

Effect of dissipative environments on Aharonov-Bohm oscillations of a particle

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A charged particle of mass M on a ring of radius R is coupled to various environments. With Monte-Carlo methods we evaluate the curvature of the Aharonov-Bohm oscillations. For Caldeira-Legget bath of oscillators we find the origin of big discrepancies between results of different groups. For a charged particle in a dirty metal environment we find a quantum phase transition at a critical R_c . At low temperatures T the curvature has the form $1/M^*R^2$ with an R independent $M^* > M$ in the $R > R_c$ phase, while M^* rapidly approached M in the $R < R_c$ phase. The approach to $T = 0$ defines diverging length scales $\sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx 1/4$ in the large and small R phases, respectively. Our preliminary results for a particle with electric dipole in a dirty metal environment for large R and low temperatures show an R independent saturation of $M^* > M$ as in a charged particle case.

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1. Introduction and theoretical model

The problem of interference in presence of a dissipative environment is fundamental for a variety of experimental systems. Interference has been monitored by Aharonov-Bohm (AB) oscillations in mesoscopic rings [1-3] or in quantum Hall edge states [4] in presence of noise from gates or other metal surfaces. Cold atoms trapped by an atom chip are sensitive to the noise produced by the chip [5-7]. In particular giant Rydberg atoms are studied [8] whose huge electric dipole is highly susceptible to such noise. An efficient tool for monitoring the effect of the environment, as proposed by Guinea [9], is to find the AB oscillation amplitude as function of the radius R of the ring. This amplitude is measured by the curvature [10-12] of the ground state energy E_0 at external flux $\phi_x = 0$, i.e.

$$1/M^*R^2 = \partial^2 E_0 / \partial \phi_x^2 \big|_0, \text{ defining an effective mass } M^*.$$

For free particles of mass M this curvature is the mean level spacing $1/MR^2$. The particle can be coupled to a variety of environments, with three systems of particular interest: (i) a Caldeira-Legget (CL) bath [9], (ii) a charged particle in a dirty metal environment [9,13] and (iii) a particle with an electric dipole in a dirty metal environment [14]. System (i) has been studied with a large variety of methods, all showing that the AB amplitude is exponentially suppressed $\sim e^{-\pi^2 \gamma R^2/4}$, i.e. a new length scale $\sim 1/\sqrt{\gamma}$ is generated by the coupling to the environment [9]. System (ii) has been studied by renormalization group (RG) methods [9,15] finding $M^* \sim R^\mu$ with a small μ , a Monte Carlo (MC) numerical method gave [13] $\mu = 1.8$, while a variational scheme [14] gave $\mu = 0$. System (iii) was also studied within the variational scheme¹⁴, leading to $\mu = 0$ as well.

In the present work we use MC methods to analyze mostly system (ii). We find that the energy cutoff used in a previous study [13] is insufficient and a higher cutoff

ω_c is needed. The low T data shows a quantum critical point at R_c at which $1/MR^2 \approx \omega_c$. For all R we find that M^* is R independent, i.e. $\mu = 0$. At $R > R_c$ we find $M^* > M$ and that even the finite T data (at $T < \omega_c$) is determined by this M^* ; at $R < R_c$ we find $M^* = M$.

The approach to $T = 0$ defines diverging length scales $\sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx 1/4$ in the large and small R phases, respectively. A related study shows that similar scales correspond to a dephasing process [16].

The time dependent angular position $\theta_m(\tau)$ of a particle on the ring has in general a winding number m so that $\theta_m(\tau) = \theta(\tau) + 2\pi m\tau$ where $\theta(0) = \theta(1/T)$ has periodic boundary condition and T is the temperature. In presence of an external flux ϕ_x (in units of the flux quantum hc/e) the partition sum has the form

$$Z = \sum e^{2\pi i m \phi} \int D\theta e^{-S(m)}$$

$$S^{(m)} = \frac{1}{2} MR^2 \int_0^{1/T} \left(\frac{\partial \theta}{\partial \tau} + 2\pi m T \right)^2 d\tau +$$

$$\alpha \int_0^{1/T} \int_0^{1/T} d\tau d\tau' \frac{\pi^2 T^2 K[(\theta(\tau) - \theta(\tau') + 2\pi m T(\tau - \tau'))]}{\sin^2 \pi T(\tau - \tau')} \quad (1)$$

