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Frequency and time domain reflection response of stratified dielectric structures

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Zeros and periodicities of the reflection coefficient and the interface impedance as well as transient response of Transverse Magnetic (TM) plane waves are studied. The symmetry in a multilayered structure shows an interesting behavior regarding the zeros of the reflection coefficient. For simple layered structures analytical solutions for the zeros and periodocities are given. Using numerical processes we see in the time domain the evolution of pulses with given characteristics. The conductivity of a dielectric layer plays a degenerative role in the shape and time evolution of the reflected pulses.

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1. Introduction

In the applied electromagnetic (EM) research the study of the EM response of a stratified dielectric structure is one of the very interesting problems. One can find a lot of works given in the past about this subject, some of the most recent being [1] – [9]. In this paper we continue and extend the study of [9], focusing mainly on the response of a multilayered dielectric structure in known waveforms of TM plane waves.

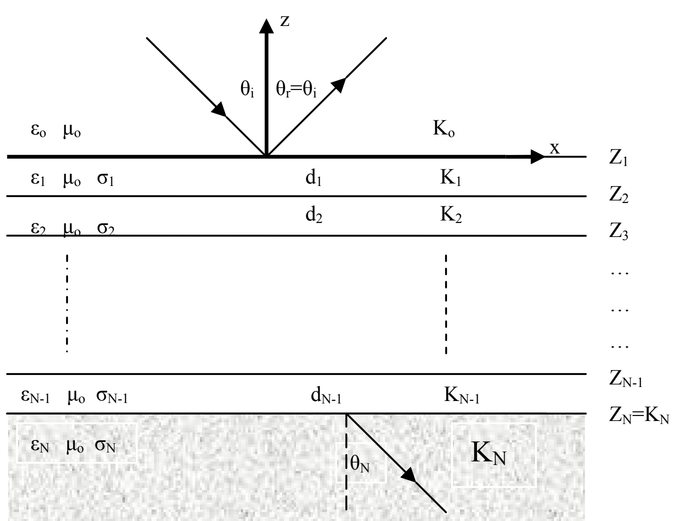

Fig. 1. An N-layer structure and an obliquely incident wave.

Fig. 1 shows a typical N-layer structure consisting of N-1 layers mounted on an N-th dielectric substrate with thickness $d_N \rightarrow \infty$. The symbols ε_i, μ_i, σ_i and d_i correspond to the dielectric constant (permittivity), magnetic permeability (assuming that it is equal to that of free space), conductivity and thickness of the i-th layer respectively. K_i and Z_i are given by the known ([10], [11]) relations

$$K_i = \eta_o \frac{\sqrt{\left(\varepsilon_{ri} - S^2\right) - j\dfrac{\sigma_i}{\omega\varepsilon_o}}}{\varepsilon_{ri} - j\dfrac{\sigma_i}{\omega\varepsilon_o}}$$

$$Z_i = K_i \frac{Z_{i+1} + K_i \tanh(u_i d_i)}{K_i + Z_{i+1}\tanh(u_i d_i)} \quad (1)$$

where

$$u_i = j\omega\sqrt{\varepsilon_o\mu_o}\sqrt{\left(\varepsilon_{ri} - S^2\right) - j\frac{\sigma_i}{\omega\varepsilon_o}}$$

$$\varepsilon_{ri} = \frac{\varepsilon_i}{\varepsilon_o}$$

for $i = 1, 2, \ldots, N$
and

$j = \sqrt{-1}$, $\omega = 2\pi f$ (f = frequency),
$K_o = \eta_o C$,

$S = \sin\theta$, $C = \cos\theta$, $\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} \approx 120\pi$, $Z_N = K_N$.

