Impacts of inclining incidence on surface plasmons excited by a focused vortex beam

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This paper reports the analysis of Surface Plasmons(SPs) field on the metal film induced by a inclining focused vortex beam, Using vectorial Debye integral theory, general angular spectrum expression for inclining incidence is given and the field intensity in the vicinity of gold film is simulated and analyzed under two kinds of opposite inclined ways relative to linearly polarized direction. The results indicate that the relationship between inclining orientation and polarized direction decides SPs field pattern and especially inclining direction coplanar with linearly polarized direction, inclined angle can adjust the amplitudes and offsets of asymmetrical field peaks but lateral FWHM of zero order field peak is expanding by and large for all two inclined way with increasing inclined angle which it is potential to decrease transverse resolution in the imaging system.

1. Introduction

Surface Plasmons [1] is excited evanescent wave that incident light of the photons interact with free electrons in the metal and dielectric interface and generate collective oscillation under the resonance condition. The SPs will quickly decay exponentially away from the interface but propagate along the metal surface. The study of SPs has been greatly focused since Ebbesen *et al*. had experimentally observed uniquely enhanced optical transmission through subwavelength hole arrays in metal films [2]. Due to extraordinary nature of the SPs, it opens up the imaging system with ultra-high resolution [3-4] and new exciting applications of SPs based circuits and optical devices [5-9] such as subwavelength waveguide components, modulators, switches and so on.

Vortex beam is special electromagnetic wave with spiral wave phase. According to its phase singularity, vortex beam may produce a dark nuclear in the spin center and has been strongly concerned. Recently, P.S. Tan and colleagues [10] reported the analysis of surface plasmon interference pattern formed by optical vortex beam, which they use a focused vortx beam to illuminate a metal/dielectric film vertically. Subsequently, their simulation indicates the SPs interference pattern generated by vortex beam has potential as a resolution enhancement technique for sub-diffraction imaging. Because in such total-internal-reflection fluorescence (TIRF) microscope, we generally consider inclined illumination, in this paper we further study that SPs standing field distribution when gold (Au) thin film is illuminated by focused vortex beam with an incline angle. Using angular spectrum representation [11], we will simulate and analyze SPs pattern under normal and inclined radiation.

2. Theory

Let us consider a gold (Au) thin film (ε_2) to be deposited on a glass substrate (ε_1) which air (ε_3) is above the film is illuminated by a focus vortex beam, as shown in Fig. 1. Vortex beam has an extraordinary intensity profile that is a ring of primary intensity accompanied by concentric outer rings of diminishing intensity. Linearly polarized vortex beam can be generated by which linearly polarized Gaussian beam pass through an azimuthally modulated phase mask and get a phase of $\exp(il\phi)$ where l is topological charge. If the beam vertically converges towards a diffraction limited spot on metal film by the lens with numerical aperture (NA)where there are a large wider spectrum of wave-vectors in the spot, thus two equipotential and opposite SPs waves with k_{SPs} in the vicinity of the metal film will generate interference. When the metal film incline an angle relative to focus vortex beam, this inclined illumination gives rise to two sets of opposed horizontal wave vector component is not equal and generate different SPs pattern in metal plane.
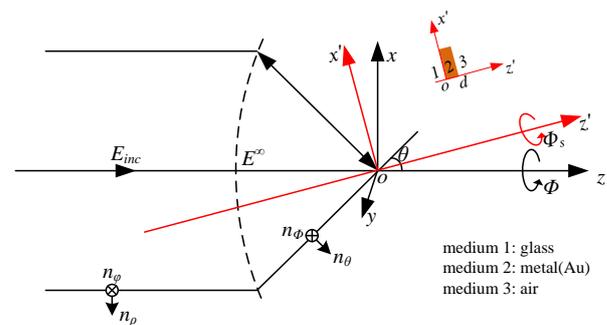
Fig. 1. schematic diagram of SPs wave generated by a inclining focusd vortex beam.

