

Fractal space-time and ball lightning as a self-organizing process in laser produced plasma

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A mathematical model of the ball lightning genesis is given in the frame of fractal space-time theory. Than, the plasmoids generated by laser ablation is similar with the BL generation through the thunderstorms-matter interaction.

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1. Introduction

Ball lightning (BL) appears as haphazard phenomena, usually associated with thunderstorms, in the form of free-floating, relatively long living flaming globes having different dimensions and colors. Because of their random appearances, a direct experimental investigation was not possible up to now. There are two different opinions on the generation of a BL. One of these starts from the hypothesis that the generation and lifetime of BL can be explained by considering the energy accumulated during its generation by a lightning stroke [1-3]. The other opinion relates the appearance and lifetime of BL with a local concentration of the radio frequency (RF) electric field energy, possible after interference of electromagnetic waves produced by atmospheric electricity [4-6].

In recent papers [7-9], the genesis and characteristics of BL can be explained in the frame of a new self-organizing physical scenario suggested both by the fractal space-time theories [10-13] and by the laboratory investigations on the self-consistent extended macroscopic space charge configurations: fireballs in dc gas discharges [7], plasmoids in gas discharges sustained by a radio frequency electric field [8, 9], plasmoids generated by laser ablation [14-18] etc. In the present paper, according to the fractal space-time theories, a mathematical model of the BL genesis is established. Than, we will assimilate the BL generation through the thunderstorms-matter interaction with the plasmoid generated by laser ablation

2. Mathematical model

Let us suppose that the motion of physical objects takes place on continuous but non-differentiable curves, i.e. the space-time is a fractal. Then, the speed field V becomes complex [8,13] and it is described by a generalized Navier-Stokes type equation (for details on the method see Ref. [13]):

$$\frac{\partial V}{\partial t} + V \cdot \nabla V - \eta \Delta V = 0 \quad (1)$$

with the viscosity $\eta = i\mathcal{D}$, and \mathcal{D} a coefficient which defines the fractal/nonfractal scale transition [10-13]. From here and from the operational relation

$$V \cdot \nabla V = \nabla(V^2/2) - V \times (\nabla \times V) \quad (2)$$

we obtain the Eq.

$$\frac{\partial V}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - V \times (\nabla \times V) - \eta \Delta V = 0, \quad (3)$$

Let us admit that the “fluid” is irrotational, i.e.

$$\Omega = \nabla \times V = 0 \quad (4)$$

Then, we can choose V of the form:

$$V = -2\eta \nabla \ln \psi = -2i\mathcal{D} \nabla \ln \psi \quad (5)$$

so that the Eq. (24) becomes:

$$\frac{\partial V}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - \eta \Delta V = \frac{\partial V}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - i\mathcal{D} \Delta V = 0 \quad (6)$$

with ψ the „complex speed potential”.

Substituting (5) in (6) and considering the identities

$$(\nabla \ln f)^2 + \Delta \ln f = \frac{\Delta f}{f}, \quad \nabla \Delta = \Delta \nabla, \quad \nabla(\nabla f)^2 = 2(\nabla f \cdot \nabla)(\nabla f) \quad (7a-c)$$

which implies

$$\nabla\left(\frac{\Delta\psi}{\psi}\right) = \Delta(\nabla \ln \psi) + 2(\nabla \ln \psi \cdot \nabla)(\nabla \ln \psi) \quad (8)$$

it results a Schrödinger type equation,

$$\mathcal{D}^2 \Delta \psi + i\mathcal{D}\partial_t \psi = 0 \quad (9)$$

up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of ψ .

Therefore, the Schrödinger type equation is obtained as an irrotational movement of fluids having a “dissipative” coefficient depending on the “ \mathcal{D} ” scale. Then, ψ simultaneously becomes wave-function and speed potential. Let us apply the previous mathematical model for the complex potential of the speed field: $\psi = \sqrt{\rho}e^{iS}$, with $\sqrt{\rho}$ the amplitude and S the phase of ψ . It results the complex speed field,

$$\mathbf{V} = \mathbf{v} - i\mathbf{u}, \mathbf{v} = 2\mathcal{D}\nabla S, \mathbf{u} = \mathcal{D}\nabla \ln \rho, \quad (10a-c)$$

Substituting (10a-c) in (6) and separating the real and imaginary parts, we obtain the Eqs.:

