

Dynamic models of hysteresis for structured media

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A new dynamic model of hysteresis for structured media is presented. This model is based on the Preisach model of hysteresis for strongly interacting ferromagnetic particle systems but can be easily generalized to other models of hysteresis. The model can be applied to the modeling and analysis of rate-dependent magnetization processes in particulate media. It is introduced in differential form, which makes it suitable for numerical implementation in standard simulators. Sample numerical results are presented and discussed.

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1. Introduction

It is well-known that the speed of variation of the applied field in magnetic systems strongly affects the output of the system [1]-[3]. For instance, in most soft magnetic ferrites [4]-[5] the major hysteresis loop can change drastically when the frequency of the applied field is increased. These phenomena are usually known as dynamic phenomena and play an important role in the behavior of magnetic circuits in high-frequency applications involving sensors and actuators, transformers, or electronic circuits in which the inductances contain hysteretic materials. So far, there exist many models that describe dynamic processes in bulk materials but more work should be done for the development of appropriate models for structured materials such as particulate media (see also [6]- [7]). In this article we try to bridge this gap by developing a new dynamic model for structured materials based on the Preisach model of hysteresis for strongly interacting ferromagnetic particulate systems (PM2) and the relaxation time approximation.

There exist two approaches to the modeling of dynamic magnetization processes in magnetic materials. The first approach applies physical models such as the Landau-Lifshitz model or various approximations of this model to describe the overall dynamics of the magnetization [8]-[9]. These approaches are usually computationally very expensive and, so far, have been applied to low dimension or simplified magnetic systems such as two-dimensional systems or three-dimensional arrays of a relatively small number of magnetic particles. The second approach uses phenomenological models of hysteresis which are modified to account for rate-dependent effects [10]. Although this approach requires special attention to calibrate the static and dynamic parameters of the model, it is usually much more computationally efficient than the first approach.

Due to their mathematical complexity, the existing dynamic models of hysteresis usually require solving

complicated differential/integral equations that results in long computation times on normal personal computers. In the following we present a dynamic Preisach-type model based on the PM2 model [11]-[12]-[13], which was previously tested against micromagnetic (Landau-Lifshitz) computations and proved to be accurate in the description of magnetization processes in structured materials with bimodal interaction field distributions. The model introduced in this paper is presented in a compact differential form which makes it suitable for numerical implementation on standard material simulators. The article is structured as follows. The basic idea of our approach is presented in Section 2. Special emphasis is given to presenting how the dynamic model can be extended to other models of hysteresis. In Section 3 we present the relationship that exists between our model and dynamic models based on the effective-field approximation. A numerical trick that avoids expensive computational simulations and sample numerical results are presented in Section 4, while conclusions are drawn in Section 5.

2. Dynamic model of hysteresis in the relaxation time approximation

Consider the definition of dynamic Preisach-type models of hysteresis presented in [1]. The total magnetization can be written as a function of time by using a time-dependent Preisach function

$P\left(H_\alpha, H_\beta, \frac{dM}{dt}\right)$ as follows:

$$M(t) = \iint_{H_\alpha > H_\beta} P\left(H_\alpha, H_\beta, \frac{dM}{dt}\right) \hat{\gamma}_{\alpha\beta} H(t) dH_\alpha dH_\beta \quad (1)$$

In this equation $\hat{\gamma}_{\alpha\beta}$ are elementary rectangular hysteresis operators with H_α and H_β as up and down switching fields and with ± 1 as saturation values. The hysteresis operators are defined by $\hat{\gamma}_{\alpha\beta}H(t) = 1$ if $H(t) > H_\alpha$, $\hat{\gamma}_{\alpha\beta}H(t) = \pm 1$ if $H_\alpha > H(t) > H_\beta$, and $\hat{\gamma}_{\alpha\beta}H(t) = -1$ if $H(t) < H_\beta$. For the sake of simplicity we have considered only the irreversible component of the Preisach distribution in (1), but the reversible component can be taken into consideration in our analysis by adding another integral to the right-hand side of equation (1). By following the technique presented in [1], we expand the Preisach distribution in power series and keep only the first two terms:

$$P(H_\alpha, H_\beta, \frac{dM}{dt}) \approx P_0(H_\alpha, H_\beta) + \frac{dM}{dt} P_1(H_\alpha, H_\beta), \quad (2)$$

where $P_0(H_\alpha, H_\beta)$ and $P_1(H_\alpha, H_\beta)$ are two material dependent functions. In this approximation, the total component of the magnetization can be computed by solving the following differential equation:

$$M(t) = \iint_{H_\alpha > H_\beta} P_0(H_\alpha, H_\beta) \hat{\gamma}_{\alpha\beta} H(t) dH_\alpha dH_\beta + \frac{dM}{dt} \iint_{H_\alpha > H_\beta} P_1(H_\alpha, H_\beta) \hat{\gamma}_{\alpha\beta} H(t) dH_\alpha dH_\beta \quad (3)$$