where the effect of environments, in each of the 3 cases, is [9,13,14]

$$\begin{aligned}
K(z) &= \sin^2 z / 2; & \alpha &= \gamma R^2 \text{ (i)} \\
&= 1 - [4r^2 \sin^2 \frac{z}{2} + 1]^{-1/2}; & \alpha &= \frac{3}{8k_F^2 l^2} \text{ (ii)} \\
&= 1 - [4r^2 \sin^2 \frac{z}{2} + 1]^{3/2}; \\
\alpha &= \frac{p^2}{e^2 l^2} \frac{3}{8k_F^2 l^2} \text{ (iii)}
\end{aligned}$$

Case (i) is the CL system where α is the coupling to a harmonic oscillator bath; case (ii) is a charge coupled to a dirty metal where k_F is the Fermi wave vector, l is the mean free path in the metal, and $r = R/l$; case (iii) is an electric dipole of strength p coupled to a dirty metal.

We are interested in the effect of the environment on particle. As a measure of this visibility we consider the curvature of the Aharonov-Bohm oscillations the visibility of quantum interference as measured by the

$$\frac{1}{M^*(T)R^2} = \frac{\partial^2 F}{\partial \phi_x^2} \Big|_{\phi_x=0} \quad (3)$$

where $F = -T \ln Z$. It is useful to consider a free particle $\alpha=0$, for which

$$\left(\frac{M}{M^*(T)} \right)_{\alpha=0} = 2\pi^2 t \sum_m m^2 e^{-\pi^2 m^2 t} / \sum_m e^{-\pi^2 m^2 t} \equiv f(t) \quad (4)$$

where $t = 2MR^2T$. This identifies the thermal length $L_T \sim 1/\sqrt{MT}$

In the interacting system a high energy cutoff can be identified by considering $\tau \rightarrow \tau'$ (corresponding to high frequencies ω) so that expansion of $K(z)$ and Fourier transform yield

$$\begin{aligned}
S^{(m)} &\rightarrow \frac{1}{2} \int \frac{d\omega}{2\pi} [MR^2 \omega^2 + 2\pi\alpha K''(0) |\omega|] |\theta(\omega)|^2 \\
&+ (2\pi m)^2 \left[\frac{1}{2} MR^2 T + \alpha K''(0) \right] \quad (5)
\end{aligned}$$

The cutoff ω_c is identified when the kinetic $\sim \omega^2$ and $|\omega|$ interaction terms are comparable, i.e.

$$\omega_c = \frac{2\pi\alpha k''(0)}{MR^2} \quad (6)$$

This ω_c replaces a possibly higher environment cutoff, since significant renormalizations start only below ω_c where the linear $|\omega|$ dispersion leads to $\ln \omega$ terms in perturbation theory and to the need for either RG treatment, or an equivalent variational scheme¹⁴. Note that $K''(0) = 1/2$; r^2 ; $3r^2$ in the 3 models above, hence

$\omega_c = \pi\gamma/M$ in case (i), while $\omega_c \sim \alpha < M/l^2$ in cases (ii) and (iii).

2. Monte Carlo simulations

For the MC numerical method we need to discretize the time axis into a Trotter number N_T of segments, i.e. the time interval of each segment is $\Delta\tau = 1/(TN_T)$. The discrete action is

$$\begin{aligned}
S^{(m)} &= \frac{1}{2} [MR^2 N_T + \alpha K''(0)] \sum_n (\theta_{n+1} - \theta_n + \frac{2\pi m}{N_T})^2 \\
&+ \frac{\alpha\pi^2}{N_T^2} \sum_{n \neq n'} \frac{K(\theta_n - \theta_{n'} + 2\pi m(n-n')/N_T)}{\sin^2(\pi(n-n')/N_T)} \quad (7)
\end{aligned}$$

