

# Enhancement of measurement accuracy in fiber Bragg grating sensors by using a nonlinear least square technique

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Wavelength detection accuracy in fiber Bragg grating sensors can be increased by use of digital signal processing after photo-detection. Nonlinear least square technique algorithms was implemented and used to improve the Bragg wavelength detection. Simulation and experimental results show that the detection accuracy of the nonlinear least square technique is about 5 times better than the FIR method.

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## 1. Introduction

Fiber Bragg gratings (FBGs) have been used as sensing elements for a number of measurands including strain, temperature and pressure [1-2]. The wavelength-encoded nature of the reflected signals from FBGs allows for absolute measurements of the measurands. In order to measure the measurands, detection of the small shift in Bragg wavelength is essential. Several schemes have been reported for detecting the Bragg wavelength shift in FBG sensors [2]. The use of an edge filter to convert the wavelength shift to intensity variation was a simple approach, but this technique was of limited wavelength detection accuracy, although the cost is low. The most sensitive technique may be to use an unbalanced two beam interferometer to convert the Bragg wavelength shift into a change in the phase difference between the two arms and then into an intensity variation in the interferometer output. This technique has demonstrated to obtain very high detection accuracy. However, the environmentally induced phase variations in the interferometer arms seriously affect the performance for static measurements and hence it is only suitable for dynamic measurements. The Fourier transform spectroscopy is also used for Bragg wavelength detection. It uses a scanning interferometer to generate an interferogram and Hilbert transform technique is used to restore the sensing information [3]. However, this technique is complicated and expensive, since an additional stable single frequency laser is needed to monitor the optical path difference (OPD) of the scanning interferometer.

Among all the wavelength detection techniques, the most popular method is the peak detection technique, which uses a tunable laser source to scan through the

spectrum of light reflected from the FBG and measure the filter wavelength corresponding to the maximum of the system output [4]. The peak detection technique can be applied to interrogate a number of FBG sensors based on wavelength division multiplexing (WDM) principle. However, the wavelength detection accuracy of the conventional peak detection technique is limited especially when the signal level of the reflected signal is low. Several signal processing methods, e.g., centroid detection algorithm coupled with curve fitting [5], spectrum correlation technique [6] and digital matched filter [7], have been proposed to improve the detection accuracy.

In previous papers we have studied the trace gas detection by OPO [8] and finite impulse response digital filter [9]. Improving of the determination accuracy in FBG sensors [10] and application of a support vector machine trace detection by temperature-tuning OPO [11] were reported. A wide tunable range fiber Bragg grating filter was described in [12].

In this paper, nonlinear least square (NLS) technique is proposed for the detection of Bragg wavelength. The principle of operation is shown in Section II. The Simulation results are presented in Section III, the experimental results are presented in Section IV and a conclusion is given in Section V.

## 2. Principle of operation

Computer simulation was carried out to demonstrate the effectiveness of digital filter techniques for noise reduction in a FBG sensor system. The system diagram used in our simulation and experiment is shown in Fig.1. The system uses a narrow band tunable laser source (TLS) and optical spectrum analyzer (OSA). The laser light from the TLS is delivered, through a variable optical attenuator

(VOA) and a 3dB directional coupler to the FBG, and the light reflected from the FBG is coupled back to the OSA through the same directional coupler. The FBG was held by a fixed stage (FS) and a translation stage (TS). Strain can be applied on the FBG by manually tuning the TS.

Assume that the reflection spectrum of the FBG is described by Gaussian functions [4]

$$R(\lambda) = R_0 \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_B}{\Delta} \right)^2 \right] \quad (1)$$

where  $R_0$ ,  $\lambda_B$ , and  $\Delta$  are the peak reflectivity, Bragg wavelength and full width at half maximum (FWHM) of the FBG, respectively. The spectrum of the reflected light at the OSA may be written as

$$I_s(\lambda) = \frac{I_0 \alpha}{4} \cdot R(\lambda) = \frac{I_0 \alpha}{4} R_0 \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_B}{\Delta} \right)^2 \right] \quad (2)$$

where  $I_0$  is the output intensity of the light source,  $\alpha$  is the intensity transmittivity of the VOA and the factor 1/4 is due to the 3dB directional coupler. The linewidth of the source has been assumed to be much narrower than that of FBG and the laser output power was regarded as constant over the FBG reflection spectrum. The purpose of the VOA is to modify the output intensity and thus change the signal-to-noise ratio (SNR).

