Fast phase retrieval by temporal phase shifting and double-digital fringe projection

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This paper presents the application of a five-step temporal phase shifting technique for recovering the phase map from fringe patterns. The fringe patterns come from a double-digital fringe projection along the object under study allowing, inclusive, the measurement of the extremes of the object, where with other techniques it is impossible due to shadows generated for the angle of projection. Experiments demonstrate that the combination of these techniques is very simple and easy to implement for engineering purposes. A few experiments are presented and the surface profiles of different objects are shown.

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1. Introduction

It is very well known that 3D shape measurement has been very important in the last several decades and today is widely applied in some many fields like manufacturing, medical image diagnosis and medical sciences, computer sciences [1,2], and other applications like 3D surface shape measurement of Micro Electro Mechanical Systems (MEMS) [3-5].

Traditionally, laser interferometry has been used with this objective because of the accuracy and stability that it offers. Together to laser interferometry, a big variety of phase-shifting algorithms have been developed for high resolution and high accuracy measurements [6] providing advantages of being less sensitive to surface reflectivity variations because they measure objects point by point.

For high accuracy measurements and because the interferometry produces fringes, a phase unwrapping algorithm is needed to obtain continuous a phase map. Many phase unwrapping algorithms have been also developed [7].

Digital fringe projection profilometry for measuring the topography of 3D objects is a very active research area in optical metrology [8-12] and one of the most important 3D shape measurement methods [13-14]. A digital fringe projection, by means of a digital projector, and a phase-shifting algorithm is able to control the phase shift and the wavelength accurately, this is because the projected fringes are digitally generated in a computer. In this research we propose a simultaneous digital fringe projection through two digital projectors and the application of a temporal phase-shifting algorithm to obtain the phase of a couple of objects.

In section 2 of this work, the theoretical description of the proposed technique is described as well as the experimental setup to be used.

Section 3 contains a simulated process. 5 fringes patterns are computationally generated and then processed by a five-step phase shifting algorithm. The wrapped and unwrapped phase retrieval is presented. In the same section, the results obtained by applying the proposed technique to two model airplanes are also shown, covering only one section of the model airplane, in this case, the tail of one of them and the fuselage of the other one.

2. Experimental set-up and theoretical description

In this proposal two single frequency fringe patterns are superposed to form a new fringe pattern of interference. Three fringe elements are considered: the two single-frequency fringes and the fringe generated by the superposing of the two linear patterns simultaneously.

In Fig. 1 the optical configuration used for this work is presented. The two fringe patterns, with different spatial frequencies, are designed in a computer and are projected over the object by a digital light projector simultaneously. A camera is used to capture an image of the deformed fringe. The resulted fringe pattern of interference is then processed by the computer to obtain the wrapped and unwrapped phase.

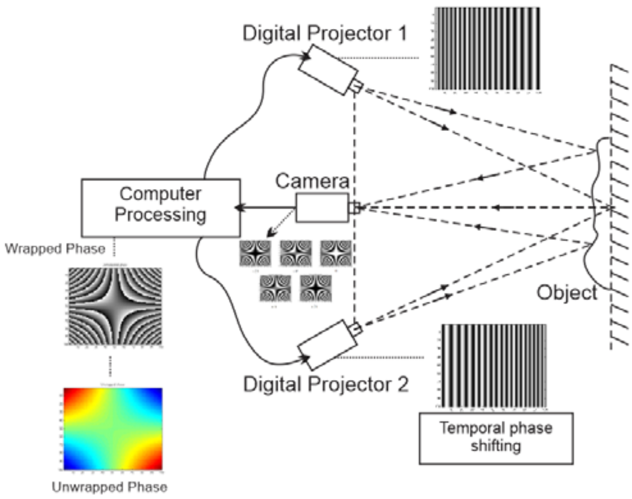


Fig. 1. Optical setup applied for the double digital fringe projection technique.

