

# Generation of vector type vortices in gradient fiber with spatial dependence of the refractive index

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The optical vortices are usually created outside the laser cavity using different optical masks and holograms. The vortex structures are characterized by helical phase fronts. Their solutions are governed by the 2D Leontovich scalar equation and admit amplitude and phase singularities. In present work we investigated the formation of vector vortex structures of optical pulses, propagating in concave and convex gradient optical fibers in nonlinear regime. The corresponding vector system of amplitude equations is solved analytically and new class analytical solutions, describing the generation of vector field vortices in such gradient optical fibers are found. These new vector vortices admit amplitude type singularities, but not phase ones. Experimentally, this will look like as a special kind of depolarization of the vector field in the spot diameter.

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## 1. Introduction

In present paper we have theoretically investigated the formation of new vector type optical vortex structures. These vector vortices are different from the standard scalar ones and have not phase singularity. The phase singularities in propagating scalar wave field were observed for the first time in [1]. The scalar optical vortices in a bulk self-defocusing Kerr nonlinear medium were experimentally observed also by authors in [2]. These optical structures generated in laser beam were later described in details in [3]. In [4] the helically phased beams with a Laguerre-Gaussian amplitude distribution carrying orbital angular momentum are found.

It is important to mention that the techniques of generation of scalar optical vortices are improved in recent years. In practice, different types of vortex structures of wave field can be created by optical holograms and various optical masks [5-9]. This gives more options for new applications as exploration of guiding dynamics of optical fibers, creation of optical tweezers, improving imaging, microscopy, sensing, frequency mixing and modulation.

All above scalar vortex structures are based on the solutions of the 2D paraxial equation of Leontovich for laser beam [10]. These solutions admit amplitude and phase singularities and also infinity of integral of the energy. Very good approach for excluding the amplitude singularities by using Gaussian pulses was presented by authors in [11]. Scalar vortices created in fibers by using various methods were reported in [12-15].

Our research is aimed at investigating the behavior of new type vector vortex structures propagating in concave and convex gradient optical fibers. Such type of fibers and their approach to optical communication systems was

investigated recently in [16-18]. The refractive index of the concave and convex gradient optical fibers can be presented by the expression:

$$\tilde{n} = n_o(\omega) + S_g(x^2 + y^2) + n_2|\vec{\Phi}|^2, \quad (1)$$

where  $\vec{\Phi}$  is the vector amplitude function, that describes the pulse envelope,  $n_o(\omega)$  and  $n_2$  are respectively the linear and nonlinear refractive indexes, characterizing the dispersion and nonlinearity of media and  $S_g$  is a constant. The term  $S_g(x^2 + y^2)$  gives the spatial dependence of the refractive index.

Depending on the sign of the constant  $S_g$  the optical fibers are divided in two types [19]:

- *Concave gradient fibers* for which the constant is bigger than zero ( $S_g > 0$ ). Towards the periphery of the waveguide it rises smoothly (Fig. 1).

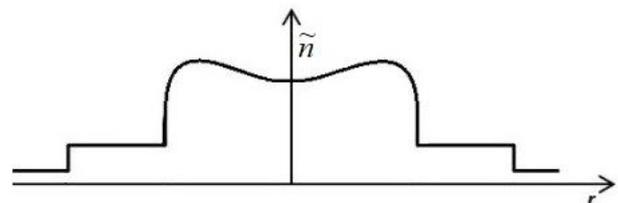


Fig. 1. Distribution of the refractive index for concave gradient fibers

Vector type vortex solutions for such kind of fibers are found for the first time in [20], where in details it was presented the mathematical algorithm for solving the nonlinear system of spatio-temporal amplitude equations, describing the evolution of the vector components. The

nonlinear dispersion relation obtained after solving this system of equations requires balance not only between diffraction and nonlinearity, but also between nonlinearity and angular distribution of the optical vector field. A number of numerical simulations of these solutions were demonstrated.

- *Convex gradient fibers* for which the constant is smaller than zero ( $S_g < 0$ ). On the fiber axis the linear refractive index has maximal value. To the periphery of the waveguides it decreases smoothly (Fig. 2).