The reflection coefficient is given by the expression ([10], [11], [14])

$$R = \frac{K_o - Z_1}{K_o + Z_1}. \tag{2}$$

2. Zeros and periodicities

Except the known zero of |R| at the Brewster angle ([12] – [14]), one can find analogue angle(s), analytically or numerically, for cases more complicated than that of a half space. In [9] such a case is referred. When we have a symmetric multilayered structure with incorporated one or more air slabs, then we can find more than two angles of incidence for having |R| = 0. A simple case is that of 3 slabs having $\varepsilon_{r1} = \varepsilon_{r3} = 4$, $\varepsilon_{r2} = \varepsilon_{rN} = 1$, $\sigma_i = 0$ and $d_i = 2.5\lambda_o$. For this case we have 8 angles of incidence, for which |R|=0 ($\theta_1 = 20.8^o$, $\theta_2 = 37.6^o$, $\theta_3 = 49.8^o$, $\theta_4 = 60.4^o$, $\theta_5 = 60.8^o$, $\theta_6 = 63.6^o$, $\theta_7 = 70.6^o$, $\theta_8 = 80.4^o$). Fig. 2 shows Re(R), Im(R) and |R| as a function of the angle of incidence. The number of zeros increases as the thickness of the slabs is getting greater. For simple cases it is easy to find analytically these angles ([9]). For the above mentioned structure it is very difficult to calculate explicitely the angles for having |R|=0.

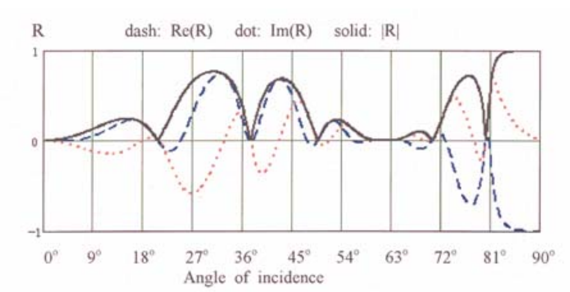

Fig. 2. Reflection coefficient for a symmetric 3-slabs dielectric structure.

Now we put into play the transmission coefficient ([10]),

$$T = \left(\frac{2K_o}{K_1 + K_o}\frac{2K_1}{K_2 + K_1}\cdots\frac{2K_{N-1}}{K_N + K_{N-1}}\right)\cdot D \cdot e^{-u_1 d_1 - u_2 d_2 - \cdots - u_{N-1} d_{N-1}} \tag{3}$$

where

$$D = \left\{\prod_{n=1}^{N-1}\left[1 - \left(\frac{K_{n-1} - K_n}{K_{n-1} + K_n}\right)\left(\frac{Z_{n+1} - K_n}{Z_{n+1} + K_n}\right)e^{-2u_n d_n}\right]\right\}^{-1}$$

In general, R and T are complex. Denoting by φ_R = angle of complex R and φ_T = angle of complex T, then, for any symmetric multilayered slab configuration we can easily see that

$$\varphi_R - \varphi_T = \left(k - \frac{3}{2}\right)\pi, \quad k = 0,1,2,3$$

or, alternatively,

$$\mathrm{Re}(R)\mathrm{Re}(T) + \mathrm{Im}(R)\mathrm{Im}(T) = 0,$$

which implies that

$$|R - T| = |R + T|.$$

This result can be justified by accounting that in symmetric configurations we have “qualitatively equivalent” reflections on interfaces from ε_i both to $\varepsilon_{i+1} > \varepsilon_i$ and to $\varepsilon_{i+1} < \varepsilon_i$. If one (or more) slab(s) of the same symmetric structure is air and the symmetry still holds, then, except the above (8) condition, it is evident that |T|=1 for more than one angles of incidence, under some restrictions on the values of ε_i and d_i ([9]).

Regarding the periodicities of R, these are passing through the periodicities of Z_1, i.e. the impedance of the first interface and they have their origin at the fact that

$$\tanh(jx) = j\tan(x) = j\tan(x + \pi).$$

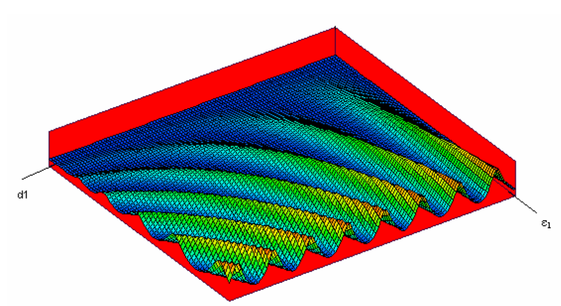

Fig. 3. The double periodicity of $|Z_1|^{-1}$ of a single dielectric slab into air.