Numerically, the resulted fringe pattern of interference can be expressed as:

$$I(x,y)=a(x,y)+b_1(x,y)\cos[\phi_1(x,y)]+ b_2(x,y)\cos[\phi_2(x,y)]+b_3(x,y)\cos[\phi_3(x,y)] \quad (1)$$

where $a(x,y)$ is the average background intensity, $b_{n=1,2,3}(x,y)$ is the intensity modulation and $\phi_{n=1,2,3}(x,y)$ denotes the phases that can be rewritten in function of carried phases and initial phases as follows:

$$\begin{aligned}\phi_1(x,y)&=\varphi_\alpha(x,y)+\varphi_1(x,y)\\ \phi_2(x,y)&=\varphi_\beta(x,y)+\varphi_2(x,y)\\ \phi_3(x,y)&=\varphi_\gamma(x,y)+\varphi_3(x,y)\end{aligned} \quad (2)$$

then, Eq. (1) can be rewritten in function of Eq. (2):

$$\begin{aligned}I(x,y)&=a(x,y)\\ &+b_1(x,y)\cos[\varphi_\alpha(x,y)+\varphi_1(x,y)]\\ &+b_2(x,y)\cos[\varphi_\beta(x,y)+\varphi_2(x,y)]\\ &+b_3(x,y)\cos[\varphi_\gamma(x,y)+\varphi_3(x,y)]\end{aligned} \quad (3)$$

However, the digital camera used in the experimental set-up has only the possibility of register one fringe pattern of interference, coming precisely from the superposition of the fringe patterns digitally projected over the object. Then, the final fringe pattern of interference can be written as:

$$I(x,y)=A(x,y)+B(x,y)\cos[\phi(x,y)] \quad (4)$$

where $A(x,y)$ is the average intensity, $B(x,y)$ is the amplitude modulation of fringes and $\phi(x,y)$ is the phase to be solve for, also known as the wrapped phase.

Considering that $B(x,y)=A(x,y)\upsilon(x,y)$, where $\upsilon(x,y)$ is the interference fringe visibility, Eq. (4) can be rewritten as

$$\begin{aligned}&I(x,y)=\\ &A(x,y)+A(x,y)\upsilon(x,y)\cos[\phi(x,y)]=\\ &A(x,y)[1+\upsilon(x,y)\cos[\phi(x,y)]]\end{aligned} \quad (5)$$

As it can be seen, there are three unknowns, $A(x,y)$, $\upsilon(x,y)$, and $\phi(x,y)$. So, in this way, it is necessary a minimum of three measurements to get the phase determined.

Temporal phase measurement methods are good options to solve the problem of determining phase, in this case, Eq. (5) can be described with a new term added to the cosine argument just as follows:

$$I(x,y)=A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)+\alpha(t)]\} \quad (6)$$

where $\alpha(t)$ is a known phase shifted between the two fringe patterns projected that will generate new fringe patterns of interference during the camera's integration time, assuming that this change has to be the same from frame to frame recorded by the camera. It means that one of the projected fringes of the optical setup shown in figure 1, must have a phase shifted according to α(t) in order to recovered the unknown phase $\phi(x,y)$

For a general case, where N measurements of I(x,y) are recorder while the phase is shifted and since the camera integrates over time, a single fringe pattern of interference might be integrated over a change in relative phase, Δ, and can be written as

$$\begin{aligned}&I_i(x,y)=\\ &\frac{1}{\Delta}\int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2}A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)+\alpha(t)]\}d\alpha(t)\end{aligned} \quad (7)$$

where i = 1 to N, this expression is applicable for any phase-shift technique because of the phase shift Δ. After integrating (7), the recorded intensity is

$$\begin{aligned}&I_i(x,y)=\\ &A(x,y)\left\{1+\upsilon(x,y)\frac{\sin\left(\frac{\Delta}{2}\right)}{\frac{\Delta}{2}}\cos[\phi(x,y)+\alpha_i]\right\}\end{aligned} \quad (8)$$

$$I_i(x,y)=A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)+\alpha_i]\} \quad (9)$$

