Generation of new doughnut beams from Li's flattened Gaussian beams

L. EZ-ZARIY, F. KHANNOUS, H. NEBDI, M. KHOUILID, A. BELAFHAL*
Laboratory of Nuclear, Atomic and Molecular Physics, Physics Department, Faculty of Sciences, Chouaïb Doukkali University, P. B.: 20, 24000 El Jadida, Morocco

Using Collins-Huygens integral formula, a method for generation of Superposition of Kummer (SK) beams as new kind of doughnut beams is demonstrated. The method is based on the conversion of phase of the non-doughnut Li's flattened Gaussian (LFTG) beams, which are the incident beams, by a Spiral Phase Plate (SPP) system. It is shown that doughnut beams can be produced from non-doughnut beams. Mathematical approaches of propagation properties of generated laser light are established for integer and fractional topological charges of SPP. The above results concerning the formation of Kummer beams by an SPP, also about propagation of LFTG and fundamental Gaussian beams through an ABCD optical system without SPP and other results are deduced from ours as particular cases when the topological charges are integer. Some numerical simulations of propagation of novel beams in free space (in the cases of an integer (χ=l=1) and a fractional topological charge (χ≠1)) and through a fractional Fourier transform (FRFT) system (for l=1) are also performed in this work.

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1. Introduction

In the end of the last century, various closed paraxial solutions of the Helmholtz equation have been the subject of several works. For the sake of their use in realization of nondiffracting beams, a particularly interesting is given to the so-called flattened Gaussian beams. Several formalisms of this family have been introduced in the literature. Among these, super Gaussian [1], weighted sums of Gaussians having different focusing parameters [2-5], super Lorentzian [2], wavelets [6], the flattened Gaussian introduced by Gori [7] and utilized by Santarsiero and Borghi [8] and Bagini et al. [9]. In 2002, the flat-toped light beams established for the first time by Li in Ref. [5], and synthesized by coherent superposition of fundamental mode Gaussian beams. Their amplitude distribution across the waist plane is similar to that of a Gaussian beam whose central region has been flattened. Recently, Kinani et al. [10] have investigated the propagation properties of these beams in turbulent atmosphere.

On other hand, generating of doughnut beams has been paid considerable attentions in the past few years. This due to its significant role for studying orbital momentum of light fields [11,12], its use in optical trapping of atoms [13,14], its application in optical tweezers [15] and optical meteorology [16]. Several methods have been realized for generating the doughnut beams, such as "fork" computer generated holograms [17], cylindrical lens mode converters [18], phase mask [19], spatial light modulators [20] and Spiral Phase Plate (SPP) [21, 22].

The SPP is widely known as a useful tool producing the doughnut laser beams [23, 24], and can permits direct conversion of incident laser beam without changing its propagation direction. In a recent work, Mawardi et al. [25] have described the propagation of doughnut (Kummer) beams produced by passage of Gaussian beams through an SPP.

The present paper aims to study the propagation of new SK beams generated by an SPP, illuminated by LFTG beams, and followed by a paraxial ABCD optical system. From our main analytical expression, concerning the integer topological charge of the SPP, the above results about the formation of Kummer beams by an SPP illuminated by Gaussian beams, and regarding the travelling of LFTG and Gaussian beams through any paraxial ABCD optical system have been deduced.

The remaining parts of this paper are organized as follows: Section 2 is reserved to the definition and plots of the intensity distribution of a LFTG beam in 3D as incident beam. Section 3 is focused on calculating the analytical expression of SK beams as new doughnut beams. Some special cases are established in this paragraph. Some numerical simulations of propagation of novel beams in free space in the case of integer (χ=l=1) and fractional topological charges (χ≠1), and through a fractional Fourier transform (FRFT) system for l=1 are treated in Section 4. A simple conclusion is outlined in Section 5.

2. Li's Flattened Gaussian beams

The electric field of propagation of a LFTG beam along the optical axis (z axis) for $z \geq 0$, in cylindrical coordinates is given by [5]

$$V_M(r,z) = A_0 \exp(-ikz)\sum_{m=1}^{M}\alpha_m \frac{\omega_m(0)}{\omega_m(z)}\exp\left\{i\left(\Phi_m(z) - \frac{kr^2}{2R_m(z)}\right) - \frac{r^2}{\omega_m^2(z)}\right\}, \tag{1}$$

where the coefficient α_m is given by

$$\alpha_m = (-1)^{m+1}\frac{M(M-1).....(M-m+1)}{m!}, \tag{2}$$

and

$$\omega_m(z) = \omega_m(0)\sqrt{1+\frac{z^2}{(z_R)_m^2}}, \tag{3a}$$

with

$$\omega_m(0) = \frac{\omega_0}{\sqrt{m\beta}}, \tag{3b}$$

where ω_0 is the beam waist of the Gaussian beam and the scaling factor β is given by

$$\beta = \sum_{m=1}^{M}\frac{\alpha_m}{m}. \tag{3c}$$

The other parameters are defined as

$$\Phi_m(z) = \arctan\left[\frac{z}{(z_R)_m}\right], \tag{4}$$

$$R_m(z) = z + \frac{(z_R)_m^2}{z}, \tag{5a}$$

and

$$(z_R)_m = \frac{z_R}{m\beta}, \tag{5b}$$

where z_R is the Rayleigh range for the beam of order m=1 which is given by

$$z_R = \pi\frac{\omega_0^2}{\lambda}. \tag{5c}$$

In the following, we will be interested only in the integer values of the beam characterization parameter *M*. In this case, Eqs. (2) and (3c) become [5]

$$\alpha_m = (-1)^{m+1}\frac{M!}{(M-m)!m!}, \tag{6a}$$

and

$$\beta = \sum_{m=1}^{M}\frac{1}{m}. \tag{6b}$$

In Fig. 1, we illustrate the intensity of incident LFTG beams for six values of the parameter M and with a beam waist of the Gaussian beam $\omega_0 = 1mm$.