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + \nabla \left(\frac{\mathbf{v}^2 - \mathbf{u}^2}{2} - \mathcal{D}\nabla \cdot \mathbf{u} \right) &= 0 \\ \frac{\partial \mathbf{u}}{\partial t} + \nabla (\mathbf{v} \cdot \mathbf{u} + \mathcal{D}\nabla \cdot \mathbf{v}) &= 0 \end{aligned} \quad (11a, b)$$

or, up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of ψ ,

$$\begin{aligned} \partial_t (\rho \mathbf{v}) + \mathbf{v} \cdot \nabla (\rho \mathbf{v}) &= -\rho \nabla \left(\frac{Q}{m} \right) \\ \partial_t \rho + \nabla \cdot (\rho \mathbf{v}) &= 0 \end{aligned} \quad (12a, b)$$

with Q the fractal potential

$$Q = -2m\mathcal{D} \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{m\mathbf{u}^2}{2} - m\mathcal{D}\nabla \cdot \mathbf{u} \quad (13)$$

and m the rest mass of the test particle of the fluid. In particular, for $\rho \nabla(Q/m) = \nabla p$, with p the internal pressure (for details see Ref. [13]), the (12a, b) system leads to the hydrodynamic model of the fractal space-time theory.

3. Plasmoids genesis by laser ablation

Let us apply the previous considerations in the numerical simulations of plasma expansion produced through the laser ablation. The strong laser pulses cause on targets the rapid heating, melting and evaporation of the surface layer. Interaction with the radiation leads to the breakdown of initial transparent vapor, the produced plasma starts absorbing the laser radiation, and as result

the radiation has no access to the surface. Thus, the processes of evaporation, the breakdown on one side and subsequent plasma evolution appear to be separated in some extent. In this case the plasma expansion can be investigated by assuming that just near the target surface a motionless thin layer of plasma is created. Such initial plasma is assumed to have a space-time distribution of the density that is strictly connected with the laser beam space-time profile. Supplementary details on the laser produced plasma can be found in Refs. [14-18].

The plasma expansion has axial symmetry and is solved in the cylindrical coordinate system induced in the region above the target surface (Fig. 1). The z -axis coincides with the laser beam axis and is directed along the outer normal to the target surface.

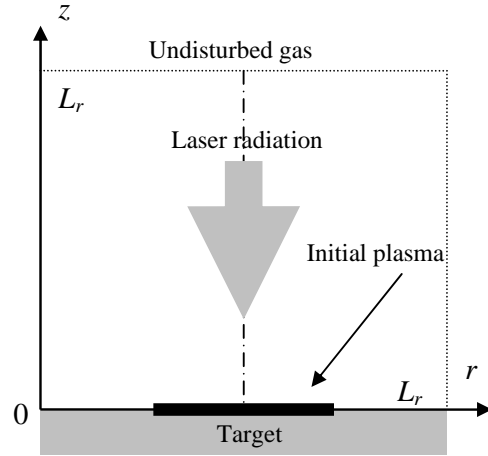


Fig. 1. The integration domain used for the numerical simulation of laser produced plasma expansion.

The plasma evolution is described with the following assumption: i) the plasma is in the state of local thermodynamical equilibrium and satisfies the quasi-neutrality condition; ii) the expansion is described in the approximation of a non-viscous non-thermo-conducting gas; iii) the release of energy of the thermal radiation is neglected and the ideal gas equations of state are considered. In such circumstances, the two-dimension gas dynamics is described by the equations system formed by the Eq. (12a, b) which are supplemented with the energy balance equation (for details see [15]),

$$\begin{aligned} \frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rnu) + \frac{\partial}{\partial z} (nv) &= 0 \\ \frac{\partial (nu)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rmu^2) + \frac{\partial}{\partial z} (nuv) &= -\frac{\partial p}{\partial r} \\ \frac{\partial (nv)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rmuv) + \frac{\partial}{\partial z} (nv^2) &= -\frac{\partial p}{\partial z} \\ \frac{\partial (ne)}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (rmue) + \frac{\partial}{\partial z} (nve) &= -p \left[\frac{1}{r} \frac{\partial (ru)}{\partial r} + \frac{\partial v}{\partial z} \right] \end{aligned} \quad (14a-d)$$

Here, t is the time, r and z the spatial coordinates, n the atoms density, u and v the velocity vector components, p

the pressure, and e the specific internal energy. The term in square brackets in the energy balance equation (14d) describes the work of pressure.