α can be change from 0 to π to generate as much as frames of fringe patterns of interference needed to measure the phase. In this work we are using the five frame technique proposed in [15] resulting 5 equations as follows

$$I_1(x,y) = A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)-2\alpha]\} \tag{10}$$
$$I_2(x,y) = A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)-\alpha]\} \tag{11}$$
$$I_3(x,y) = A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)]\} \tag{12}$$
$$I_4(x,y) = A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)+\alpha]\} \tag{13}$$
$$I_5(x,y) = A(x,y)\{1+\upsilon(x,y)\cos[\phi(x,y)+2\alpha]\} \tag{14}$$

After some mathematical processing, and according to [15], these equations can be combined as follows:

$$\begin{aligned}&\frac{I_2(x,y)-I_4(x,y)}{2I_3(x,y)-I_5(x,y)-I_1(x,y)}\\ &=\frac{\sin[\phi(x,y)]\sin(\alpha)}{\cos[\phi(x,y)]-\cos[\phi(x,y)]\cos(2\alpha)}\\ &=\frac{\sin[\phi(x,y)]\sin(\alpha)}{\cos[\phi(x,y)][1-\cos(2\alpha)]}\\ &=\tan[\phi(x,y)]\frac{\sin(\alpha)}{1-\cos(2\alpha)}\end{aligned} \tag{15}$$

As the phase $\phi(x,y)$ is the wanted term, it can be written from Eq. (15),

$$\phi(x,y)= \tan^{-1}\left[\frac{I_2(x,y)-I_4(x,y)}{2I_3(x,y)-I_5(x,y)-I_1(x,y)}\frac{1-\cos(2\alpha)}{\sin(\alpha)}\right] \tag{16}$$

If $\alpha = \frac{\pi}{2}$, as it has been taken for this work, Eq. 15 turns into

$$\phi(x,y)=\tan^{-1}\left\{\frac{2[I_2(x,y)-I_4(x,y)]}{2I_3(x,y)-I_5(x,y)-I_1(x,y)}\right\} \tag{17}$$

3. Simulated and experimental results

According to the theoretical description presented above, Fig. 2 shows 5 simulated fringe patterns of interference as it is described in Eqs. (10-14). For this simulation we consider a phase step of $\alpha = \frac{\pi}{2}$ between each fringe pattern.

Note that the phase step introduced to the original fringe pattern of interference gives changes on its own properties, in other words, each fringe pattern is clearly different to the rest of them. Then the wanted phase, $\phi(x,y)$, is computed according to Eq. 17 and the resulted wrapped phase is shown in figure 3, together to the unwrapped phase that represents the surface countering of the object under study.