The $1/2\alpha K''(0)$ term comes from the $n = n'$ interaction term by expanding $K(z)$ around $z = 0$. A key issue in our MC study is the choice of energy cutoff $1/\Delta\tau$ and the corresponding Trotter number $N_T = 1/(T\Delta\tau)$. The correct choice is such that the free kinetic term dominates over the single $n = n'$ interaction term, i.e. $N_T \geq \omega_c/T$; this corresponds to an energy cutoff of ω_c , as anticipated above. A previous MC study on the charge problem¹³ has chosen $N_T = 2/(MR^2T)$, i.e. an energy cutoff of $\approx 1/MR^2$. For large r this cutoff is much smaller than ω_c and is therefore insufficient. Eqs. (1,3) identify $1/M^*(T)R^2 = 2\pi^2 t \sum_m m^2 e^{-\pi^2 m^2 t} / \sum_m e^{-\pi^2 m^2 t} \equiv f(t)$ so that the MC evaluates the fluctuations in winding number (m^2) at external flux $\phi_x = 0$. The procedure is to start with some m , update θ_n at a time position n to $\theta_{n'}$ and accept or reject the change according to the MC rule with probability $\exp[S^{(m)}\{\theta_n\} - S^{(m)}\{\theta_{n'}\}]$. After the N_T points are successively updated, the winding number is shifted to $m' = m \pm 1$ and the shift is accepted or rejected with the probability $\exp[S^{(m')}\{\theta_n\} - S^{(m')}\{\theta_n\}]$.

An update of θ_n is done randomly with a step size that produces an acceptance ratio of about 50%¹¹.

A. CL model

We start to present our data by showing in Fig. 1 the dependence of M/M^* on the number of iterations for the CL model with $t = 0.01$, $\gamma = 0.5$. To estimate errors we evaluate the correlation function between different paths for a given run and deduce a correlation length ξ . We discard the initial 105 MC iterations and then evaluate the standard deviation σ of the average data; the error is then¹⁷ $\sigma\sqrt{2\xi + 1}$. The reason for large errors shown in Fig. 1 and the necessity to discard a very large number of initial iterations is a huge correlation length (typically, a few thousand units), and we, therefore, need a few millions of iterations in order to decrease the standard deviation of the average data. We have checked a few points of the CL model at low temperatures for different γ and found a good agreement with the results presented in¹¹.

Extrapolation to zero temperature for the CL model with $\gamma = 0.5$ is shown in Fig. 2.

B. A charged particle coupled with a dirty metal

We now present results for the main subject of our research: a charged particle coupled with a dirty metal.

The inset in Fig. 3 shows the N_T dependence of M/M^* for the charge problem with $r = 5$, $t = 0.2$, $\alpha = 0.019$.

The choice¹³ $N_T = 2/(MR^2T) = 0.8$ is clearly insufficient; saturation sets in around $N_T \approx 100$ which is of order of $\omega_c/T = 30$. In the following we choose our N_T , in the charge problem, to be $N_T = 40\alpha^2/t = 10\omega_c/(\pi T)$, i.e. $N_T = 95$ for the inset parameters. For the dipole case, where ω_c is 3 times higher we choose

$N_T = 120\alpha^2/t = 10\omega_c/(\pi T)$. Fig. 3 shows that for $r = 5$, $t = 0.2$, $\alpha = 0.02$ (red squares) saturation indeed sets in near $N_T = 300$.

This high value of N_T restricts realistic MC studies. We have noticed, however, that this high N_T is necessary only in the vicinity of $n = n'$ in the double sum of (7), where the summand is rapidly varying. Hence the double sum is taken over all points only in vicinity of the singularity, i.e. for $|n - n'| < 0.03N_T$.

For points that are further separated we coarse grain the sum with fewer points, corresponding to an effective $N_T = 1/t$.

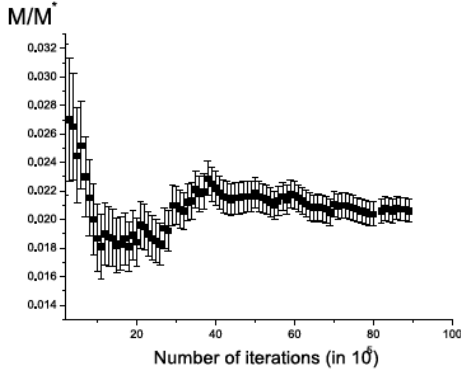


Fig. 1. Dependence of the effective mass on the number of iterations for CL model with $t=0.01$, $\gamma=0.5$

The results of this procedure are shown by the green circles in Fig. 3, and are in agreement with the full calculation that includes all N_T points.