In practice, random noise may exist during the detection and hence the measured FBG spectrum may be expressed as

$$I(\lambda) = I_s(\lambda) + n(\lambda) = \frac{I_0 \alpha}{4} R_0 \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_B}{\Delta} \right)^2 \right] + n(\lambda) \quad (3)$$

Where  $n(\lambda)$  is the noise contribution to the signal. The SNR of  $I(\lambda)$  is then  $10 \log(1/\sigma)$ , where  $\sigma$  is the standard deviation of the Gaussian white noise.

The principle of the NLS technique [8] is explained as follows. The NLS method assumes that the function of the reflection spectrum is of the known analytical form, namely Gaussian shape in this case, depending on three parameters of  $A$ ,  $\lambda_B$  and  $\Delta$ . To simplify our discussion, the spectrum of the reflected light can be written as

$$I_s(\lambda; A, \lambda_B, \Delta) = A \exp \left[ -4 \ln 2 \left( \frac{\lambda - \lambda_B}{\Delta} \right)^2 \right] \quad (4)$$

where  $A$  is equal to  $I_0 R_0 \alpha / 4$ . For the measured noisy Gaussian spectrum, the function  $I_s(\lambda)$  are tabulated at  $m$  values  $y_1 = I_s(\lambda_1) \dots y_m = I_s(\lambda_m)$ . Now consider the overdetermined set of  $m$  equations

$$y_1 = I_s(\lambda_1; A, \lambda_B, \Delta)$$

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$$y_m = I_s(\lambda_m; A, \lambda_B, \Delta) \quad (5)$$

It is desired to solve these equations to obtain the values  $A$ ,  $\lambda_B$ ,  $\Delta$ , which best satisfy this system of equations. In the beginning, the values of  $(A, \lambda_B, \Delta)$  are randomly selected and then define

$$d\beta_i = y_i - I(\lambda_i; A, \lambda_B, \Delta) \quad (6)$$

Now obtain a linearized estimate for the changes ( $dA$ ,  $d\lambda_B$ ,  $d\Delta$ ) needed to reduce  $d\beta_i$  to 0,

$$d\beta_i = \frac{\partial I_s}{\partial A} \cdot dA + \frac{\partial I_s}{\partial \lambda_B} \cdot d\lambda_B + \frac{\partial I_s}{\partial \Delta} \cdot d\Delta \Bigg|_{\lambda_i} \quad (7)$$

This can be written in component form as

$$d\beta_i = B_{ij} d\phi_j \quad (8)$$

where  $\phi = (A, \lambda_B, \Delta)$  and  $B$  is the  $m \times 3$  matrix

$$B_{ij} = \begin{pmatrix} \frac{\partial I_s}{\partial A} \Big|_{\lambda_1} & \frac{\partial I_s}{\partial \lambda_B} \Big|_{\lambda_1} & \frac{\partial I_s}{\partial \Delta} \Big|_{\lambda_1} \\ \vdots & \vdots & \vdots \\ \frac{\partial I_s}{\partial A} \Big|_{\lambda_m} & \frac{\partial I_s}{\partial \lambda_B} \Big|_{\lambda_m} & \frac{\partial I_s}{\partial \Delta} \Big|_{\lambda_m} \end{pmatrix} \quad (9)$$

In more concise matrix form

$$d\beta = B d\phi \quad (10)$$

Applying the matrix transpose of  $B$  to both sides gives

$$B^T d\beta = (B^T B) d\phi \quad (11)$$

Defining  $a = B^T B$  and  $b = B^T d\beta$  then gives matrix equation

$$a d\phi = b \quad (12)$$

which can be solved for  $d\phi = (dA, d\lambda_B, d\Delta)$  using standard matrix techniques such as Gaussian elimination. This offset is then applied to  $\phi$  and then a new  $d\beta$  is calculated. By iteratively applying this procedure until the elements  $d\phi$  become smaller than some prescribed limit, a solution is obtained [13].