It should be noted that, if the rate of variation of the applied field is very low, dM/dt is negligible and model (3) becomes the classical Preisach model of hysteresis. Therefore, the first term in the right-hand side of equation (3) can be identified as the magnetization computed in the framework of the classical Preisach model at very low rates of variation of the applied field. The second term in the right-hand side of equation (3) is due to the dynamic effects. By introducing the notations:

$$M_0(t) = \iint_{H_\alpha > H_\beta} P_0(H_\alpha, H_\beta) \hat{\gamma}_{\alpha\beta} H(t) dH_\alpha dH_\beta, \quad (4)$$

$$\tau(t) = \iint_{H_\alpha > H_\beta} P_1(H_\alpha, H_\beta) \hat{\gamma}_{\alpha\beta} H(t) dH_\alpha dH_\beta, \quad (5)$$

equation (3) becomes $M(t) = M_0(t) + \tau(t) \frac{dM}{dt}$, which leads to the following differential equation for M :

$$\frac{dM}{dt} = \frac{M(t) - M_0(t)}{\tau(t)}. \quad (6)$$

It is important to remark that, although equation (6) was deduced in the framework of Preisach-type models of hysteresis, it can be easily generalized and applied to most phenomenological model of hysteresis. In the numerical implementation $M_0(t)$ is the static magnetization and $\tau(t)$ is a history dependent relaxation time that should be found by using appropriate calibration techniques. In the first-order approximation (“relaxation time approximation”), $\tau(t)$ can be considered constant,

$$\tau \frac{dM}{dt} = M(t) - M_0(t). \quad (7)$$

and the magnetization can be integrated numerically by using standard numerical integration procedures. In the following we present a few extensions of the dynamic model presented above to a three widely spread models of hysteresis from the literature: the PM2 model, the Energetic model [15], and the Jiles-Atherton model [14]. We have chosen these models of hysteresis because they are representative for most other models in the literature: the PM2 model is Preisach-type model in which the magnetization is given as in integral equation, the Jiles-Atherton model is a purely differential model of hysteresis, and the Energetic model is an analytic model of hysteresis (in which the magnetization is expressed as a function of the applied field as an analytic equation).

2.1 Extension to the PM2 model of hysteresis

In the case of the PM2 model, the Preisach distribution can be written as the product between the coercive field distribution and the interaction field distribution: $P_0(H_\alpha, H_\beta) = P_c(h_c) \times P_i(h_i)$, where $h_c = (H_\alpha - H_\beta)/\sqrt{2}$ and $h_i = (H_\alpha + H_\beta)/\sqrt{2}$. The interaction field distribution is bimodal and has variable variance and mean values. In our simulations we consider:

$$P_i(h_i) = \frac{1}{\sqrt{2\pi}h_{\sigma i}} \left\{ \frac{M_s + M}{2} \exp\left[-\frac{(h_i - h_{i0})^2}{2h_{\sigma i}^2}\right] + \frac{M_s - M}{2} \exp\left[-\frac{(h_i + h_{i0})^2}{2h_{\sigma i}^2}\right] \right\}, \quad (8)$$

while the coercive field distribution is:

$$P_c(h_c) = \frac{1}{\sqrt{2\pi}h'_{\sigma c}h_c} \exp\left[-\frac{(\ln h_c - h'_{c0})^2}{2h_{\sigma c}^2}\right], \quad (9)$$

with $h_{\sigma c}^2 = \ln(h_{\sigma c}/h_{c0} + 1)$ and $h'_{c0} = \ln h_{c0} - h'_{\sigma c}/2$, where h_{c0} and $h_{\sigma c}$ are the average value and standard deviation of the distribution. The differential equation for the magnetization becomes:

$$\tau \frac{dM}{dt} = M - \frac{1}{2\pi h_{\sigma i} h'_{\sigma c} h_c} \iint_{H_c > H_\beta} \exp\left[-\frac{(\ln h_c - h'_{c0})^2}{2h_{\sigma c}^2}\right] \times \left\{ \frac{M_s + M}{2} \exp\left[-\frac{(h_i - h_{0i})^2}{2h_{\sigma i}^2}\right] + \frac{M_s - M}{2} \exp\left[-\frac{(h_i + h_{0i})^2}{2h_{\sigma i}^2}\right] \right\} \hat{\gamma}_{\sigma\beta} H(t) dH_\alpha dH_\beta \quad (10)$$