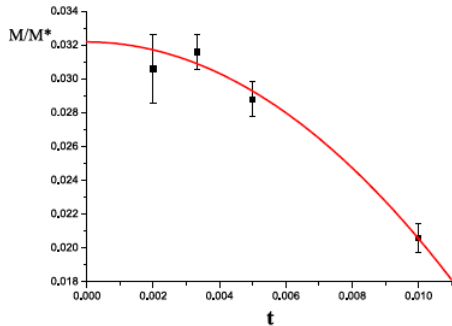


Fig. 2. Extrapolation to zero temperature for the CL model with $\gamma=0.5$.

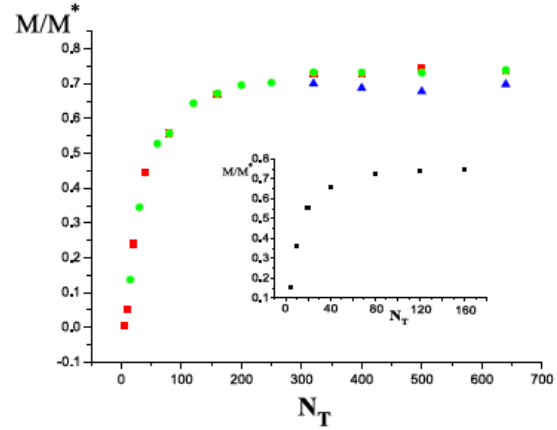


Fig. 3. Trotter number dependence of the effective mass for the dipole case with $r = 5$, $t = 0.2$, $\alpha = 0.02$, using (i) all N_T points in the double sum Eq. (7) – red squares, (ii) For points $|n - n'| > 0.03N_T$ sum is coarse grained (see text) – green circles, (iii) the whole sum is coarse grained – blue triangles. Inset: The charge case with $r = 5$, $t = 0.2$, $\alpha = 0.019$ using all N_T points in the ums

The double sum has then $\approx 1/2 \cdot 10^{-3}N_T^2 + 1/2 \cdot t^{-2}$ terms, much less than the $1/2N_T^2$ terms of the full calculation. We also show data where the double sum is coarse grained at all points, including those near $n = n'$, by blue triangles. Here the double sum has only $1/2 \cdot t^{-2}$ terms; this data has significant deviations from the full calculation. Before we present main results of this research, we wish to stress that there are two independent large parameters in the MC procedure: a Trotter number N_T and a number of iterations; the convergence of the results should therefore be checked separately for each of those two parameters.

We proceed to present our results on $M/M^*(T)$. At low temperatures we evaluate $\langle m^2 \rangle$, and the average involves typically many values of m . To estimate errors we evaluate the correlation function for a given run and deduce a correlation length ξ . We discard the initial 10^4 MC iterations and then evaluate the standard deviation σ of the average data; the error is then¹⁷ $\sigma \sqrt{2\xi + 1}$.

We typically find a short correlation length of a few units and we run till an error of $\sim 2\%$ is achieved; the number of iterations is then $\approx (1 - 2) \cdot 10^5$, where each iteration is an update of N_T values of the θ_n . At high temperatures $t > 1$, where $M/M^* \leq 10^{-3}$, the probability of $m \neq 0$ becomes extremely small so that just $m = \pm 1$ determine the outcome¹¹. Hence we evaluate $\langle m^2 \rangle = 2(e^{S_1} - S_0)_0$, averaging with e^{-S_0} . In this method we find a rather long correlation length of 10^3 , yet there is no need to vary m and a 2% accuracy can be achieved after $\approx (1 - 2) \cdot 10^5$ iterations.

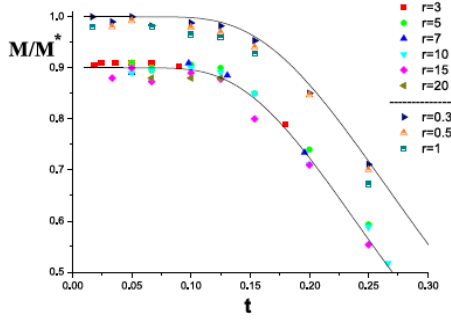


Fig. 4. AB curvature as function of reduced temperature with $\alpha = 0.019$. All $r \geq 3$ values fit the renormalized form $0.9f(t/0.9)$ – the lower curve. At $r \leq 1$ the data approaches $f(t)$ of a free particles – the upper curve.