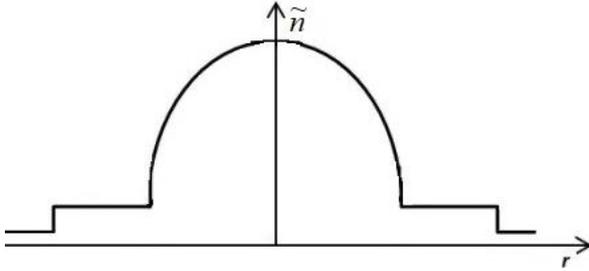


Fig. 2. Distribution of the refractive index for convex gradient fibers

## 2. Basic equations

In present investigation we will use the normalized nonlinear amplitude equation, describing the propagation of a linear polarized amplitude electrical field  $\vec{\Phi} = (U, V, 0)$ . It is written in the form [16]:

$$i \frac{\partial \vec{\Phi}}{\partial z} + \frac{1}{2} \left( \Delta_{\perp} \vec{\Phi} - S_d \frac{\partial^2 \vec{\Phi}}{\partial t^2} \right) + S_g (x^2 + y^2) \vec{\Phi} + \gamma |\vec{\Phi}|^2 \vec{\Phi} = 0, \quad (2)$$

where  $\gamma = \frac{1}{2} k_0^2 r_{\perp}^2 n_2 |\Phi_0|^2$ ,  $\Delta_{\perp} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ .

The parameter  $\gamma$  presents the nonlinearity of the medium and  $\Delta_{\perp}$  is the transverse Laplace operator. The constant  $S_d$  characterizes the dispersion of the media. The additional third term gives the spatial dependence of the linear refractive index. As we already mentioned, in [20] it is found an exact analytical solution for vector vortex structures in optical fiber with concave profile ( $S_g > 0$ ) of the spatial refractive index. An approximate solution of the equation above in scalar form for gradient fibers ( $S_g < 0$ ) is described in [19].

In this paper we are looking for exact analytical solution in vector form of equation (2) in the case of  $S_g < 0$  for vortex structures, generated in gradient fibers. This condition corresponds to optical waveguides and photonic crystals with gradient convex profile of the refractive index (Fig. 2). Such types of optical fibers have a number of applications in different nonlinear devices, controlling and manipulating laser light. They are used in modern optical sensors and communication systems, as parts of optical computers, lenses and mirrors in thin-film optics [21-26].

We are looking for vortices solutions of the partial differential equation (2) for the vector amplitude function  $\vec{\Phi}(x, y, z, t) = (U, V, 0)$ .

The following system of scalar equations for each of the components  $U$  and  $V$  of the vector amplitude function is formed:

$$\frac{\partial U}{\partial z} + \frac{1}{2} \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) - S_d \frac{\partial^2 U}{\partial t^2} + S_g (x^2 + y^2) U + \gamma |U^2 + V^2| U = 0, \quad (3)$$

$$\frac{\partial V}{\partial z} + \frac{1}{2} \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) - S_d \frac{\partial^2 V}{\partial t^2} + S_g (x^2 + y^2) V + \gamma |U^2 + V^2| V = 0, \quad (4)$$

where

$$U = U_x(x, y, z, t), V = U_y(x, y, z, t). \quad (5)$$

In order to find a solution of the system (3) and (4) we use the following mathematical algorithm:

- We will work in cylindrical coordinates:

$$\begin{aligned} x &= r \cos \theta, \quad y = r \sin \theta, \\ r &= \sqrt{x^2 + y^2}, \quad \theta = \arctan(y/x), \\ \Delta_{\perp} A &= \left( \frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} \right), \end{aligned} \quad (6)$$

- After the applied transformations, the system of equations (3) and (4) takes the form:

$$i \frac{\partial U}{\partial z} + \frac{1}{2} \left( \frac{1}{r} \frac{\partial U}{\partial r} + \frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} \right) - \frac{S_d}{2} \frac{\partial^2 U}{\partial t^2} - S_g r^2 U + \gamma |U^2 + V^2| U = 0, \quad (7)$$

$$i \frac{\partial V}{\partial z} + \frac{1}{2} \left( \frac{1}{r} \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V}{\partial \theta^2} \right) - \frac{S_d}{2} \frac{\partial^2 V}{\partial t^2} - S_g r^2 V + \gamma |U^2 + V^2| V = 0. \quad (8)$$