In the coming section, using the Collins-Huygens integral formalism for diffraction, we discuss the propagation of a LFTG beam through an SPP followed by an ABCD optical system and two analytical expressions of a novel kind of beams, one for the integer topological charge and the other for the fractional case, are performed. The novel beam generated is referred as SK beams.

3. ABCD-treatment of the LFTG beams: Generation of SK beams

Let us consider that the SPP which located in *z'* is illuminated with a LFTG beam which described by Eq. (1) (see Fig. 2). Assuming that the beam hits exactly at the center of the SPP, the field distribution behind this optical system is given by

$$E(\rho',\varphi',z') = A_0 e^{-ikz'}\sum_{m=1}^{M}\alpha_m\frac{\omega_m(0)}{\omega_m(z')}e^{i\Phi_m(z') - i\frac{k\rho'^2}{2R_m(z')}}e^{-\frac{\rho'^2}{\omega_m^2(z')}}\exp(i\chi\varphi') \tag{7}$$

where χ is the topological charge which can be integer or fractional. According to Collins-Huygens integral, which allows one to calculate the field distribution of the emerging light from an initial plane *z'* to any desired plane z, the out-put field distribution after propagation through any optical system described by an ABCD transfer matrix can be established as [26]

$$\begin{aligned} E(\rho,\varphi,z) = -\frac{i}{\lambda B}\exp(ikz)\int_0^{\infty}\int_0^{2\pi}E(\rho',\varphi',z')\times\exp\left[\frac{ik}{2B}\left(A\rho'^2 + D\rho^2\right)\right] \\ \times\exp\left[-\frac{ik\rho\rho'\cos(\varphi-\varphi')}{B}\right]\rho' d\rho' d\varphi' \end{aligned} \tag{8}$$

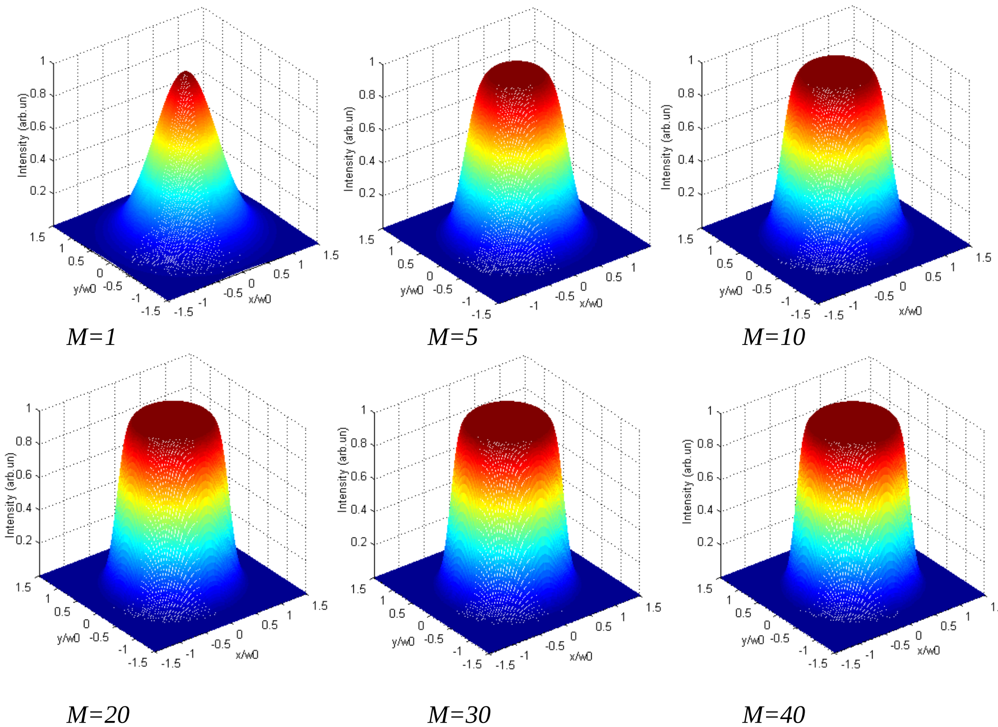


Fig. 1. Intensity of incident LFTG beams for different beam order M with $\omega_0 = 1mm$.

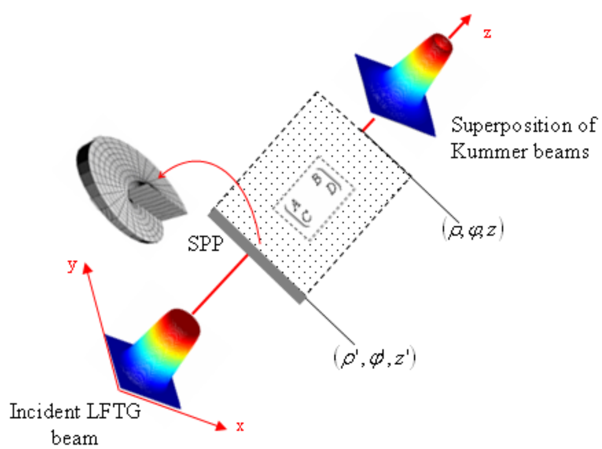


Fig. 2. Schematic LFTG beam passing through an SPP followed by an ABCD optical system for producing the SK beams.