In Fig.4 we show our data at low temperatures, $t < 0.3$. We discuss first the data for $r \geq 3$, where we observe saturation at $M/M^* \approx 0.9$, independent of r . The possible dependence of $M^*(r)$ at $T = 0$ of interest as a means of monitoring anomalies in the ground state^{9,13}. Previous studies proposed $M^* \sim r^\mu$ with either^{9,15} a small μ or¹³ $\mu = 1.8$ for $\alpha > 1$ or¹⁴ $\mu = 0$. Our MC is consistent with the $\mu = 0$ prediction¹⁴, though the numerics cannot exclude a small $\mu \leq 0.05$. The $\mu = 0$ result shows that the AB curvature $\sim 1/r^2$ is the same as for free particles, i.e. the ground state has no anomaly and is fully described by an r independent M^* . Furthermore, Fig. 4 shows that M^* determines the finite temperature behavior, as long as $T < \omega_c$. Thus if we replace $M \rightarrow M^* = M/0.9$ in Eq. (4) we obtain the lower curve $0.9f(t/0.9)$ in Fig. 2 which is a good fit to the data. The thermal length is then $L_T \sim 1/\sqrt{M^* T}$. We turn now to discuss the $r \leq 1$ data in Fig. 4. We find here that the data at low t saturates at $M^* = M$, i.e. no renormalization at all. The transition from $M^* = M/0.9$ to $M^* = M$ at low T is at $r_c \approx 2$. We propose that this transition occurs when the free particle level spacing $1/2MR^2$ is comparable with ω_c , leading to $r_c \approx 1/\sqrt{4\pi\alpha} \approx 2$. At $r < r_c$ the bath is not effective in coupling excited states of the particle and an approach to $M^* = M$ is expected. However, the sharp transition between the two regimes is surprising.

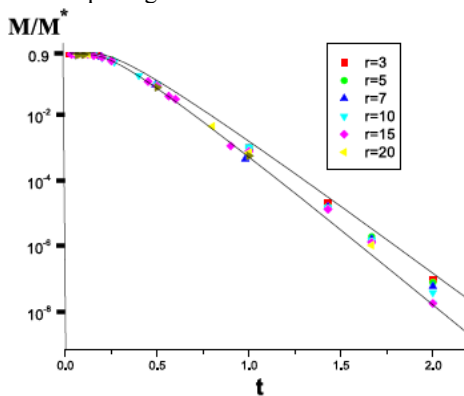


Fig. 5. AB curvature including high temperatures with $\alpha = 0.019$. All data fall in between the upper line $f(t)$ and the lower line $0.9f(t/0.9)$.

In Fig. 5 we show our $r \geq 3$ data up to $t = 2$. The data falls in between two lines: $0.9f(t/0.9)$ and $f(t)$. The lower curve $0.9f(t/0.9)$ corresponds to the renormalized system and fits data with $T \ll \omega_c$, i.e. $t \ll 4\pi\alpha r^2$. For a fixed t as r decreases T approaches ω_c and the data approaches the upper curve which is the unrenormalized free particle form $f(t)$.

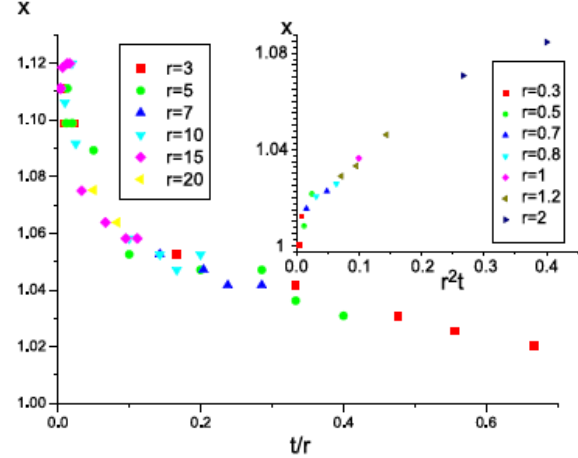


Fig. 6. Scaling of the x variable in $M/M^* = f(tx)/x$. Note the distinct scaling forms of the large and small r phases.

