ ), the interaction field falls off rapidly having a value of  $H_{\text{int}} = 0.37 M_S$  at  $s=6a$ .

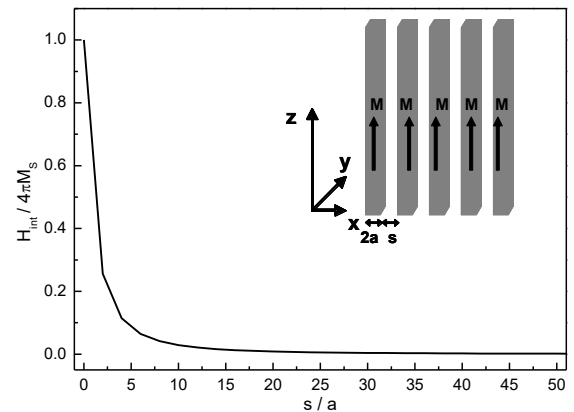


Fig. 2. Calculated normalized dipolar interaction field as a function of  $s/a$  for  $M_S // z$ .

Fig. 2 shows the results obtained when the orientation of the magnetization inside the nanowires is parallel to the long nanowire axis  $M_S // z$ . Similarly to the first case, the dipolar interaction field also decreases with increasing the separation between the nanowires.

For a separation  $s=0$  between the nanowires the interaction field is given by Eq. (11). Considering that  $H_{\text{self}}=0$  the interaction becomes:  $H_{\text{self}}=4\pi M_S$ . If the separation between the nanowires increases the interaction field rapidly decreases and for  $s=6a$ , one can get  $H_{\text{int}}=0.75M_S$ .

#### 4. Conclusions

We have presented a model for the interaction field in large and uniformly magnetized ferromagnetic nanowire chain. The effective interaction field has been modeled within a mean field approach with the interaction field expressed as a function of nanowire separation.

The model predicts that, independently of the wire diameter and material, the dipolar interaction field decreases with the increase of the separation. The decreasing of the interaction field is more pronounced when the magnetization is oriented perpendicular to the wire axis.

#### References

- [1] K. Nielsch, *Appl. Phys. Lett.* **79**, 1360 (2001).
- [2] C. M. Niemeyer, *Angew.Chem.Int.Ed* **40**, 4128 (2001).
- [3] E. Nogales, *Anu. Rev. Biochem.* **69**, 277 (2000).
- [4] R. Skomski, H. Zeng, M. Zheng, and D. J. Sellmyer, *Phys. Rev. B* **62**, 3900 (2000).
- [5] J. Velazquez, C. Garcia, M. Vazquez, A. Hernando, J. *Appl. Phys.* **85**, 2768 (1999).
- [6] L. Sampaio, E. H. C. P. Sinnecker, G. R. C. Cernichiaro, M. Knobel, M. Vazquez, J. Velazquez, *Phys. Rev. B* **61**, 8976 (2000).
- [7] L. Belliard, J. Miltat, A. Thiaville, S. Dubois, J. Duvail, L. Piraux, *J. Magn. Magn. Mater.* **190**, 1 (1998).
- [8] M. Darques, A. Encinas, L. Piraux, A. Popa, P. Bayle-Guillemaud, U. Ebels, *Appl. Phys. Lett.* **86**, 72508 (2005).
- [9] A. Encinas, M. Demand, L. Piraux, U. Ebels, *Phys. Rev. B*, **63**, 104414 (2001).
- [10] A. Hubert, R. Schafer „Magnetic domains”, Ed. Springer- Verlag, Berlin, (1998).

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