Substituting Eq. (7) into Eq. (8), one obtains

$$E(\rho,\varphi,z) = -\frac{iA_0}{\lambda B} e^{ik(z-z')} e^{i\frac{kD}{2B}\rho^2} \sum_{m=1}^{M} \alpha_m \frac{\omega_m(0)}{\omega_m(z')} e^{+i\Phi_m(z')} \int_0^\infty \left(\int_0^{2\pi} e^{\left(i\chi\varphi' - i\frac{k\rho\rho'}{B}\cos(\varphi-\varphi')\right)} d\varphi' \right) \times e^{-\left(\frac{1}{\omega_m^2(z')} + i\frac{k}{2R_m(z')} - i\frac{kA}{2B}\right)\rho'^2} \rho' d\rho' . \quad (9)$$

In the following, we will evaluate Eq. (9) for an integer and a fractional topological charge.

3.1. Generated SK beams for an integer topological charge

In this case, we set that $\chi=l$ when l is an integer, and with the help of the following integrals [27]

$$\int_0^{2\pi} e^{il\varphi'} e^{-i\frac{k\rho\rho'}{B}\cos(\varphi-\varphi')} d\varphi' = 2\pi(-i)^{|l|} e^{il\varphi} J_{|l|}\left(\frac{k\rho\rho'}{B}\right), \quad (10)$$

and after some developments, Eq. (9) can be further simplified as

$$E(\rho,\varphi,z) = A_0 \frac{2\pi(-i)^{|l|+1}}{\lambda B} e^{ik(z-z')} e^{i\frac{kD}{2B}\rho^2} \sum_{m=1}^{M} \alpha_m \frac{\omega_m(0)}{\omega_m(z')} e^{i\Phi_m(z')} \times \int_0^{\infty} \rho' e^{-\left[\frac{1}{\omega_m^2(z')}+i\frac{k}{2R_m(z')}-i\frac{kA}{2B}\right]\rho'^2} J_{|l|}\left(\frac{k\rho}{B}\rho'\right) d\rho' . \quad (11)$$

If we set that

$$\gamma_m = \frac{1}{\omega_m^2(z')} + i\frac{k}{2R_m(z')} - i\frac{kA}{2B}, \quad (12)$$

and by using the following integral [28],

$$\int_0^{\infty} x^{\mu} e^{-\alpha x^2} J_{\nu}(\beta x) dx = \frac{\beta^{\nu}\Gamma\left(\frac{1}{2}\nu+\frac{1}{2}\mu+\frac{1}{2}\right)}{2^{\nu+1}\alpha^{\frac{1}{2}(\mu+\nu+1)}\Gamma(\nu+1)} {}_1F_1\left(\frac{\mu+\nu+1}{2};\nu+1;-\frac{\beta^2}{4\alpha}\right), \quad (13)$$

the final analytical expression of the converted beam by an SPP has the from

$$E(\rho,\varphi,z) = A_0 \frac{\pi(-i)^{|l|+1}}{\lambda B 2^{|l|}} e^{i\left(k(z-z')+l\varphi\right)} e^{i\frac{kD}{2B}\rho^2} \left(\frac{k}{B}\right)^{|l|} \rho^{|l|} \frac{\Gamma\left(\frac{|l|}{2}+1\right)}{\Gamma(|l|+1)} \times \sum_{m=1}^{M} \alpha_m \frac{\omega_m(0)}{\omega_m(z')} e^{i\Phi_m(z')} \gamma_m^{-\frac{|l|}{2}-1} {}_1F_1\left(\frac{|l|}{2}+1;|l|+1;-\frac{k^2\rho^2}{4\gamma_m B}\right). \quad (14)$$

where $\Gamma(.)$ is the gamma function and ${}_1F_1(a;b;x)$ is the confluent hypergeometric (Kummer) function. That can allows referring SK beams produced with the help of an SPP illuminated by a LFTG beams.

Our main result established in Eq. (14) can be considered as a generalization of the above studies concerning the generated doughnut beams by an SPP illuminated by Gaussian ones [25] and concerning the passage of LFTG and Gaussian beams through an ABCD optical system without SPP.

3. 2. Generation of SK beams by using a fractional topological charge χ

The subject of this section is to see what happens if the SPP is illuminated with a LFTG beam with a wavelength $\lambda_\chi \neq \lambda$. When one illuminates the SPP with a wavelength λ_χ , the SPP introduces a topological charge χ to the beam not equal to 1 but is a fractional quantity. This topological charge χ can be calculated from the equation $\chi = (\Delta n.h)/\lambda_\chi$, where h is the step height. Δn denotes the difference of refractive index between the SPP and its surrounding. λ_χ is the wavelength of the incident beam.

When the topological charge χ is not an integer (fractional), the integration over the azimuthal angle in Eq. (8) is not taken by applying directly the well known closed form integral established in Eq. (10). However, in this case the phase factor $\exp(i\chi\varphi')$ can be expressed as a Fourier series as [29]

$$e^{i\chi\varphi'} = (-i)^{\chi} \frac{\sin(\pi\chi)}{\pi} \sum_{n=-\infty}^{n=+\infty} \frac{\exp(in\varphi')}{\chi - n}. \quad (15)$$

According to Collins-Huygens integral, which allows one to calculate the field distribution of the emerging light from an initial plane z' to any desired plane z, the out-put field distribution after propagation through an optical system described by an ABCD matrix can be established as

$$E(\rho,\varphi,z)=-\frac{iA_0}{\lambda B}e^{ik(z-z')}e^{i\frac{kD}{2B}\rho^2}\frac{\sin(\pi\chi)}{\pi}\sum_{m=1}^{M}\alpha_m\frac{\omega_m(0)}{\omega_m(z')}e^{i\Phi_m(z')}\sum_{n=-\infty}^{n=+\infty}\frac{1}{\chi-n}\int_0^{\infty}\int_0^{2\pi}e^{\left(in\varphi'-i\frac{k\rho\rho'}{B}\cos(\varphi-\varphi')\right)}d\varphi'$$
$$\times e^{-\left(\frac{1}{\omega_m^2(z')}+i\frac{k}{2R_m(z')}-i\frac{kA}{2B}\right)\rho'^2}\rho' d\rho'. \tag{16}$$