We therefore parameterize our data by a function $x(r, t)$ such that $M/M^* = f(tx)/x$. In the $r \geq r_c$ phase we expect $x(r, t)$ to vary between $x = 1$ at high t and

$x = 1/0.9$ at low t , while for $r \leq r_c$ we expect the opposite trend as t varies. Fig. 6 shows that $x(r, t)$ has a scaling form that is markedly distinct in the two phases.

In the $r > r_c$ phase the scaling variable is t/r while in the $r < r_c$ it is $r^2 t$. The scaling for $r \geq r_c$ is consistent with a high t expansion¹³ that yields $x = 1 + 4\alpha t/(\pi t)$, though we have not tested scaling with α . Temperature affects both renormalizations: due to the bath, controlled by T/ω_c , and due to the free particle spectra, controlled by $t = 2MR^2 T$ via Eq.(4). To focus on the bath renormalization, it is useful to study scaling of r with t fixed, rather than with T fixed. In both phases, increasing r at a fixed t leads to a larger M^* , which is expected since more degrees of freedom become coupled. However, since the scaling variables are t/r and $r^2 t$ in the two phases, a remarkable result follows that the t dependence is opposite. In particular, at $T = 0$ the renormalization parameter x is maximal for $r > r_c$ while it is minimal ($x = 1$) for $r < r_c$. We can interpret r_c as an unstable fixed point, with RG flow for $r > r_c$ to $r \rightarrow \infty$ leading to fully renormalized M^* , while for

$r < r_c$ the flow is to $r \rightarrow 0$ leading to an unrenormalized $M^* = M$ at $T = 0$. Recalling that $t = 2MR^2 T$, we conclude that the data shows length scales

$r_M \sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx \frac{1}{4}$ in the large and small r

phases, respectively. At scales $r > r_M$ the system approaches its $T = 0$ fixed point, which depends on the

initial r . Similar length scales, with the same exponents as above, were recently identified as dephasing lengths¹⁶.

C. A dipole coupled with a dirty metal

Finally, we present preliminary data for a third model: a particle with an electric dipole coupled with a dirty metal environment. As has been mentioned above we need a Trotter number N_T 3 times higher than for a corresponding value of α in a charged particle case (see

Fig. 3). Fig. 7 represents results for large R and low temperatures for a model with $\alpha = 0.02$. It shows an R independent saturation of $M^* > M$ as in a charged particle case.

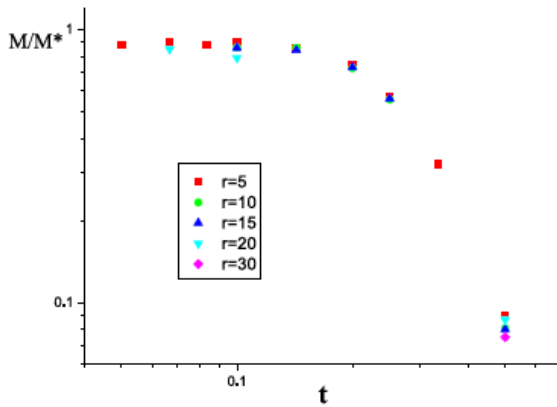


Fig. 7. Dependence of the effective mass on the temperature for a dipole model with $\alpha = 0.02$.

3. Conclusion

We summarize but noting that we have found out the reason for large errors in the CL model: a huge correlation radius for successive paths makes it necessary to use a very large number of iterations. For a charged particle model we have found an unexpected phase transition between two phases of model (ii) with distinct T dependence and renormalization properties. In both phases the ground state corresponds to an R independent M^* while the approach to this ground state is via distinct scaling laws. We have identified the exponents for the length scales as $\sim T^{-\eta}$ with $\eta \approx 1$ and $\eta \approx \frac{1}{4}$ in the two

phases. We have also found that in both phases there appears a thermal length with a critical exponent $\eta \approx 1/2$, which is the remainder of the free particle behavior. We speculate that this last exponent can account for the experimental data observed in⁷. More data for larger values of the coupling constant α is necessary in order to clarify dependence of the critical radius r_c . As for a dipole

model, our preliminary results resemble the corresponding results for a charged particle model.

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