- We make the following substitutions:

$$U = K_1(z, t) R_1(r) T_1(\theta), \quad (9)$$

$$V = K_2(z, t) R_2(r) T_2(\theta), \quad (10)$$

where  $K_1$ ,  $K_2$ ,  $R_1$ ,  $R_2$ ,  $T_1$  and  $T_2$  are new unknown amplitude functions of the variables  $z$ ,  $t$ ,  $r$  and  $\theta$ .

We assume that  $T$  is given by the expression:

$$T = T_1 = T_2 = e^{in\theta}. \quad (11)$$

After the substitution of expressions (9), (10) and (11) in equations (7) and (8) we obtain:

$$\left( i \frac{\partial K_1}{\partial z} - \frac{S_d}{2} \frac{\partial^2 K_1}{\partial t^2} \right) \frac{1}{K_1} + \frac{1}{2} \left( \frac{1}{r} R_1' + R_1'' - \frac{n^2}{r^2} R_1 - 2S_g r^2 R_1 \right) \frac{1}{R_1} + \gamma |R_1^2 K_1^2 + R_2^2 K_2^2| = 0, \quad (12)$$

$$\left( i \frac{\partial K_2}{\partial z} - \frac{S_d}{2} \frac{\partial^2 K_2}{\partial t^2} \right) \frac{1}{K_2} + \frac{1}{2} \left( \frac{1}{r} R_2' + R_2'' - \frac{n^2}{r^2} R_2 - 2S_g r^2 R_2 \right) \frac{1}{R_2} + \gamma |R_2^2 K_2^2 + R_1^2 K_1^2| = 0. \quad (13)$$

To find the solutions of the equations (12) and (13) it is necessary to divide the variables. In the obtained system first term in brackets isn't function of  $r$  and the last two terms are not functions of  $t$  and  $z$ . In order to keep the equality it is necessary the expressions in the brackets, on both sides of the system, to be equal to zero.

$$\left( i \frac{\partial K_1}{\partial z} - \frac{S_d}{2} \frac{\partial^2 K_1}{\partial t^2} \right) \frac{1}{K_1} = 0, \quad (14)$$

$$\frac{1}{2} \left( \frac{1}{r} R_1' + R_1'' - \frac{n^2}{r^2} R_1 - 2S_g r^2 R_1 \right) \frac{1}{R_1} + \gamma |R_1^2 K_1^2 + R_2^2 K_2^2| = 0, \quad (15)$$

$$\left( i \frac{\partial K_2}{\partial z} - \frac{S_d}{2} \frac{\partial^2 K_2}{\partial t^2} \right) \frac{1}{K_2} = 0, \quad (16)$$

$$\frac{1}{2} \left( \frac{1}{r} R_2' + R_2'' - \frac{n^2}{r^2} R_2 - 2S_g r^2 R_2 \right) \frac{1}{R_2} + \gamma |R_2^2 K_2^2 + R_1^2 K_1^2| = 0. \quad (17)$$

According to our mathematical algorithm, we assume that the phase function has the form:

$$K = K_1 = K_2 = e^{i\psi}, \quad \psi = az + bt, \quad (18)$$

where  $a$  and  $b$  are constants:  $\frac{\partial \psi}{\partial z} = a$ ;  $\frac{\partial \psi}{\partial t} = b$ ;  $\frac{\partial^2 \psi}{\partial t^2} = 0$ .

After applying the substitution above in equations (14) and (16), we obtain the following connection between the unknown constants:

$$a = b^2 \frac{S_d}{2}. \quad (19)$$

Our next step is to make another substitution for the functions  $R_1$  and  $R_2$ :

$$R_1 = \frac{A}{r} ch(mr^2), \quad R_2 = i \frac{A}{r} sh(mr^2), \quad (20)$$

where  $A$  is a constant.