Taking in account the parameter γ_m, given by Eq. (12) and by using the well known integrals of Eqs. (10) and (13) and recalling the next relationship

$$J_{-n}(z)=(-1)^n J_n(z), \tag{17}$$

and after some calculations, the final analytical expression of the converted beam by an SPP has the from

$$E(\rho,\varphi,z)=A_0\frac{\pi(-i)^{\chi+1}}{\lambda B}e^{ik(z-z')}e^{i\frac{kD}{2B}\rho^2}\frac{\sin(\pi\chi)}{\pi}\sum_{m=1}^{M}\alpha_m\frac{\omega_m(0)}{\omega_m(z')}e^{i\Phi_m(z')}$$
$$\times\left[\sum_{n=-\infty}^{n=-1}\frac{1}{\chi-n}(-i)^n e^{in\varphi}(-1)^{-n}\left(\frac{k\rho}{B}\right)^{-n}\frac{\Gamma\left(-\frac{n}{2}+1\right)}{2^{-n+1}\gamma_m^{\frac{-n}{2}+1}\Gamma(-n+1)}{}_1F_1\left(-\frac{n}{2}+1;-n+1;-\frac{k^2\rho^2}{4B^2\gamma_m}\right)\right.$$
$$\left.+\sum_{n=0}^{n=+\infty}\frac{1}{\chi-n}(-i)^n e^{in\varphi}\left(\frac{k\rho}{B}\right)^{n}\frac{\Gamma\left(\frac{n}{2}+1\right)}{2^{n+1}\gamma_m^{\frac{n}{2}+1}\Gamma(n+1)}{}_1F_1\left(\frac{n}{2}+1;n+1;-\frac{k^2\rho^2}{4B^2\gamma_m}\right)\right]. \tag{18}$$

In the coming sub-paragraph, from our generalized analytical expression of production of SK beams in case of integer topological charge l, established in Eq. (14), and under some conditions on parameters M and l, some above studies concerning the formation of Kummer beams by an SPP, about the propagation of LFTG and Gaussian beams through an ABCD optical system without SPP, are discussed as particular cases. We will also derive the propagation of the SK beams through an ABCD optical system for the small ρ and moderates B.

3. 3. Special cases

Case 1

If we set that $M=1$, Eq. (1) reduces to

$$V_1(r,z)=A_0\exp(-ikz)\frac{\omega_0}{\omega_1(z)}\exp\left\{i\left(\Phi_1(z)-\frac{kr^2}{2R_1(z)}\right)-\frac{r^2}{\omega_1^2(z)}\right\}, \tag{19a}$$

where

$$\omega_1(z)=\omega_0\sqrt{1+\frac{z^2}{z_R}}, \tag{19b}$$

$$\Phi_1(z)=\arctan\left[\frac{z}{z_R}\right], \tag{19c}$$

and

$$R_1(z)=z+\frac{z_R}{z}. \tag{19d}$$

Eq. (19a) characterizes the analytical expression of the normalized fundamental Gaussian beam. In this condition, Eq. (14) becomes

$$E(\rho,\varphi,z)=A_0\frac{\pi(-i)^{|l|+1}}{\lambda B 2^{|l|}}e^{ik(z-z')}e^{i\frac{kD}{2B}\rho^2}e^{il\varphi}\left(\frac{k}{B}\right)^{|l|}\rho^{|l|}\frac{\Gamma\left(\frac{|l|}{2}+1\right)}{\Gamma(|l|+1)}$$
$$\times\frac{\omega_1(0)}{\omega_1(z')}e^{i\Phi_1(z')}\gamma_1^{-\frac{|l|}{2}-1}{}_1F_1\left(\frac{|l|}{2}+1;|l|+1;-\frac{k^2\rho^2}{4\gamma_1 B^2}\right). \tag{20}$$

This is the output optical field distribution of Kummer beams passing through an ABCD paraxial optical system for any integer topological charge l. This is a doughnut beam generated from a fundamental Gaussian beam converted by an SPP. The beam profile can be calculated at any position z. In that case if l=1, and after tedious calculations it's easy to show that

$$E(\rho,\varphi,z)=\frac{\pi\sqrt{\pi}A_0}{4\lambda B}\times\frac{\omega_1(0)}{\omega_1(z')}e^{i\Phi_1(z')}e^{ik(z-z')}e^{i\frac{kD}{2B}\rho^2}e^{i\varphi}\left(\frac{k\rho}{B}\right)$$
$$\gamma_1^{-\frac{3}{2}}\exp\left(-\frac{k^2\rho^2}{8\gamma_1 B^2}\right)\left(I_0\left(\frac{k^2\rho^2}{8\gamma_1 B^2}\right)-I_1\left(\frac{k^2\rho^2}{8\gamma_1 B^2}\right)\right). \tag{21}$$

This last equation is the same of the main result of Ref. [25] concerning the generation of Kummer beam by an SPP illuminated by a Gaussian beam.

Then, our main result can be considered as a generalization of study in Ref. [25].

Case 2

If $\chi=l=0$, Eq. (14) reduces to

$$E(\rho,\varphi,z)=\frac{-i\pi A_0}{\lambda B}e^{ik(z-z')}e^{i\frac{kD}{2B}\rho^2}\sum_{m=1}^{M}\alpha_m\frac{\omega_m(0)}{\omega_m(z')}e^{i\Phi_m(z')}\gamma_m^{-1}\exp\left(-\frac{k^2\rho^2}{4\gamma_m B^2}\right). \tag{22}$$

This equation describes the propagation of LFTG beams through an ABCD optical system without SPP system. The output field distribution in this case is a non-doughnut beam.