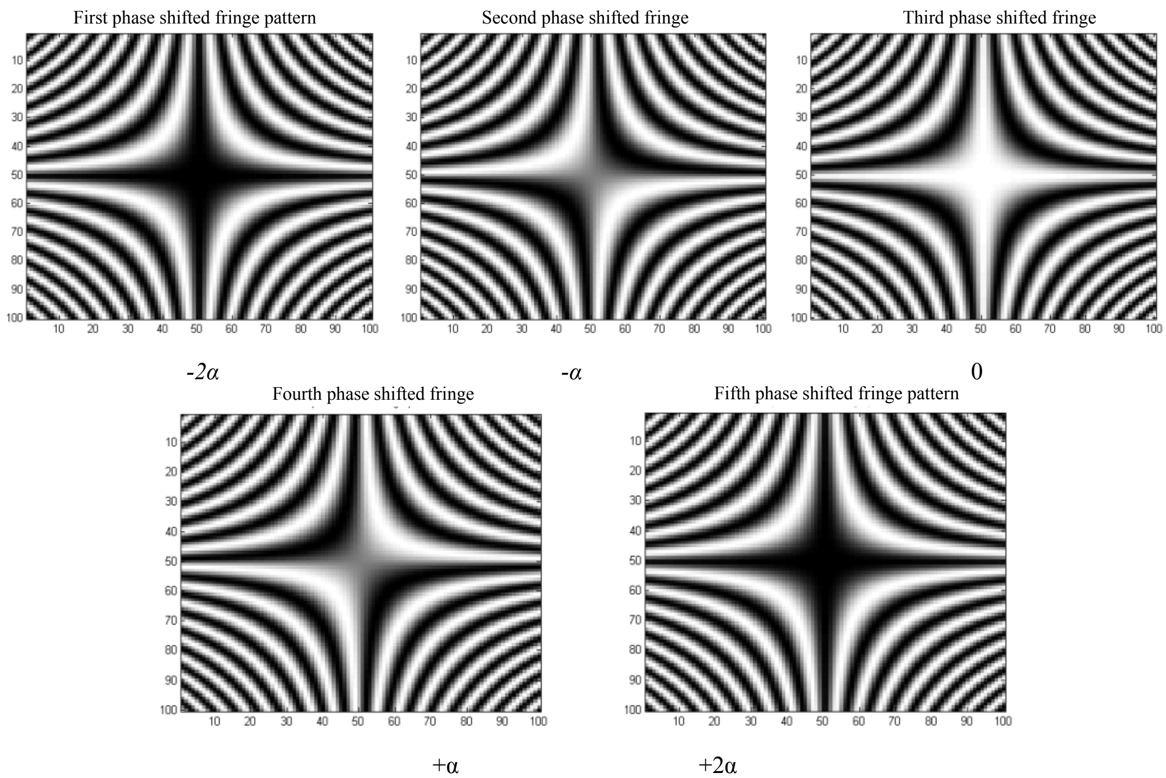
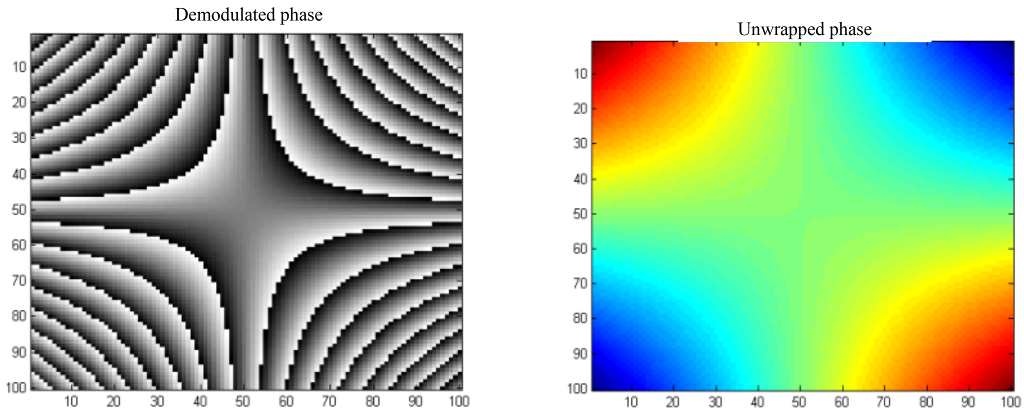


Fig. 2. Simulated frames of fringe pattern of interference with a phase step of $\alpha = \frac{\pi}{2}$ between each other.



a *b*

Fig. 3. Simulated phase retrieval with a temporal phase shifted algorithm. (a) Wrapped phase and (b) Unwrapped phase recovered from the simulated fringe.

Once the simulation is done, a model airplane is placed in front of the camera according to the experimental setup shown in Fig. 1. Part of the tail section is selected for this first experiment.

Fig. 4 shows the 5 fringe patterns of interference taken for the phase reconstruction process presented in this work.