2.2 Extension to the Energetic model of hysteresis

In the case of the Energetic model of hysteresis the static magnetization M_0 is related to the applied field H by the following analytical equation [14]:

$$H = N_r M_0 + \text{sgn}(M_0) h \left[\left[\left(1 + \frac{M_0}{M_s} \right)^{\frac{M_0}{M_s}} \left(1 - \frac{M_0}{M_s} \right)^{-\frac{M_0}{M_s}} \right]^{\frac{\kappa}{2}} - 1 \right] + \text{sgn}(M_0 - M_{0r}) \left(\frac{k}{\mu_0 M_s} + c_r H_r \right) \times \left[1 - \kappa \exp\left(-\frac{q}{\kappa M_s} |M_0 - M_{0r}|\right) \right], \quad (11)$$

where M_{0r} is the value of the magnetization at the last reversal point and κ is a parameter that depends on the history of the applied filed:

$$\kappa = 2 - \kappa_r \exp\left(-\frac{q}{\kappa_r M_s} |M_0 - M_{0r}|\right), \quad (12)$$

where κ_r is the value of κ computed at the last reversal point. The first term of equation (10) describes the (linear) demagnetizing filed, the second term represents nonlinear material behavior, and the third term describes (irreversible) hysteresis effects. The initial magnetization curve starts with $M = 0$, $H = 0$, $M_{0r} = 0$, and $\kappa = 1$. The dynamic model is given by the following differential equation:

$$\tau \frac{dM}{dt} = M - M_0, \quad (13)$$

where M_0 can be computed numerically from equation (11).

2.4 Extension to the Jiles-Atherton model of hysteresis

In the case of the Jiles-Atherton model of hysteresis the static magnetization M_0 is related to the applied field by the following differential equation [15]:

$$\frac{dM_0}{dH} = (1-c) \delta \frac{L(H + \alpha M_0) - M_0}{k(1-c) \text{sgn}(\dot{H}) - \alpha [L(H + \alpha M_0) - M_0]} + c \frac{dL(H + \alpha M_0)}{dH} \quad (14)$$

where L is the anhysteretic curve, which is usually assumed to be a Langevin function:

$$L(H) = M_s \left[\coth\left(\frac{H}{a}\right) - \frac{a}{H} \right] \quad (15)$$

and δ is defined as follows:

$$\delta = \begin{cases} 0, & \text{if } \text{sgn}(\dot{H}) = -1 \text{ and } L(H + \alpha M_0) - M_0 \geq 0; \\ 0, & \text{if } \text{sgn}(\dot{H}) = 1 \text{ and } L(H + \alpha M_0) - M_0 \leq 0; \\ 1, & \text{otherwise.} \end{cases} \quad (16)$$

a , c , α , and k are some parameters related to the shape of the major hysteresis loop. The dynamic model can be written as a second order nonlinear differential equation (assuming τ constant) that is obtained by introducing (14) in (6) and taking the derivative with respect to time.

$$\tau \frac{d^2 M}{dt^2} = \frac{dM}{dt} - \left\{ (1-c) \delta \frac{L(H + \alpha M_0) - M_0}{k(1-c) \text{sgn}(\dot{H}) - \alpha [L(H + \alpha M_0) - M_0]} + c \frac{dL(H + \alpha M_0)}{dH} \right\} \dot{H} \quad (17)$$

This equation can be solved by using standard numerical integration techniques.

3. Comparison with the effective-field approximation technique

In this section we compare the dynamic model (6) with another dynamic model of hysteresis based on the effective field approximation technique [16]. We try to analyze under which conditions the dynamic model (6) can be cast into (i.e. completely described by) an effective-field approximation, in which the dynamic effects are modeled by using the following expression for the effective magnetic field:

$$H_{eff} = H + F(\dot{M}), \quad (18)$$

where F is a function of the derivative of the magnetization with respect to time, \dot{M} called generalized moving function. Such models have been applied to the modeling of dynamic effects in ferrites and particulate media. Despite the fundamental differences that exist between the effective filed model (18) and the dynamic model (6) we shall see that there are very interesting similarities between the two models.