### 3. Simulation results

Consider now an FBG sensor of an ideal Gaussian reflection spectrum corrupted by additive Gaussian white noise. The FBG has a Bragg wavelength of 1550 nm and a FWHM of 0.2 nm. The noise was randomly generated by a computer program. Fig. 2 shows the ideal Gaussian FBG spectrum without noise, the FBG spectra before and after the use of a NLS method. The FBG spectrum with noise was sampled from 1549.5 nm to 1550.5 nm by 200 points. The introduction of Gaussian white noise greatly destroyed the FBG spectrum and made it difficult to identify the location of the peak reflectivity, namely Bragg wavelength. The recover of the Bragg wavelength was achieved by using NLS method. It should be noted that convergence is often greatly improved by picking initial values close to the best-fit value. Therefore, in order to obtain good initial values, the measured data were initially filtered by a moving average window with a window size of 25. After filtering, the amplitude, the wavelength corresponding to the reflection maximum and the FWHM were then used to generate the initial values  $A$ ,  $\lambda_B$  and  $\Delta$ , respectively. Before applying the NLS, the SNR of the spectral signal is 3dB and the wavelength corresponding to the peak of the spectrum is difficult to identify. After the NLS, the Bragg wavelength is obtained and the waveform is much smoother.

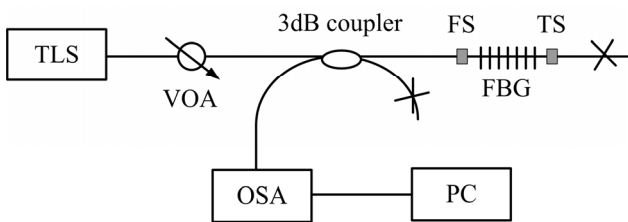


Fig. 1. Schematic of simulation and experimental setup. TLS: tunable laser source; VOA: variable optical attenuator; OSA: optical spectrum analyzer; FS: fixed stage; TS: translation stage; PC: personal computer; FBG: fiber Bragg grating.

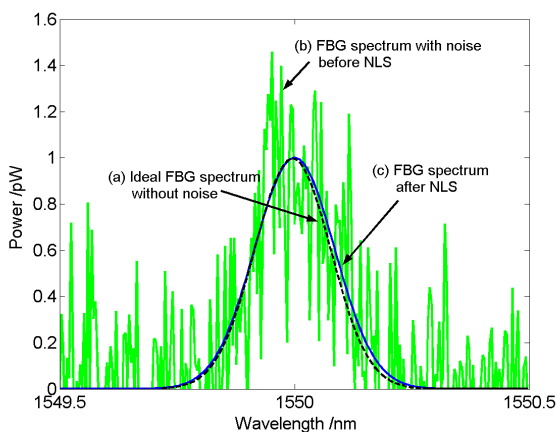


Fig. 2. (a) Ideal Gaussian Bragg spectrum without noise(dash line); (b) FBG spectrum with noise (solid line); (c) signal after NLS (solid line).

The performance of the NLS was studied when the number of sampling points was increased from 200 to 8000. The root-mean square (RMS) errors of the estimated Bragg wavelength as a function of the number of sampling points are demonstrated in Fig. 3. In order to calculate the RMS error, the wavelength measurement was repeated 20 times for each number of sampling points. For the number of sampling points ranging from 200 to 1000, the wavelength measurement is greatly influenced by the step size. Increasing the number of sampling points, i.e. reducing the step size, can reduce the wavelength measurement errors to a great extent. When the number of sampling points exceeds 2000, the improvement of the Bragg wavelength detection accuracy cannot be further increased. When the number of sampling points approaches 8000, the RMS error approaches 0.2pm. However, the performance of the NLS method cannot be further enhanced by simply increasing the number of sampling points, due to the effects of the noise.