As shown in Fig. 1, assuming polarized direction along the x axis ($E_{inc}\cos\phi\,\mathbf{n_x}$) in Cartesian coordinate(x, y, z) and the focus point located in the origin of coordinate, according to vectorial Debye integral theory of Richards and Wolf[11] and the literature[10], the angular spectrum representation of the transmitted field under normal illumination is given by

$$\mathbf{E}_t(x,y,z) = A \iint_{k_x k_y} \mathbf{E}^{\infty}(k_x, k_y) \frac{1}{k_z} \exp[i(k_x x + k_y y + k_z z)] dk_x dk_y$$

(1)

Where A is a constant, $\mathbf{E}^{\infty}$ is the asymptotic far field in the direction of the unit vector $\mathbf{s}=(k_x/k, k_y/k, k_z/k)$.

$$\mathbf{E}_t(\rho,\varphi,z) \propto \int_0^{\theta_{max}} \int_0^{2\pi} t^p E_{inc} \cos\phi \begin{bmatrix} \cos\phi\cos\theta \\ \sin\phi\cos\theta \\ -\sin\theta \end{bmatrix} (\cos\theta)^{1/2} \exp[i(l\phi + k_{z3}z + k_\rho\rho\cos(\phi-\varphi))]$$
$$\times \sin\theta d\theta d\phi$$

(2)

where (ρ,φ,z) is the corresponding cylindrical coordinate of the coordinate system (x, y, z), $E_{inc}\exp(il\phi)$ represents incident vortex beam, θ is the divergence angle of the conjugate ray which $\theta_{max} = \sin^{-1}(NA/n_1)$ depends on the numerical aperture of the objective lens and the index of refraction of the surrounding medium, the transmission coefficient t^p can be derived by continuous boundary conditions for tangential magnetic field [12].

$$t^p = \frac{4\exp[i(k_{z2}-k_{z3})d]}{(1+p_{12})(1+p_{23})[1+r_{12}r_{23}\exp(i2k_{z2}d)]}$$

$$p_{ij} = \frac{\varepsilon_i k_{zj}}{\varepsilon_j k_{zi}}, r_{ij} = \frac{1-p_{ij}}{1+p_{ij}}, k_{zj} = \sqrt{k_j^2 - k_\rho^2} \quad (3)$$

where d is the thickness of the gold layer, medium 1 = glass, 2 = Au and 3 = air. So when vortex beam vertically focuses on the upper surface of gold layer, tangential wavevector k_ρ is equal to $k_1\sin\theta$, k_{zj} is the longitudinal wave vector for jth medium.

When xoz plane or metal plane rotates around y-axis at β degree which is equivalent that vortex beam has inclining incidence in the $x'oz'$ plane of cartesian coordinate(x', y, z'), the electric field $\mathbf{E}'$ and incident wavevector $\mathbf{k_1'}$ in the new coordinate can be expressed as

$$\mathbf{E}' = R(\alpha,\beta,\gamma)\mathbf{E} = R_x(\alpha)R_y(\beta)R_z(\gamma) \left[E_{inc}\cos\phi \begin{bmatrix} \cos\phi\cos\theta \\ \sin\phi\cos\theta \\ -\sin\theta \end{bmatrix} \right] \quad (4)$$

$$\mathbf{k_1'} = \begin{bmatrix} k_{x1}' \\ k_{y1}' \\ k_{z1}' \end{bmatrix} = R(\alpha,\beta,\gamma)\mathbf{k_1} = R_x(\alpha)R_y(\beta)R_z(\gamma) \begin{bmatrix} k_1\sin\theta\cos\phi \\ k_1\sin\theta\sin\phi \\ k_1\cos\theta \end{bmatrix}$$

(5)

where $R(\alpha,\beta,\gamma)$ is coordinate transformation matrix and $R_x(\alpha), R_y(\beta), R_z(\gamma)$ describe the transformation of the Jones vector for x, y, z respectively. From the fig1.(a), $R = R_y(\beta)$.