Let us denote the static magnetic susceptibility computed at relatively low variation rates of the applied field by $\chi_0 = \frac{dM_0}{dH} = \frac{dM}{dH_{eff}}$. By taking the derivate of (18) with respect to time we obtain:

$$\frac{dH_{eff}}{dt} = \dot{H} + F_{\dot{M}} \frac{d^2 M}{dt^2}, \quad (19)$$

which can be rewritten as:

$$\frac{d^2 M}{dt^2} = \frac{\frac{dH_{eff}}{dt} - \dot{H}}{F_M(\dot{M})} = \frac{\frac{dH_{eff}}{dt} \frac{dM}{dH_{eff}} - \chi_0 \dot{H}}{\chi_0 F_M(\dot{M})} = \frac{\dot{M} - \dot{M}_0}{\chi_0 F_M(\dot{M})}, \quad (20)$$

where $F_M(\dot{M}) = \frac{dF(\dot{M})}{d\dot{M}}$. Similarly, by taking the derivative of (7) with respect to time we obtain:

$$\frac{d^2 M}{dt^2} = \frac{\dot{M} - \dot{M}_0}{\tau}. \quad (21)$$

It is apparent from equations (20) and (21) that the relaxation time dynamic model for structured media is equivalent to the effective-field dynamic model if the relaxation time is:

$$\tau = \chi_0 F_M(\dot{M}). \quad (22)$$

It should be mentioned that due to the rough approximations used in the proof of equation (7), the last equation gives only a first-order approximation for the relaxation time and a more detailed analysis is required to find a better approximation.

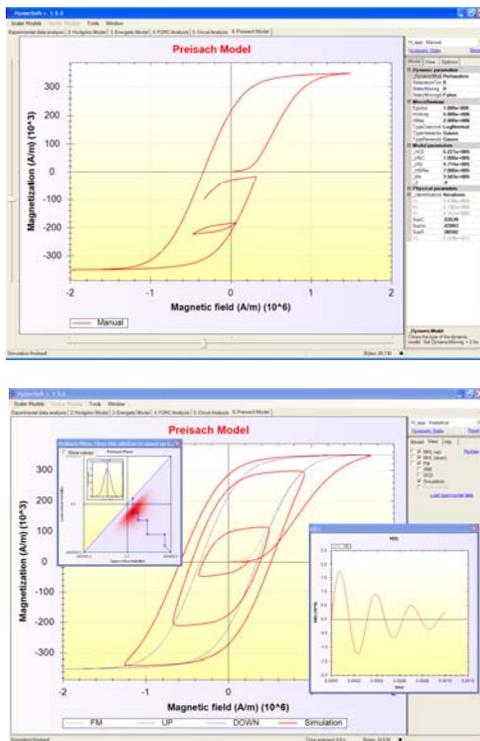


Fig. 1. HysterSoft – simulation framework for phenomenological models of hysteresis. The magnetic field is defined at runtime for the simulation presented in the left side. On the right side is presented a dynamic simulation at $f = 10\text{K Hz}$ for a MnZn ferrite.

4. Numerical implementation and results

In order to find the magnetization as a function of time, equation (6) should be integrated numerically in one-dimension. In this equation $M_0(t)$ should be computed by using an additional two-dimensional numerical integration, which leads to an equivalent three-dimensional integration problem. This fact makes the technique presented in the previous section computationally very expensive. For this reason, in the following, we develop a technique that reduces the computational cost substantially.

It is straightforward to set-up a differential equation for the static magnetization as a function of the applied field. This equation can be written in terms of the (static) magnetic susceptibility χ_0 as follows:

$$\frac{dM_0(t)}{dt} = \chi_0(H, M, t) \frac{dH}{dt}, \quad (23)$$

where dH/dt is the rate of variation of the applied field. The last equation can be coupled with equation (6) to form a system of two differential equations with unknowns $M_0(t)$ and $M(t)$. This system should be subject to appropriate boundary conditions and integrated numerically to compute the total magnetization as a function of time. It is apparent that the numerical overhead required to solve this system is smaller than the numerical overhead required to integrate equations (4) and (6). Indeed, the integration of coupled equations (6) and (23) requires the knowledge of the magnetic susceptibility, which can be usually computed by a one-dimensional integration, while the integration of equation (6) alone requires the knowledge of the static magnetization, which involves a two-dimensional integration in the framework of Preisach-type models of hysteresis.