After the substitution of expressions (20) in equations (15) and (17) we obtain:

$$\frac{1-n^2}{2r^2} + (4m^2 - 2S_g)r^2 + \frac{\gamma}{r^2} A^2 = 0, \quad (21)$$

$$\frac{1-n^2 + 2\gamma A^2}{2r^2} + (4m^2 - 2S_g)r^2 = 0. \quad (22)$$

From the equation (21) we can define the constant  $A$ :

$$A = \sqrt{\frac{n^2 - 1}{2\gamma}}, \quad (23)$$

and the constant  $m$  is connected with the spatial refractive index of the optical fiber:

$$m = \pm \sqrt{\frac{S_g}{2}}. \quad (24)$$

Thus, going back through all the substitutions and assumptions, we have found exact analytical solutions of the system of equations (3) and (4), describing the formation of vortices in convex gradient optical fiber in the case of  $b=1$ :

$$U = \sqrt{\frac{n^2 - 1}{2\gamma}} \frac{1}{r} ch \left( r^2 \sqrt{\frac{S_g}{2}} \right) e^{i \left( t+z \sqrt{\frac{S_d}{2}} \right)} e^{in\theta}, \quad (25)$$

$$V = i \sqrt{\frac{n^2 - 1}{2\gamma}} \frac{1}{r} sh \left( r^2 \sqrt{\frac{S_g}{2}} \right) e^{i \left( t+z \sqrt{\frac{S_d}{2}} \right)} e^{in\theta}. \quad (26)$$

In the case of a concave gradient fiber ( $S_g > 0$ ) following the same mathematical algorithm, we can find new solutions. These new solutions have the same constants as those for gradient fiber with a positive  $S_g$  number. In contrast to the solutions found in [20] here the amplitude function  $\Phi$  is expressed not by hyperbolic functions, but as simple trigonometric functions.

$$U' = \sqrt{\frac{n^2 - 1}{2\gamma}} \frac{1}{r} \cos\left(r^2 \sqrt{\frac{S_g}{2}}\right) e^{i\left(t+z\sqrt{\frac{S_d}{2}}\right)} e^{in\theta}, \tag{27}$$

$$V' = \sqrt{\frac{n^2 - 1}{2\gamma}} \frac{1}{r} \sin\left(r^2 \sqrt{\frac{S_g}{2}}\right) e^{i\left(t+z\sqrt{\frac{S_d}{2}}\right)} e^{in\theta}, \tag{28}$$

where  $n$  is the vortex number,  $n > 1$ ,  $n = 2, 3, 4, \dots$

The solutions (27) and (28) present the components ( $U'$  and  $V'$ ) of the vector electrical field  $\vec{\Phi}$  of the amplitude function. The parameter  $n$  corresponds to the angular distribution of the vortex structure. This vortex parameter needs to have values other than zero in order to form vortex structures. In the case of  $n=0$  there are no solutions of the basic vector system of nonlinear equations.

The usual scalar theory of optical vortices is linear and that's why the amplitude constants are not presented in the obtained linear dispersion relation. The solutions (25) - (28) of the corresponding nonlinear vector system (7) - (8) arise new nonlinear dispersion relations where the balance is not only between diffraction and nonlinearity, but also between nonlinearity and angular distribution of the optical vector field.

### 3. Numerical calculations

We have performed a couple of numerical calculations, based on the solutions (27) and (28) of the system of equations (3) and (4). Pulses propagating in nonlinear dispersive fibers with spatial dependence of the refractive index and different values of vortex number  $n$  are presented in the following figures. The intensities of the optical vortices according to the analytical solutions (27) and (28) for negative sign of  $S_g$  parameter and vortex number  $n=2$  are of the kind:

$$|U|^2 = \frac{3}{2} \frac{ch^2\left(r^2 \sqrt{\frac{S_g}{2}}\right)}{r^2}, \tag{29}$$

$$|V|^2 = \frac{3}{2} \frac{sh^2\left(r^2 \sqrt{\frac{S_g}{2}}\right)}{r^2}. \tag{30}$$

In Fig. 3 (a) and (b) are shown the profiles of the components  $|U|^2$  and  $|V|^2$  of the amplitude function of the optical vortex for  $S_g < 0$ . It is observed a symmetrical distribution of the pulse intensity.