Case 3

Where $M=1$ and $\chi=l=0$, Eq. (14) reduces to

$$E(\rho,\varphi,z)=\frac{-i\pi A_0}{\lambda B}e^{ik(z-z')}e^{i\frac{kD}{2B}\rho^2}\frac{\omega_1(0)}{\omega_1(z')}e^{i\Phi_1(z')}\gamma_1^{-1}\exp\left(-\frac{k^2\rho^2}{4\gamma_1 B^2}\right). \tag{23}$$

This equation describes the receiver field at a plane located at position z. It was the output fundamental Gaussian beam travelling through a paraxial ABCD optical system without an SPP system.

Case 4

Near optical axis at small ρ and moderates B ($\rho/B<<1$), Eq. (14) reduces to

$$E(\rho,\varphi,z)\approx A_0\frac{\pi(-i)^{|l|+1}}{\lambda B 2^{|l|}}e^{i(k(z-z')+l\varphi)}e^{i\frac{kD}{2B}\rho^2}\left(\frac{k}{B}\right)^{|l|}\rho^{|l|}\frac{\Gamma\left(\frac{|l|}{2}+1\right)}{\Gamma(|l|+1)}\sum_{m=1}^{M}\alpha_m\frac{\omega_m(0)}{\omega_m(z')}\gamma_m^{-\frac{|l|}{2}-1}e^{i\Phi_m(z')}. \tag{24}$$

This last equation confirms explicitly that the formed beams are hollow ones with a dark central intensity.

4. Numerical simulations

In this section, we are interested in analyzing the propagation of SK beam using Eqs. (14) and (17) for two particular cases: free space and Fractional Fourier Transform (FRFT) system in the case of integer topological charge $\chi=l$, respectively. We will also study the propagation through the SPP followed by a free space with a fractional topological charge.

4. 1. Propagation in free space

As first example, we consider the propagation through a free space for which the transfer matrix coefficients are: $A=1$, $B=z-z'$, $C=0$ and $D=1$. By using Eq. (13) and taking into account the above transfer coefficients and the following parameters: $\omega_0=1mm$, $\lambda=0.84948\,\mu m$, $l=1$ and $z'=1400mm$, we present in Fig. 3, the intensity of superposition of Kummer beams travelling in a free space. This figure presents the evolution of three dimensional intensity distributions of SK beams versus r propagating in a free space for fixed LFTG order beam M and for different propagation distances z. From the plots of this figure, we remark that the maximum intensity of the generated doughnut beam decreases gradually with the propagation distance. Also, we note that, for a fixed M, as long as the propagation distance decreases as much as the intensity of generated beam increases and the radius of dark region of the doughnut beam become finer and the intensity reaches zero quickly. This explains that the profile of the intensity of generated beam becomes very fine with the decreasing of propagation distance z. Note that the radius of the dark region becomes small. As far as z increases away from the SPP as far as the maximum intensity profile decreases. Also, we remark that the spot size expands and the radius of the centre dark region becomes important because of the phenomenon of diffraction.

Fig. 4 presents the intensity distribution in 3D of the SK beam at propagation distance $z=1850\ mm$ and for the fixed parameters cited above (z', ω_0) for different beam orders M. The plots show that the maximum of intensity increases slightly by increasing the order M.