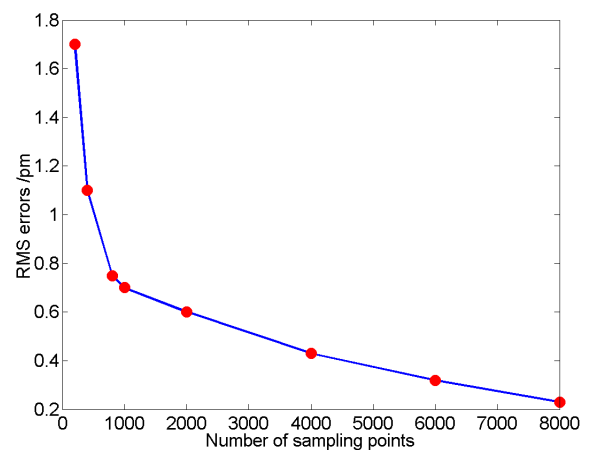


Fig. 3. RMS errors of the Bragg wavelength measurement vs. number of sampling points.

Fig. 4 shows the RMS errors of the estimated Bragg wavelength for different values of SNR. For comparison, the RMS errors for the windowed linear phase FIR digital filter were also demonstrated. The FIR filter utilized a 501th order Kaiser window. The number of sampling points was chosen to be 1000 for both methods. It is clear the performance of the NLS method is much better than the FIR methods, especially for the large noise, i.e. small value of SNR. As SNR increase, the RMS errors for both methods are greatly reduced. The RMS errors for both methods tend to be a constant when SNR is larger than 25dB. The RMS error for the NLS method reaches 0.04pm as the SNR approaches 30dB, whereas the RMS error for the FIR method comes close to 0.21pm.

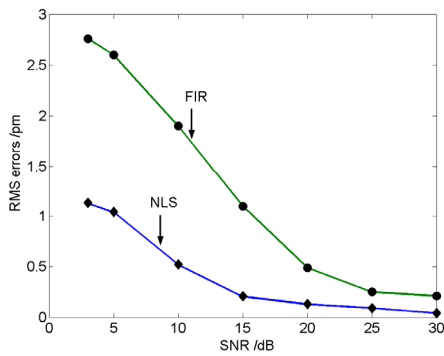
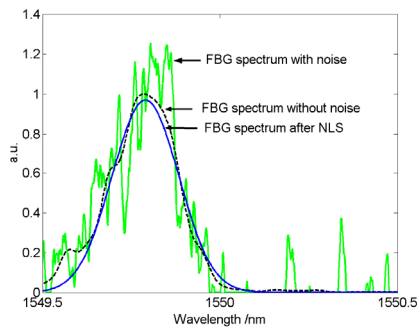


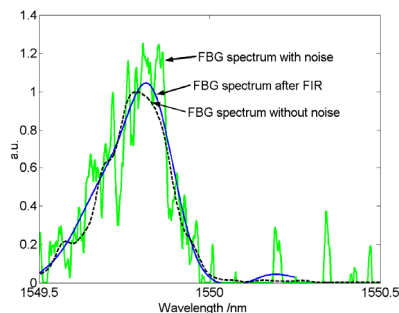
Fig. 4. RMS errors for the Bragg wavelength measurement for different SNR.

#### 4. Experimental results and discussion

Experiments were conducted using the same setup shown in Fig. 1. The TLS has the power about  $30\mu\text{W}$ . The wavelength range of the OSA was set from 1549.5 to 1550.5nm and sampled by 1000 points, corresponding to a step size of 1pm. The ‘ideal’ reflection spectrum was measured by setting the VOA to 0dB, which corresponds to the large SNR. The noisy reflection spectrum was measured by setting the VOA to 20dB, which effectively reduces the SNR value and thus give rise to a noisy spectrum.



(a)



(b)

Fig. 5(a). FBG spectra before and after NLS filtering; (b) FBG spectra before and after FIR filtering.