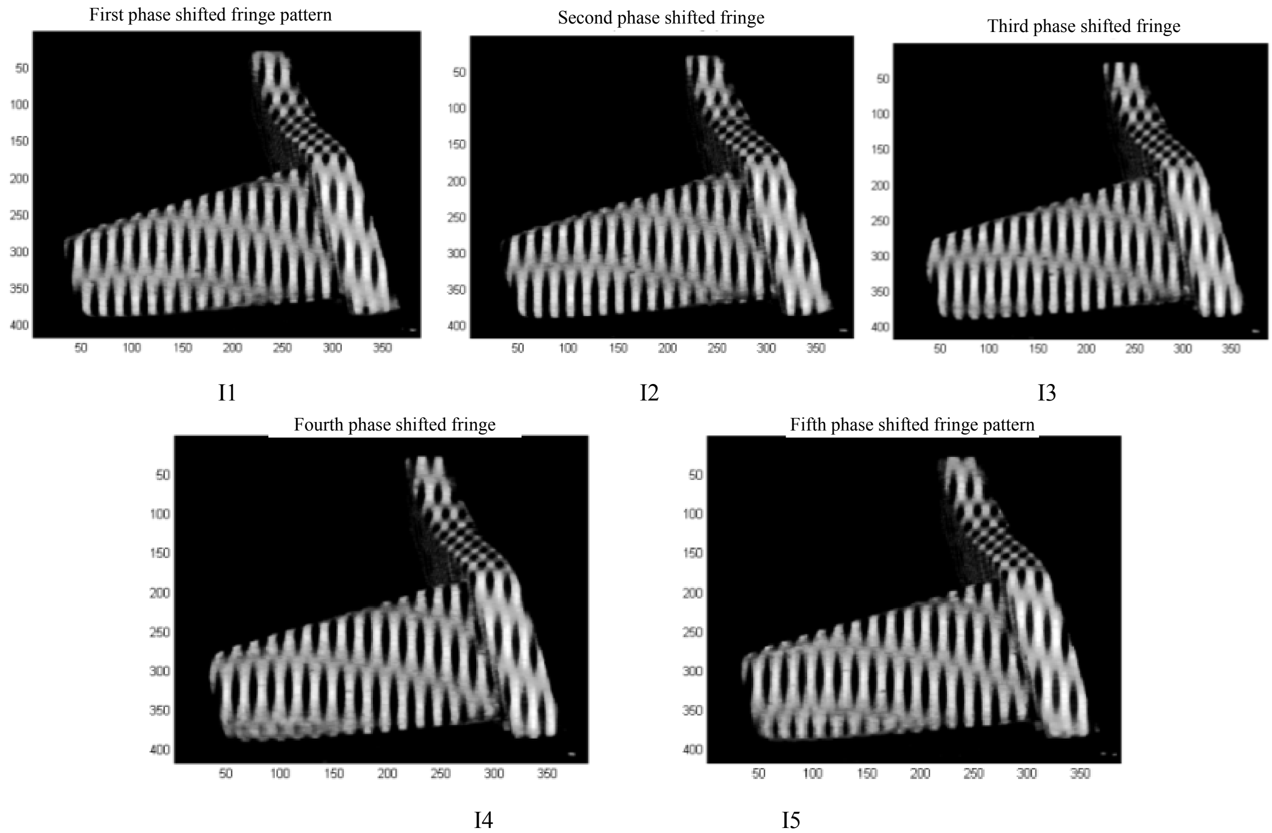
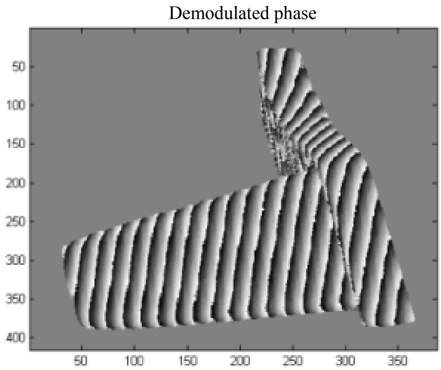


Fig. 4. Fringe patterns to be processed by eq. (16) taken from experiment 1.