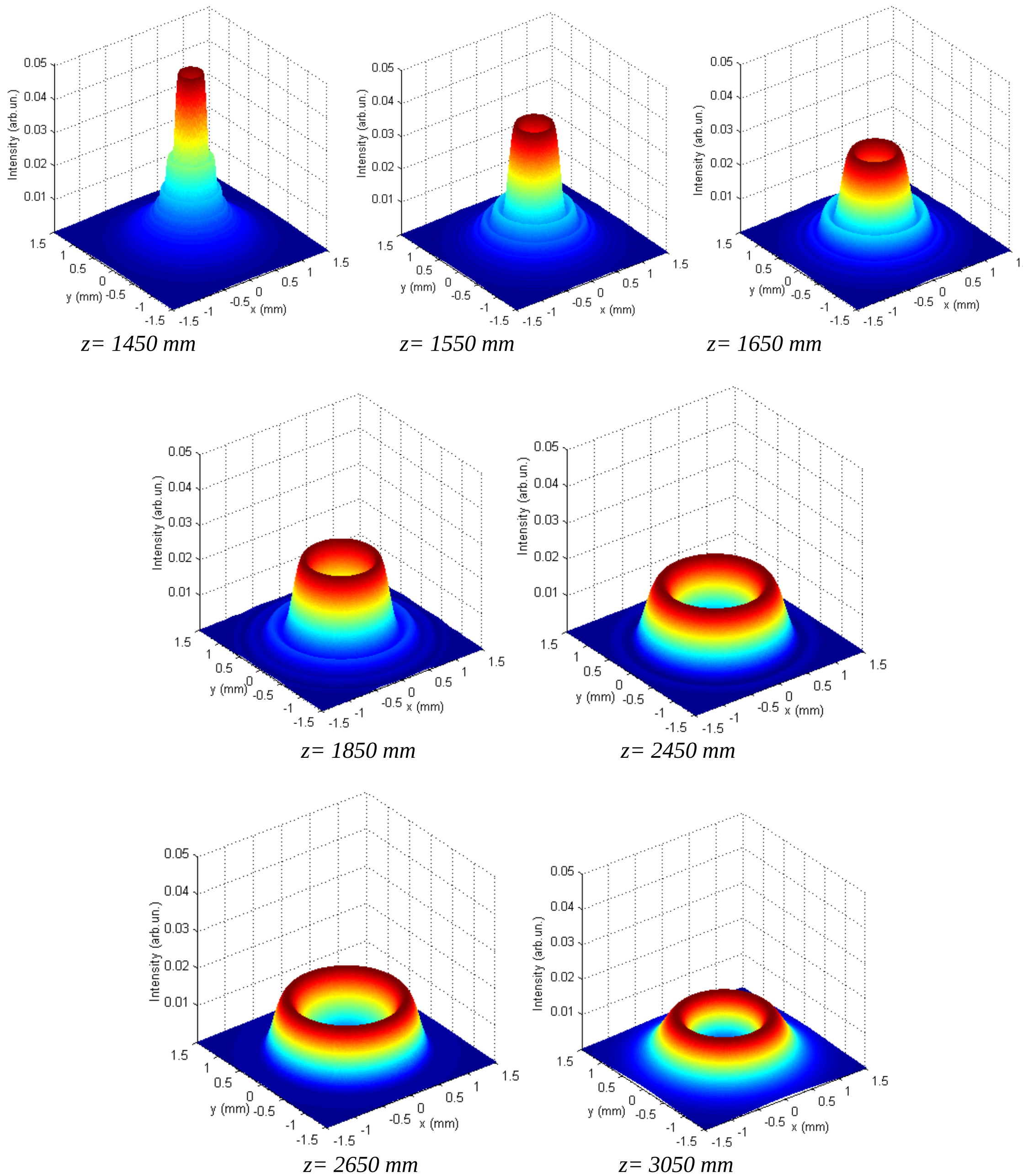


Fig. 3:, Evolution of the intensity distribution of SK beams after passing through a free space, for topological charge l=1, ω_0=1 mm, z'=1400 mm, M=40, λ=0.849 μm and for different propagation distances z.

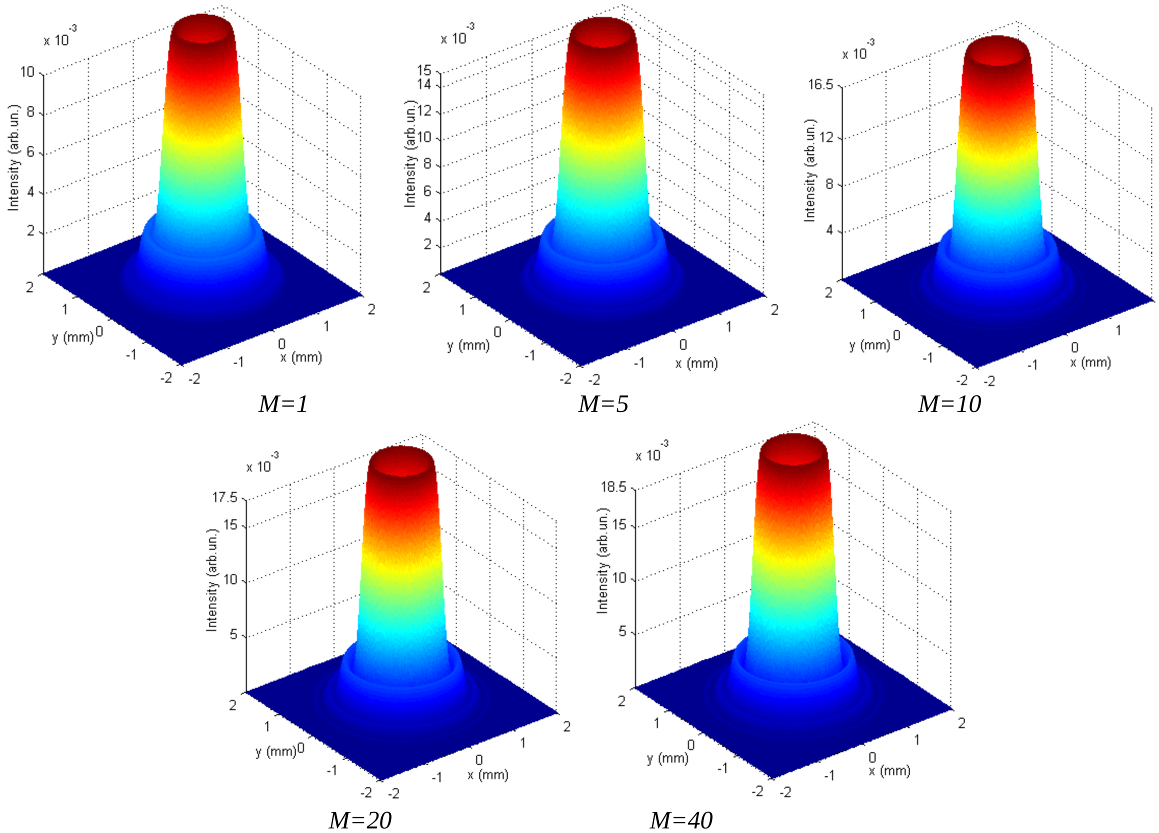


Fig. 4: Evolution of the intensity distribution of SK beams travelling a free space with topological charge l=1, ω_0=1 mm, z'=1400 mm and z=1850 mm for different beam orders M.

4.2 Propagation through a FRFT system

The FRFT case is regarded as a generalization of the conventional Fourier transform. It was defined in mathematics by Namias [30], and introduced in optics by Mendlovic and Ozaktas [31, 32]. There are at least two ways to implement optically the FRFT: the first is based on the use of the graded index medium and the second uses the combinations of lenses and space [33]. The ABCD matrix associated with the FRFT p-order is given by