Fig. 5(a) shows the spectra before and after NLS filtering when the FBG was unstrained. For comparison, the filter results by using FIR filter are demonstrated in Fig. 5(b). The ‘ideal’ reflection spectrum was demonstrated in the figures with 20dB reduction in intensity for comparison. It is seen that both methods have good filter results and make the wavelength corresponding to the peak amplitude easier to identify. The reflection spectrum after NLS filtering has a perfect Gaussian shape and the Bragg wavelength is close to the ideal one. However, in the reflection spectrum after FIR filtering, the measured wavelength corresponding to the peak amplitude is more subject to the noise.

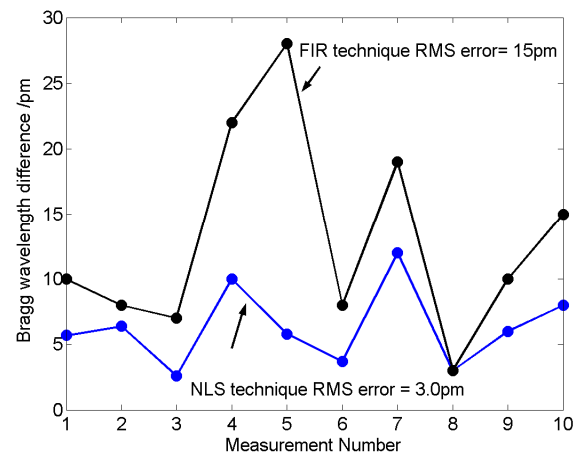


Fig. 6. Bragg wavelength difference obtained from 10 independent measurements with the NLS and FIR techniques.

In order to compare these two methods, the absolute values of measured Bragg wavelength differences with these two techniques are shown in Fig. 6. The measurement was repeated 10 times within 3min. Within this time period, the Bragg wavelengths of the FBG can be regarded to be constant. The measurement wavelength errors by using NLS method are much smaller than those by using FIR technique. In addition, the performance of the NLS method is more stable than the FIR method, for the reason that the FIR method is easily influenced by the noise. This can be seen from their difference in the RMS errors. The RMS error is 3pm for the NLS technique, whereas 15pm for the FIR method. The detection accuracy of the NLS technique is thus 5 times better than the FIR method.

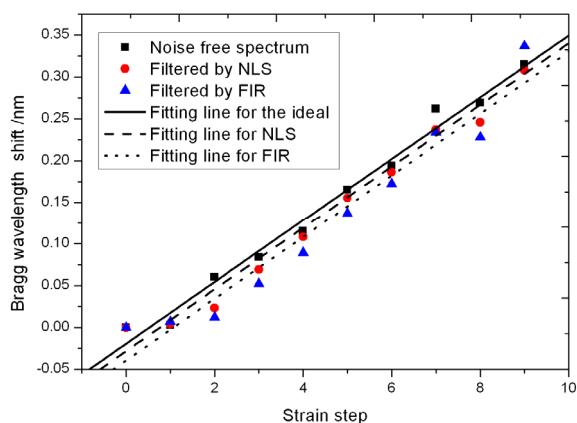


Fig. 7. Bragg wavelength shift for the applied strain.

The performance of the NLS is further investigated in FBG strain sensor. Similarly, the 'ideal' signals were obtained by setting the attenuator to 0dB, while the 'estimated' signals were obtained by setting the attenuator to 20dB and then filtered by the NLS method. FIR method was also used as a reference. In this experiment, the Bragg wavelength was only measured once for each strain step and the strain step used is  $25\mu\epsilon$ . Fig. 7 shows the Bragg wavelength shift for the applied strain. It is seen that the filtered results by using NLS are much closer to the ideal one. The standard deviations are 13pm, 15pm and 27pm for the noise free spectrum, NLS method and FIR method, respectively. This further proves that the NLS method can significantly improve the measurement accuracy since its performance is not considerably affected by the white noise, whereas the FIR method is.

## 5. Conclusion

A digital post-processing technique based on NLS technique has been developed and applied to improve the wavelength detection accuracy in FBG sensors. Computer simulations and experiments with LED source with a conventional OSA show the appropriate designed digital filters can improve the measurement significantly. The

NLS algorithm performs better than the FIR algorithm for this application. The digital techniques can readily be used in combination with other interrogation schemes for accurate wavelength measurement in FBG sensing system.

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