Fig. 5 shows the resulted wrapped and unwrapped phase obtained by means of the temporal phase shifted algorithm and the double digital fringe projection.



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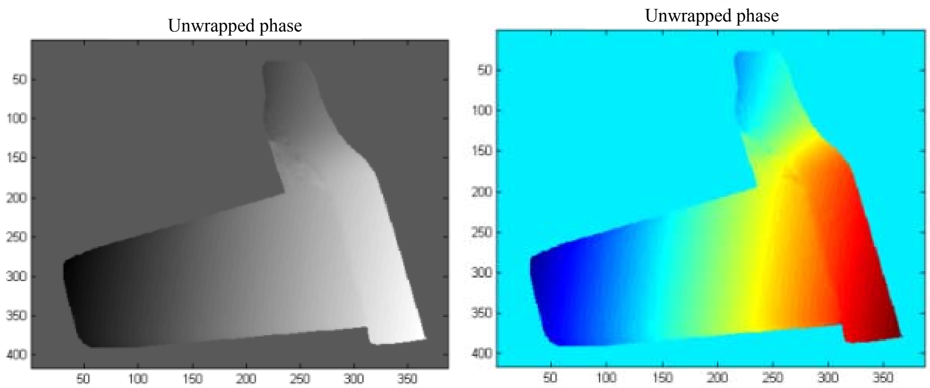
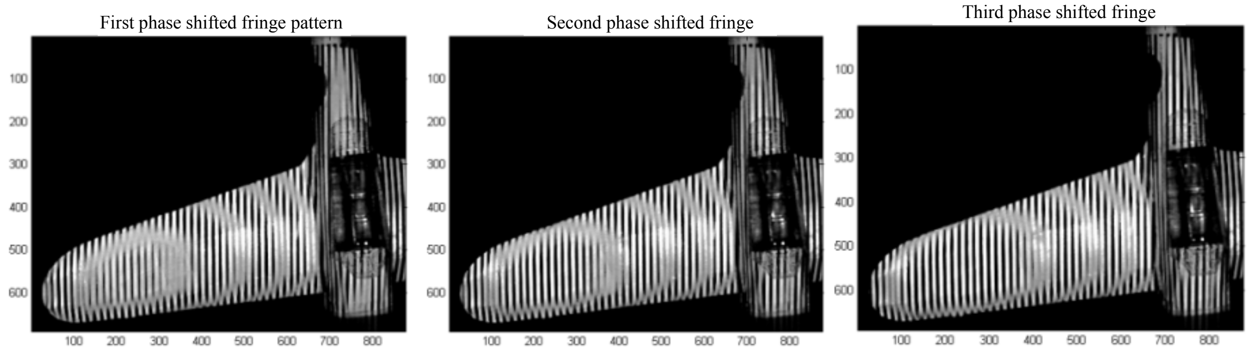


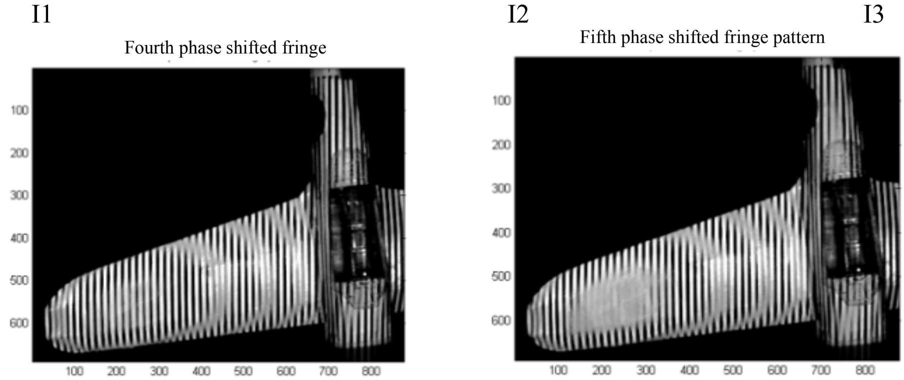
b

Fig. 5. Phase retrieval of a tail section of a model airplane. (a) Unwrapped phase and (b) Wrapped phase.