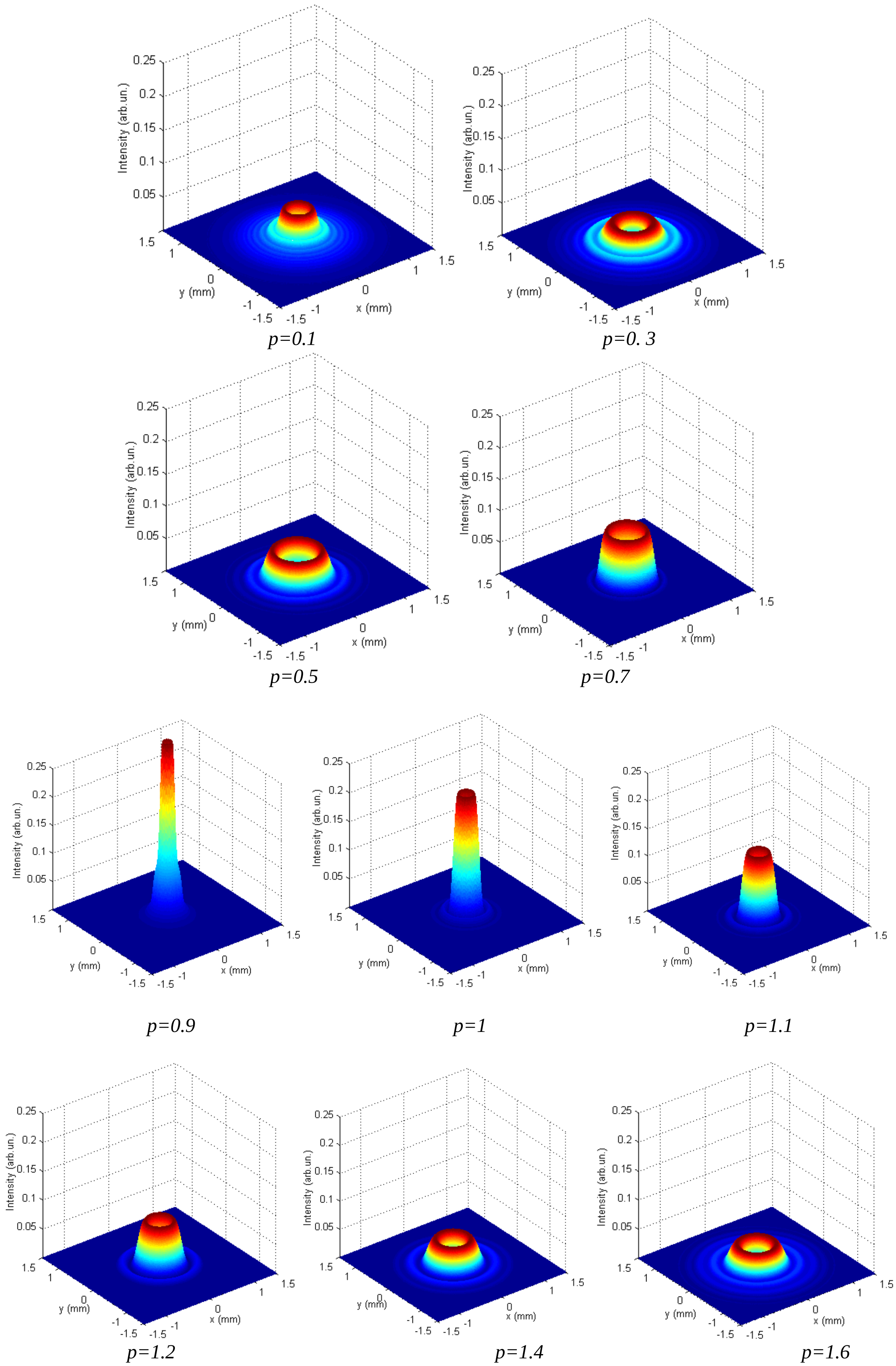
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos\phi & f_s \sin\phi \\ -\frac{1}{f_s}\sin\phi & \cos\phi \end{pmatrix}, \quad (25.a)$$

with

$$\phi = p\pi/2\,, \quad (25.b)$$

and f_s is the standard focal length.

By inserting the coefficients of the matrix given by Eqs. (25) into Eq. (14) one can obtain the intensity distributions of SK beams through a FRFT system for different fractional order *p*. The intensity distributions of SK beam travelling different FRFT planes with various fractional orders *p* are depicted in Fig. 5. From the illustrations of this figure, it's provided that the intensity distribution keeps the same profile whatever the FRFT order p. From *p=0.1* to *0.9* with increasing of FRFT order p, the radius of the centre dark region decreases and the maximum of intensity distributions increases, thus enough to *p=0.9*. Then, in this interval the doughnut outgoing beam becomes quite fine and intense with increased *p*. In the followed interval (from *p=0.9* to *1.8*), the maximum of intensity distribution of the output beam decreases and radii of dark region increases respectively with increasing of the FRFT p-order. Like to the first interval, in the third one (from *1.8* to *2.9*), more the parameter *p* increases the intensity maximum increases, and the beam propagates in space following r also increases.


Intensity (arb.un.)
x (mm)
y (mm)
p=0.1
p=0. 3
p=0.5
p=0.7
p=0.9
p=1
p=1.1
p=1.2
p=1.4
p=1.6