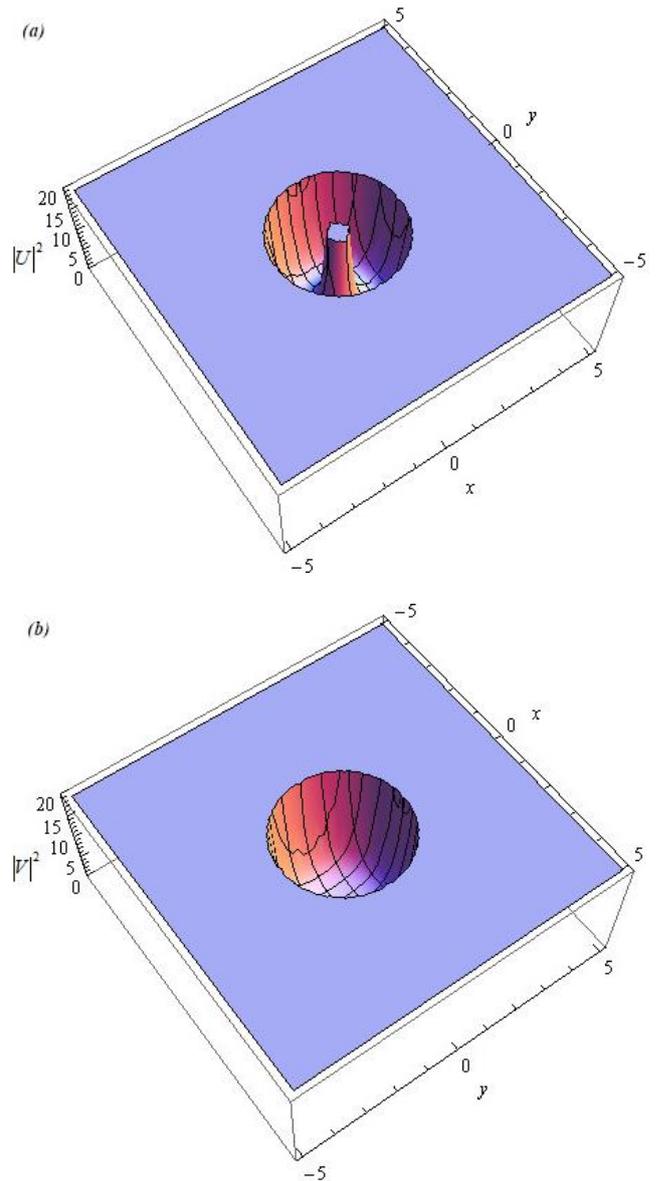


Fig. 3. Intensity profiles of the components (a)  $U$  and (b)  $V$  in the case of  $S_g < 0$  and  $n=2$  (color online)

On Fig. 4 it is presented the intensity profile of the vortex structure. It is also observed a symmetrical distribution of the vortex energy in the pedestal.

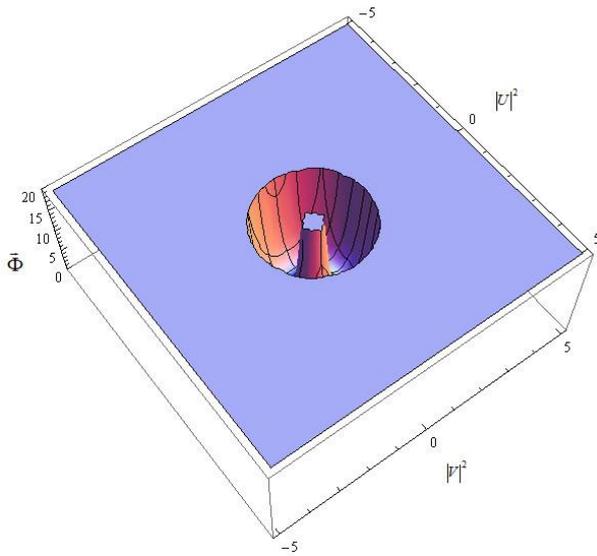


Fig. 4. Intensity profile of the optical vortex in the case of  $S_g < 0$  and  $n=2$  (color online)