Fig. 3 shows the double periodicity of $|Z_1|^{-1}$ (direct consequence of the periodicity of $|Z_1|$) with respect to d_1/λ_o and ε_1 for a single slab into air, having $\sigma_1 = 0$. It is obvious the existence of a double periodicity, especially in cases that d_1/λ_o and/or ε_1 are large.

Another interesting case is that of $\tanh(u_1 d_1) \rightarrow \infty$ for a slab configuration with $\sigma_1=0$ into air. For this case we have

$$u_1 d_1 = (2k+1)\frac{\pi}{2} \quad (4)$$

which leads to

$$\frac{d_1}{\lambda_o} = \frac{2k+1}{4\sqrt{\varepsilon_{r1} - S^2}} , \quad k = 0,1,2,... \quad (5)$$

Substituting (4) into the expressions of R (2), T (3) and Z_1 (1) we have

$$R = \frac{K_o^2 - K_1^2}{K_o^2 + K_1^2} \quad (6)$$

$$T = -j\frac{2K_o K_1}{K_o^2 + K_1^2} \quad (7)$$

and

$$Z_1 = \frac{K_1^2}{K_o} . \quad (8)$$

From (6) and (7) it is easily obtained that

$$|R|^2 + |T|^2 = 1$$

which holds for any symmetric structure. Equ. (8) denotes that Z_1 shows a pure resistive character for dielectrics with $\sigma=0$ and thickness given by (5).

3. Transient response

All the above analysis is in the frequency domain and also we didn't make mention of the special characteristics of the incident wave. Let's suppose now that we have an incident TM plane wave having a specific form, which in general is denoted by p(t), in the time domain, with $P(\omega)$ its Laplace Transform (L.T.). The response of a dielectric structure to an obliquely incident TM plane wave in the frequency domain is

$$H(\omega) = R(\omega)P(\omega), \quad (9)$$

where $R(\omega)$ is the reflection coefficient (2). The inverse L.T. of (9) gives the same response in the time domain, i.e.

$$h(t) = L^{-1}\{H(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\omega)P(\omega)e^{j\omega t}d\omega . \quad (10)$$

Since, in general, the integral (10) cannot be analytically found, we can, using a proper numerical integration code, to compute h(t) in the desired time domain. This h(t) will depend on the dielectric structure as well as on the characteristics of the incident pulse.

Before start studying a specific pulse, let's see first the behavior of the reflection coefficient in the time domain. To express the same thing, let's examine the time domain reflection response to the unit impulse function (or Dirac delta function)

$$\delta(t) = \begin{cases} \infty , & t = 0 \\ 0 , & t \neq 0 \end{cases} ,$$

although, mathematically speaking, such a function does not exist. Since

$$L\{\delta(t)\} = 1 ,$$

from (10) we can write the reflection coefficient in the time domain, i.e.

$$R(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\omega)e^{j\omega t}d\omega .$$

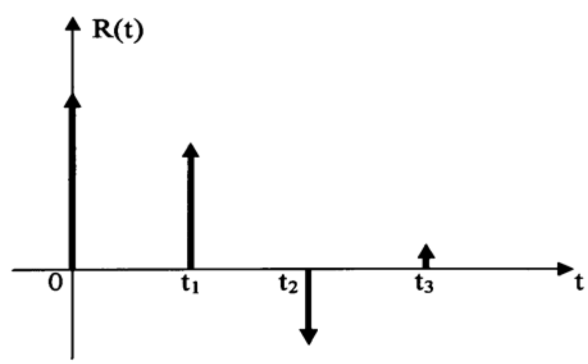

Fig. 4. Reflection coefficient in the time domain.

Fig. 4 shows R(t) for a simple case of a dielectric layer (with specific values of ε_1, d_1 and σ_1=0) on an other dielectric substrate (having ε_s, $d_s \rightarrow \infty$ and σ_s=0). As we can see from Fig. 4, the first (t=0) response to the unit impulse function is due to the first, direct reflection of this impulse on the top of the structure. At $t=t_1$ we have a second response, which comes after the first down and up of the impulse, with a first reflection on the down interface of the structure. The negative behavior of the third response is due to the first interior reflection of the impulse on the air – dielectric interface. This negative character vanishes after a second reflection on the same interface but it is reappeared after an odd number of interior reflections on the interface air – dielectric. We can compute t_1, t_2, ... using the simple relation ([11])

$$t_n = \frac{2nd_1}{c}\sqrt{\varepsilon_{r1} - S^2} \ ,$$

where n=0, 1, 2, ... and $c=(\varepsilon_o\mu_o)^{-1/2}$ is the velocity of light into free space.