It is remarkable that equations (6)-(23) provide a universal framework for the analysis of dynamic magnetic processes for any static model of hysteresis. Moreover, since many models of hysteresis (for instance, see [14]) give an analytical equation for the magnetic susceptibility and the magnetization is usually unknown, the numerical technique presented above is very efficient and can be easily implemented numerically.

The technique described in the previous section has been numerically implemented in HysterSoft [17], which is a simulation framework for the modeling and simulation of magnetization processes by phenomenological models of hysteresis (see Fig. 1). In the following we present sample numerical results obtained by using our technique. We consider a strongly interacting magnetic system in which the interaction field distribution is modeled by (8). The parameters of the PM2 model were carefully

identified by using Landau-Lifshitz computations. For more details related to the calibration technique we recommend [11].

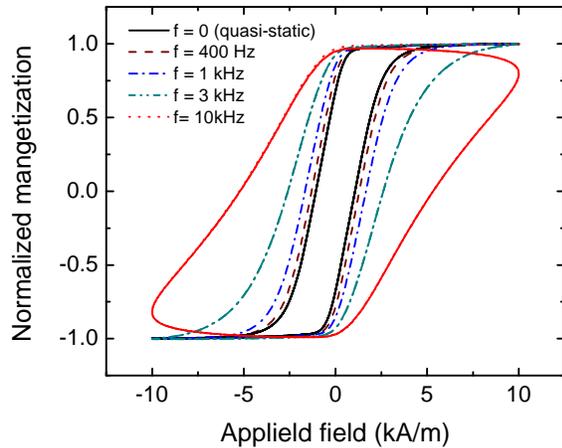


Fig. 2 Hysteresis loops obtained at different frequencies for a particulate media.

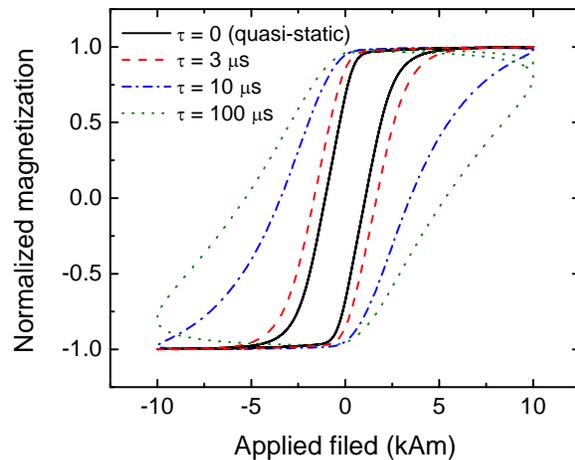


Fig. 3 Hysteresis loops computed for different values of the relaxation time parameter.

In the simulations presented below we use the following values for the parameters of the PM2 model, which were obtained by using micromagnetic simulations: $h_{i\sigma} = 1131$ Oe, $h_{c\sigma} = 750$ Oe, $h_{c0} = 1272$ Oe, and $h_{i0} = 929$ Oe. The reversible component of the PM2 model was chosen exponential with standard deviation $h_{r\sigma} = 3000$ Oe. The weight of the irreversible component was given by $S = 0.95$.

In Figure 2 we present the major hysteresis loop computed at various frequencies of the applied magnetic field: $f = 0$ Hz (quasi-static variations), $f = 400$ Hz, $f = 1$ kHz, $f = 3$ kHz, and $f = 10$ kHz. In Figure 3 we present the major hysteresis loop computed for different values of the relaxation parameter: $\tau = 0$ Hz (quasi-static variations), $\tau = 3 \mu\text{s}$, $\tau = 10 \mu\text{s}$, and $\tau = 100 \mu\text{s}$. Increasing the relaxation time is equivalent to increasing the frequency of the applied field. Therefore, one can produce various simulation results by running only one simulation and scaling the time variable properly.

5. Conclusions

A new dynamic model of hysteresis is presented for the description of rate-dependent magnetization processes in structured materials. The model is based on the relaxation time approximation and is applied to the Preisach model of hysteresis for strongly interacting ferromagnetic particulate systems (PM2). The numerical implementation of the model is discussed in detail. It is shown that our model can be extended to other phenomenological models of hysteresis and can be efficiently implemented numerically for the analysis of dynamic processes.

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