The system of equations is supplemented by the initial and boundary conditions. The box integration domain is initially filled with undisturbed gas,

$$\begin{aligned} t=0: \quad u=v=0, \quad n=n_0, \\ T=T_0 \quad 0 \leq (r \times z) \leq (L_r \times L_z) \end{aligned} \quad (15)$$

where T is the temperature.

A plasma source produced by the laser beam is considered on the target surface, having a Gaussian space-time distribution,

$$\begin{aligned} z=0: \quad u=v=0, \quad T=T_{plasma}, \\ n=n_{max} \exp\left[-\frac{(t-\tau)^2}{(\tau_L/2)^2}\right] \exp\left[-\frac{(r-L_r)^2}{(d_L/2)^2}\right], \end{aligned} \quad (16)$$

with d_L , τ_L similarly with the laser beam space-time full widths, and T_{plasma} the initial plasma temperature. The maximum atoms density n_{max} is taken according with the critical electron density of the laser wavelength and the average ions charge state $\bar{Z}=2$. Such charge state is usually obtained by using laser fluencies of order 10^{11} W/cm² (e.g. for commercial Nd:YAG laser, $\lambda=532$ nm) and aluminum targets [19]. The electron plasma temperature of order $T_{plasma}=20$ eV was experimentally evidenced (for details see [19]). The symmetry condition is added,

$$\begin{aligned} r=0, L_r: \quad u(0)=u(L_r), \quad v(0)=v(L_r), \\ n(0)=n(L_r), \quad T(0)=T(L_r) \end{aligned} \quad (17)$$

and the undisturbed gas is considered on the upper boundary,

$$z=L_z: \quad u=v=0, \quad n=n_0, \quad T=T_0 \quad (18)$$

The equations system (14) with the conditions (15-18) is numerically solved using finite differences [20] and the following parameters: $L_r=L_z=300\mu\text{m}$, $\tau_L=10$ ns, $d_L=100\mu\text{m}$, $n_{max}=10^{21}\text{ cm}^{-3}$, $n_0=n_{max}/1000$, $T_0=0.1$ eV.

In Figs. 2a)-c) the 2D-numerical solutions for the total atom density at the time moments: (a) $t=10$ ns (maximum laser energy), (b) $t=13$ ns and (c) $t=16$ ns, and in Figures 3a)-c) its corresponding contour curves are given.

It must be noted that, by comparison with the classical methods [14-18], in our model the fluid equations are valid at all space-time scales.

4. Discussions and conclusions

By analyzing the results of the numerical simulation, the followings can be concluded:

i) the plasma core is divided in two solitons; according to Refs. [7,9], the solitons corresponds to the structures of double layer type;

ii) the speed of the first soliton is imposed by the speed of the vaporization front [21],

$$v_v = \frac{F(t)}{\rho(\lambda_v + cT_v)}$$

where ρ , λ_v , c , and T_v are the density, vaporization heat, specific heat and vaporization temperature, respectively. $F(t)$ represents the light energy flux that is absorbed at the t time moment. In particular, for aluminum, by using $\rho=2702\text{ kg/m}^3$, $\lambda_v=293\text{ kJ/mol}$, $c=0.9\text{ J/g K}$ and $T_v=2792\text{ K}$, $F(t)=5 \times 10^{14}\text{ W/m}^2$, it results $v_v=7.3 \times 10^4\text{ m/s}$. This is in good agreement with the experimental observations [15])