The reflection, defraction and transmission in general of a pulse does not affect the characteristics of the pulse, i.e. its shape and duration, provided that all the dielectrics have zero conductivity. The whole process acts catalytically on the time of transmission of the pulse.

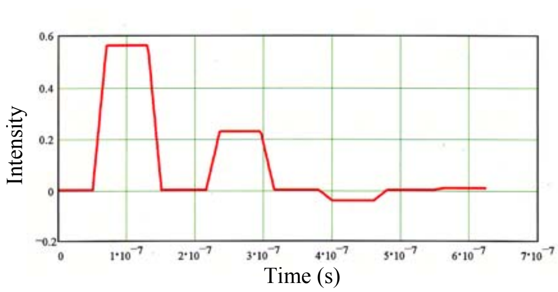

Fig. 5. Response in time domain of a two-layer structure to a trapezoid pulse.

Let's take a pulse with clear geometrical characteristics, e.g. a trapezoid shape pulse, which is described by

$$p(t) = \begin{cases} \frac{1}{\tau}(t-a) & , \quad a \le t < a+\tau \\ 1 & , \quad a+\tau \le t < b-\tau \\ \frac{1}{\tau}(b-t) & , \quad b-\tau \le t < b \\ 0 & , \quad \text{elsewhere} \end{cases} \qquad (11)$$

The LT. of (11) is

$$P(\omega) = e^{-ja\omega}\left[1+\left(\frac{1}{j\omega}-1\right)e^{-j\tau\omega}\right] + e^{-jb\omega}\left[1-\left(\frac{1}{j\omega}+1\right)e^{j\tau\omega}\right].$$

Fig. 5 shows the response of the same structure as that which gave R(t) (Fig. 4). The difference between the two situations of Figs. 4 and 5 is that in Fig. 4 we took a=0, i.e. the whole response was shifted left for t=a. We see from Fig. 5 the conservation of the main characteristics of the trapezoid pulse. The first pulse is due to the immediate reflection of the incident wave at t=a. The second pulse is coming from the reflection on the second interface. The third pulse is due to three successive interior reflections, first on the second interface, then on the first interface and after this on the second interface again. This is the reason that this third pulse of Fig. 5 is reversed. The fourth pylse of Fig. 5 – it is not seen clearly – arises coming after 5 interior reflections and the negative character is cancelled, since a second interior reflection takes place on the first interface.

In the following we will see the evolution of a semi-sinusoidal pulse in the time domain as a function of some characteristic values of a multilayered dielectric structure. A semi-sinusoidal pulse with amplitude equal to 1 is described by (26)

$$h(t) = \begin{cases} \sin\left[\frac{\pi(t-a)}{b-a}\right] & , \quad a \le t \le b \\ 0 & , \quad \text{elsewhere} \end{cases} \qquad (12)$$

where a,b correspond to the time of rising and ending the pulse respectively. The L.T. of (12) is

$$H(\omega) = \frac{\pi(b-a)\left(e^{-ja\omega} + e^{-jb\omega}\right)}{\pi^2 - (b-a)^2\omega^2} \ .$$

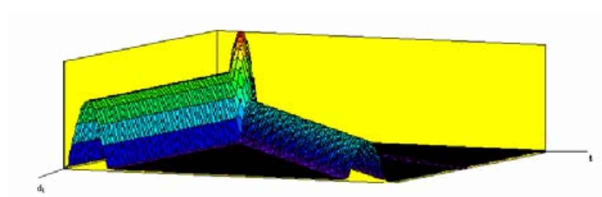

Fig. 6. Transient response of a two-layer half-space to a semi-sinusoidal pulse.