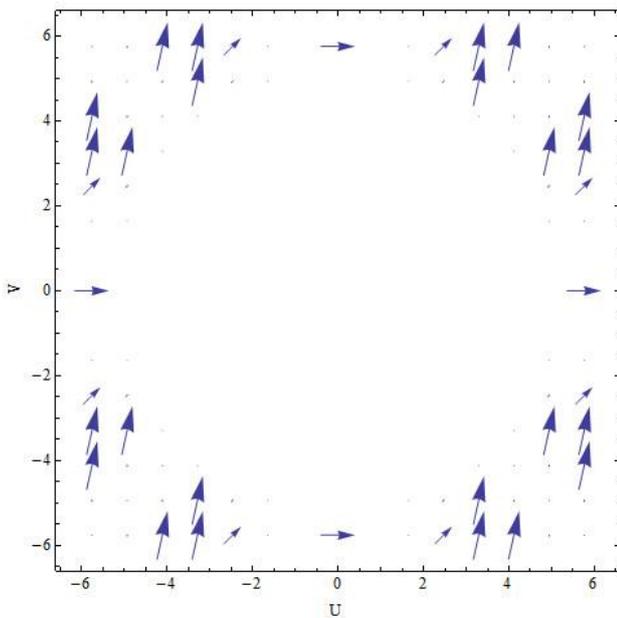


Fig. 5. Diagram of the vector amplitude function for  $n=2$  in the case of  $S_g < 0$ . Rotation of the vector of the electrical field along the periphery is observed (color online)

On Fig. 5 it is shown the diagram of the vector amplitude function of the optical vortex in the case of  $S_g > 0$  for  $n=2$ . Significant rotation of the vector of the electrical field along the periphery of the vortex is observed.

In the second case we consider the vortex solutions (27) and (28) with a positive sign of the parameter  $S_g$  ( $S_g > 0$ ). They are of the kind:

$$|U'|^2 = \frac{3}{2} \frac{\cos^2 \left( r^2 \sqrt{\frac{S_g}{2}} \right)}{r^2}, \tag{31}$$

$$|V'|^2 = \frac{3}{2} \frac{\sin^2 \left( r^2 \sqrt{\frac{S_g}{2}} \right)}{r^2}. \tag{32}$$

In Fig. 6 (a) and (b) the profiles of the components  $|U'|^2$  and  $|V'|^2$  of the amplitude function of the optical vortex for  $n=2$  are presented. In this case the distribution of the intensity of the pulse is also symmetrical.

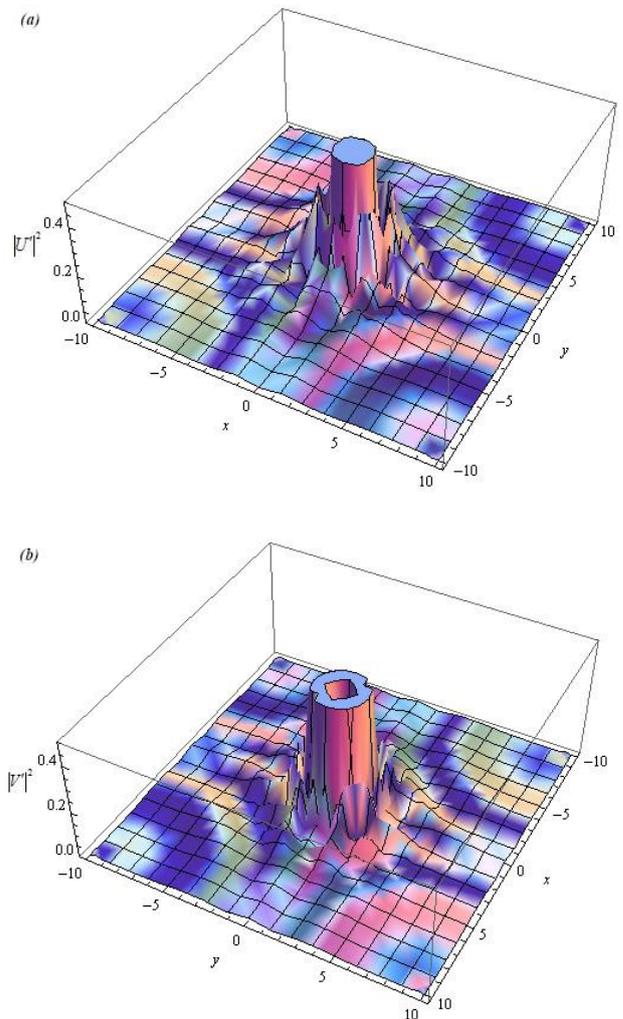


Fig. 6. Intensity profiles of the components (a)  $U'$  and (b)  $V'$  in the case of  $S_g > 0$  and  $n=2$  (color online)