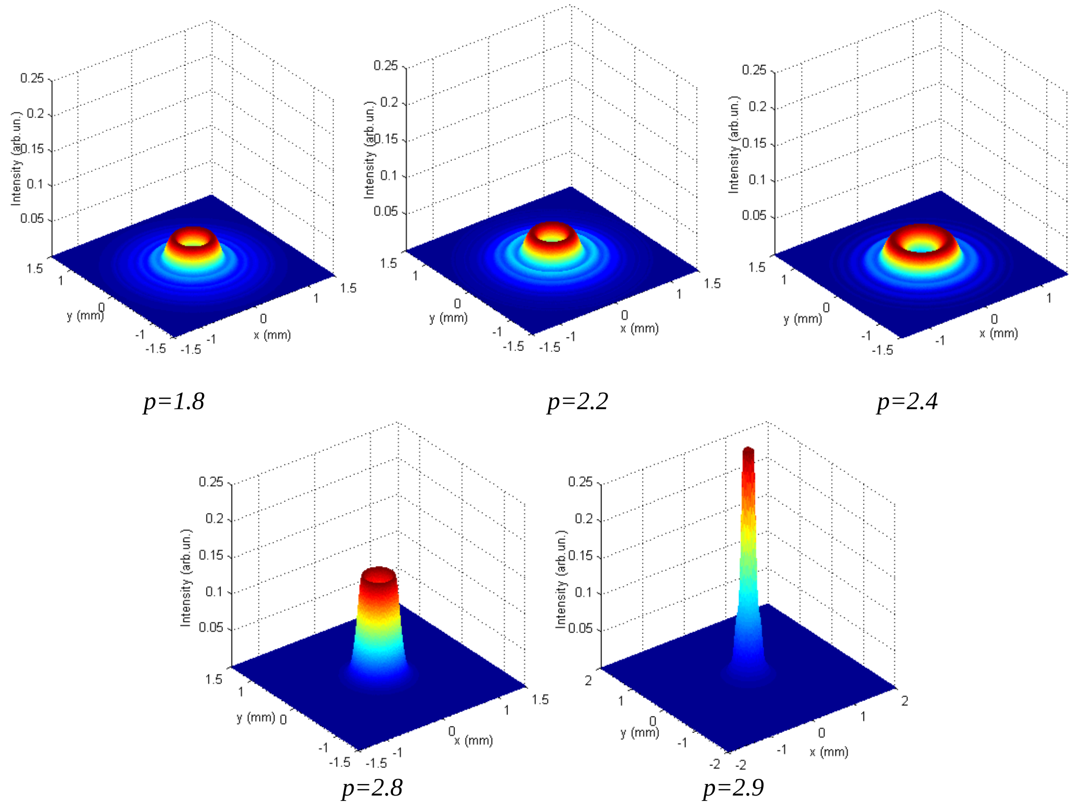


Fig. 5: Evolution of the intensity distribution of SK beams after propagation through a FRFT system for: l=1, ω_0=1 mm, z'=1400 mm, f_s=500 mm, and M=40 for different FRFT orders p.

4. 3. Propagation through an SPP with fractional topological charge

The aim of this paragraph focuses on the study of the response of the SPP when it's illuminated by a LFTG beam of wavelength $\lambda_\chi \neq \lambda = 849.9\ nm$ (where λ is the wavelength corresponding to an integer topological charge *l=1*). In this condition, if the SPP lighted up by a LFTG beam of $\lambda_\chi \neq \lambda\ (\lambda = 849.9\ nm)$, it introduces a fractional topological charge χ. The wavelengths and their corresponding topological charges used in the numerical simulations of this subsection are listed in the following Table 1.

Table 1. Fractional topological charges used by the SPP when illuminated with a beam of wavelength $\lambda_\chi \neq \lambda(= 849.9\ nm)$.

λ_χ (nm)	867.24	858.48	849.9	841.48	833.23
χ	0.98	0.99	1	1.01	1.02

The calculated intensity distributions for SK beams generated with different fractional topological charges χ are shown in Fig. 6 using the analytical expression of Eq. (18). The plots of this figure illustrate that the intensity maxima decreases with the increasing of χ. For showing the intensity oscillations in the case of $\chi \neq 1$, we give in this figure a zoom in the region [0,2mm].

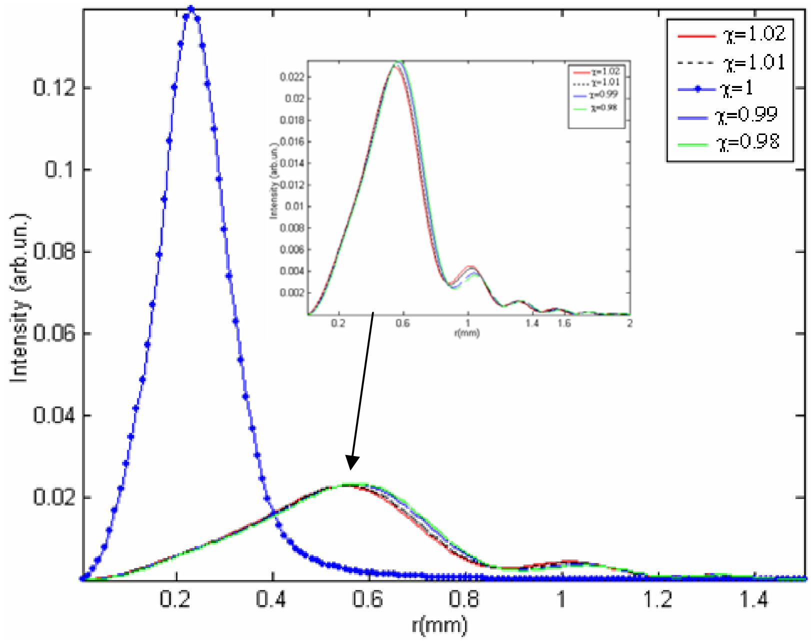


Fig. 6. Evolution of the intensity distribution of SK beams for different fractional topological charges χ (=0.98; 0.99; 1; 1.01; 1.02).

5. Summaries

We have suggested and demonstrated a novel analytical expression of superposition of Kummer beams as new doughnut beams generated by an SPP of integer and fractional topological charges which converts the phase of Li's flattened Gaussian beams. In the case of an integer topological charge of SPP, the generation of Kummer beams by an SPP system illuminated by fundamental Gaussian beams and the propagation of the LFTG and Gaussian beams travelling a paraxial ABCD optical system without SPP are deduced as special cases of our generalized study. Numerical calculations are performed to illustrate the paraxial propagation of these beams through free space (in the both cases: integer and fractional topological charges) and through a fractional Fourier transform system (in the case of integer one (l=1)).

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*Corresponding author: belafhal@gmail.com