Fig. 6 shows the time response to the above sinusoidal pulse of a dielectric structure consisting of a substrate with $d_s \to \infty$, ε_{rs}=10 and σ_s=0 over which there is a dielectric layer with ε_{r1}=5, σ_1=0 and d_1 taking values from 0 up to $4.5\lambda_o$. As we can see from this figure, the first pulse train corresponds to the first, immediate reflection of the incident pulse on the first interface at t=a. The second pulse starts rising at

$$t = a + \frac{2d_1}{c}\sqrt{\varepsilon_{r1} - S^2} \ .$$

For every value of d_1, this time is linearly dependent on d_1 and for some – few – small values of d_1 the starting time is smaller than the end time of the first pulse train. That' s the reason of the peak of the pulses for t<b, where the second pulse is added with the first one. A third pulse – not clearly seen in the figure – is reversed and the reason is the first (interior) reflection on the interface air – dielectric.

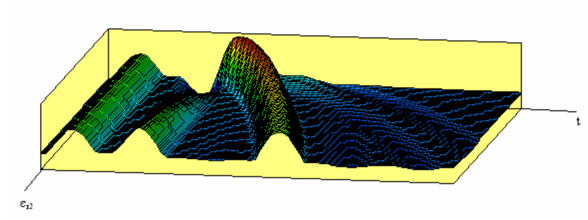

Fig. 7. Transient response of a three-layer half-space to a semi-sinusoidal pulse.

Fig. 7 corresponds to a more complicated case than the previous one. Here the dielectric structure consists of two layers having $d_1 = d_2$, $\sigma_1 = \sigma_2 \neq 0$, $\varepsilon_{r1} = 4$ and ε_{r2} taking values from 1 up to 15. The two layers are seated on a dielectric substrate with $\varepsilon_{rs} = 80$ and $\sigma_s = 0$. It is interesting to comment here that the second pulse train is reversed for $\varepsilon_{r2} < \varepsilon_{r1}$ and this is not due to multiple interior reflections but only to one interior reflection on the upper interface dielectric – air.The third pulse train starts rising at

$$t = a + \frac{2d_1}{c}\left(\sqrt{\varepsilon_{r1} - S^2} + \sqrt{\varepsilon_{r2} - S^2}\right)$$

and, of course, the time delay is not linearly dependent on ε_{r2}. The influence of $\sigma_1 = \sigma_2 \neq 0$ on the pulse characteristics is degenerative and it is not clearly seen in this figure, because of the small values of the conductivities σ_1 and σ_2.

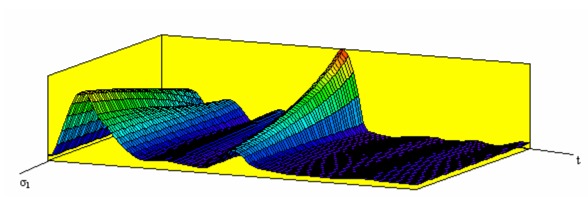

Fig. 8. The degenerative role of the conductivity to the transient response of a four-layer half-space.

The degenerative role of the conductivity is seen clearly in Fig. 8, where we have the reflected pulses on a dielectric structure with 3 layers mounted on a dielectric substrate with infinite thickness. All the dielectrics are assumed having zero conductivities except the upper one, which has conductivity taking values from 0 up to 0.01 S/m. The influence of this conductivity is obvious to all the reflected pulses, which tend to deteriorate for large values of σ_1, except the first pulse, which is generated after the first, immediate reflection on the upper interface.

4. Conclusions

Zeros, periodicities and transient phenomena of reflection present a special interest studying the response of dielectrics to EM waves. In both frequency and time domains, this response is strongly dependent on the physical characteristics of the dielectric structures – which are supposed to be multilayered in this work – as well as on the nature of the incident wave and the angle of incidence. In pure (σ=0) dielectrics, we can use an optical ray technique and for simple cases it is possible to have analytically obtained results concerning the zeros and the periodicities of reflection and/or transmission coefficient and of the impedance of partial interfaces. In cases of conductive dielectrics, the above technique fails and only numerical processes give solution to problems concerning the frequency and time response of dielectric structures. In general, the existence of conductivity acts degeneratively on the reflected pulses. Applications of the studied reflection processes can be found in designing and developing EM absorbers ([1], [2], [15]), in constructing earth models for subsurface investigation ([16]) and in optical biosensing ([8]).

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