In Fig. 7 it is presented the intensity profile of the vortex structure. It is observed a symmetrical distribution of the vortex energy in the pedestal.

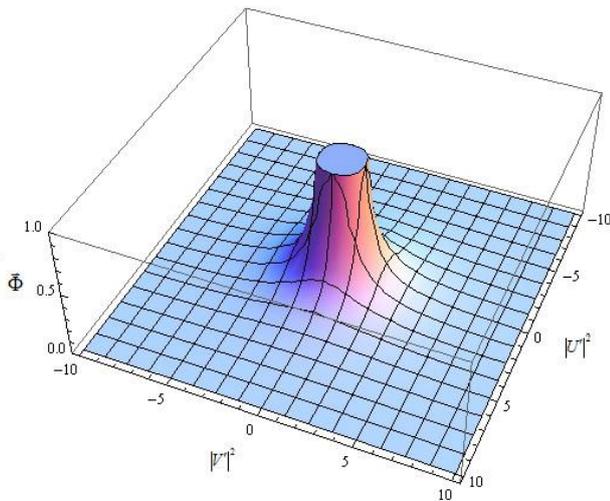


Fig. 7. Intensity profile of the optical vortex in the case of  $S_g > 0$  and  $n=2$  (color online)

On Fig. 8 it is shown the diagram of the vector amplitude function of the optical vortex in the case of  $S_g > 0$  for  $n=2$ . Significant rotation of the vector of the electrical field in the center of the vortices is observed.

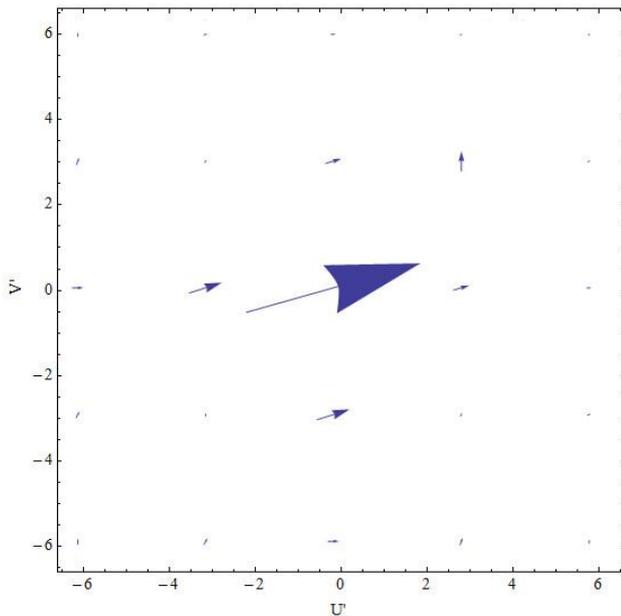


Fig. 8. Diagram of the vector amplitude function for  $n=2$  in the case of  $S_g > 0$ . Significant rotation of the vector of the electrical field in the center of the vortices is observed (color online)

#### 4. Conclusions

In the present paper a new class of vector optical vortices in convex gradient fibers with spatial dependence of the refractive index is presented. These vector solutions have no singularity in the phase.

Numerical simulations of the exact analytical solutions were made in two cases, depending on the

coefficient  $S_g$ , taking into account the spatial dependence of the refractive index. Again, as in our previous work [20], the nonlinear dispersion relations of these vector vortex solutions are obtained as balance between diffraction, nonlinearity and angular distribution of the field. Thus, a stability of these vortices can be expected. In an experiment these vector vortices will look like a special (vortex) type of depolarization of the vector field in the spot diameter.

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