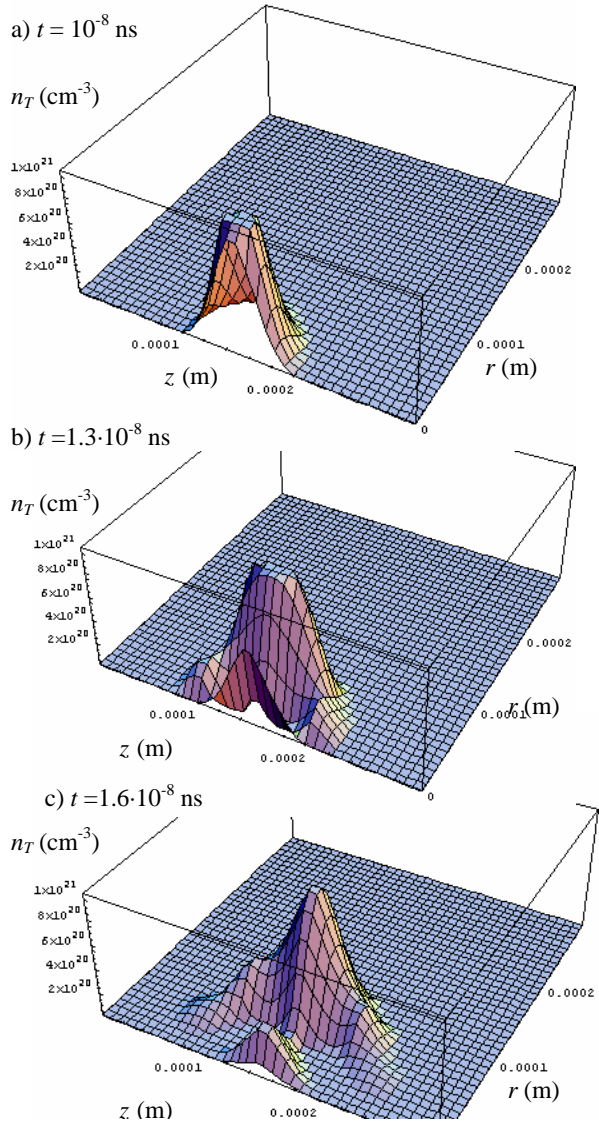


Fig. 2. Numerical solutions of expanding plasma density.

iii) the speed of the second soliton is imposed by means the average speed of the shock wave [21],

$$v' = \sqrt{2(\gamma^2 - 1) \frac{W_L}{\Delta t \pi r^2 \rho}},$$

that for the laser energy $W_L=50$ mJ/pulse, absorption time $\Delta t = 10$ ns, beam radius $r=0.5$ mm and adiabatic index $\gamma=5/3$ gives $v'=2.3 \times 10^3$ m/s. Such experimental physical scenario is also observed in Ref. [15].

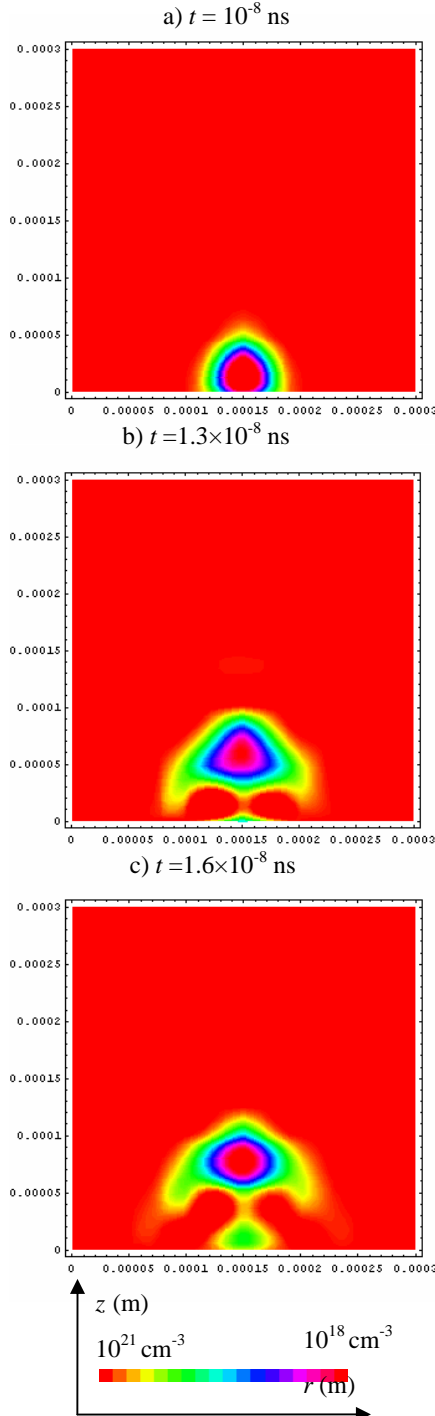


Fig. 3. The contour plot of plasma density for various time moments.

iv) the plasma characteristics (density, electron and ion temperatures, average charge state etc.), depend both the laser parameters and by the target type, and they are involved in our simulations by means of the initial and boundary conditions;

v) Taking into account our theoretical results given in Paragraph 2, it results that the model can be applied at any space-time scale. Consequently, the BL which could appear through the thunderstorm-matter interaction can be treated in a similar manner as plasmoids generation by laser ablation. In this way, a possible answer on the controversial problem concerning the BL genesis is given.

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