In figure 6, the fuselage section of a different model airplane is selected for this study.



I1 I2 I3

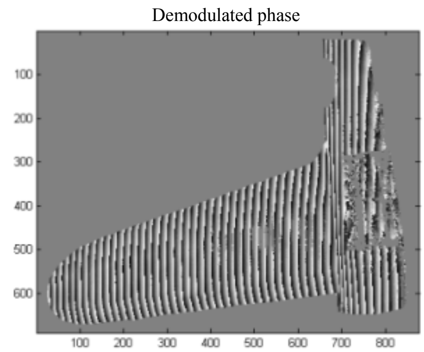


I4 I5

Fig. 6. Fringe patterns to be processed by eq. (16) taken from experiment 2.

It is important to notice that the cabin of this model airplane is made of a transparent plastic but has a well-defined shape that could not be well recovered by this technique.

Fig. 7 shows the resulted wrapped and unwrapped phase obtained by the proposed technique.



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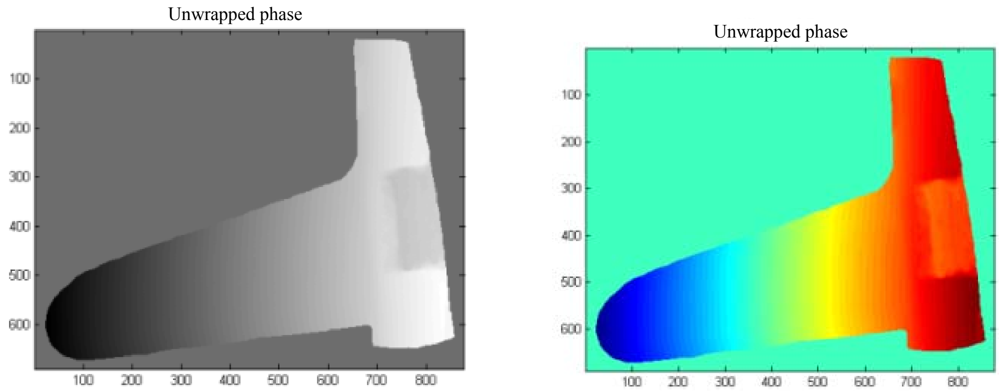

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Fig. 7. Phase retrieval of the fuselage section of a second model airplane for experiment 2. (a) Unwrapped phase and (b) Wrapped phase.

As it can be seen, the proposed technique works well to phase retrieval of the second model airplane, even in the cabin, which gives us the intention of a future work for measuring transparent objects.

4. Conclusions

In this work we have presented a fast way to obtain the phase retrieval of an object. The proposed method doesn´t need of calibration, synchronization or other kind of experimental consideration to work properly. Double-digital fringe projection for phase retrieval is a technique that offers a good approximation to the profile of an object that can be under extremal conditions. The application of this technique is very easy and allows fast measurements, so, it is an excellent option for engineering applications.

With traditional fringe projection, the complete shape of an object, inside the camera plane, was impossible to retrieval due to shadows generated on the extreme of the object according the illumination angle. The digital projectors give a good quality and uniform illumination over the object, but, because the light projected is coming from different directions, the fringe pattern obtained over the object exhibits its shape considering even its shadow areas and can be seen in real time without the need of any computer processing, just by watching and interpreting the form of the fringes.

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