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \quad (6)$$

From Eq.(5), we know inclining gives rise to the change of transverse wavevector on the metal interface where its azimuth angle turns from ϕ to ϕ'

$$\tan\phi' = \frac{k_{y1}}{k_{x'1}} = \frac{\sin\theta\sin\phi}{\sin\theta\cos\phi\cos\beta + \cos\theta\sin\beta} \quad (7)$$

here, we can know transverse azimuth angle ϕ' depends on $k_{x'1}, k_{y1}$. In fact, planar integration over k_{x1}, k_{y1} has been transformed into planar integration over $k_{x'1}, k_{y1}$ and in order to replace the planar integration over $k_{x'1}, k_{y1}$ by a spherical integration over θ, ϕ we should transform the differentials as

$$\frac{1}{k_{z'1}}dk_{x'1}dk_{y1} = \frac{1}{k_{z'1}}dk_{x1}dk_{y1} = \frac{1}{k_{z'1}}d(k_{x1}\cos\beta + k_{z1}\sin\beta)dk_{y1}$$
$$= \frac{k_1^2}{k_{z'1}}(\cos\beta - \frac{k_{x1}\sin\beta}{k_{z1}})\sin\theta\cos\theta d\theta d\phi$$

(8)

Thus the angular spectrum representation of transmitted field under inclining, can be expressed in cylindrical coordinate by

$$\mathbf{E}_t(\rho_s,\varphi_s,z') \propto \int_0^{\theta_{max}} \int_0^{2\pi} t^p \mathbf{E}'(\cos\theta)^{1/2} \exp[i(l\phi + k_{z'3}z' + k_\rho'\rho_s\cos(\phi'-\varphi_s))]$$
$$\times \frac{1}{\cos\theta\cos\beta - \sin\theta\sin\beta\cos\phi}(\cos\beta - \frac{\cos\phi\sin\theta\sin\beta}{\cos\theta})\sin\theta\cos\theta d\theta d\phi$$

(9)

where (ρ_s, φ_s, z')is the cylindrical coordinate of the (x', y, z') coordinate system.

$$k_\rho' = \sqrt{(k_{x'1})^2 + (k_{y1})^2}, k_{z'3} = \sqrt{(k_3)^2 - (k_\rho')^2}$$

Subsequently, the vertical component of transmitted field is

$$E_t(\rho_s, \varphi_s) \propto \int_0^{\theta_{\max}} \int_0^{2\pi} t^P E_{inc} \cos\phi (\sin\beta\cos\phi\cos\theta + \cos\beta\sin\theta)(\cos\theta)^{1/2}$$

$$\times \exp[i(l\phi + k_{z'3}z + k'_\rho \rho_s \cos(\phi' - \varphi_s))]\sin\theta d\theta d\phi$$

(10)

Furthermore, there is the other different situation that when xoz plane or metal plane rotates around x-axis at α degree. It only needs change the Jones vector and differentials to derive formulas again. Due to Limited paper, these formulas will no longer be given.

3. Numerical analysis

Assumed that the dielectric constant of glass ε_1, gold film ε_2 and air ε_3 are 2.31, -5.28+0.2i, 1 respectively, incident wavelength is 532nm, the gold film thickness is 45nm, $\theta_{\max}$ is 1.2rad and topological charge l is 1, when a linearly polarized incident vortex beam with polarized direction along the x axis vertically ($E_{inc}\cos\phi$, $\alpha = 0^o$) focuses on the upper surface of gold layer, the numerical simulation of transmitted vertical field intensity on the lower surface has been calculated. In Fig. 2(a), there are discrete SPs interference fringes with rotational symmetry originate from the interfering of electromagnetic wave with opposite wave-vectors on the Au film which is

surface plasmons self-interference without using protrusions or holes, and we can observe the brightness of the fringes is decaying quickly with increasing lateral distance. This may result from the dispersion relation of SPR. The Fig. 2(b) shows the numerical result of SPs intensity on the an Au/air interface under normal illumination when topological charge l is −1, which the rotational direction of SPs fringes are contrary to the direction of those fringes in Fig. 2(a) and indicates that rotational direction is connected with the sign of topological charge l.

If we adopt the incoherent superposition of two vortex beams with opposite topological charge $l = \pm 1$ to excite gold film, there are symmetrical SPs interference fringes on the film as shown in Fig. 3(a). The Fig. 3(b) shows the distribution of normalized $|E_z|^2$ along the x-direction, which the maximum field value locates in the center of metal layer and the fringes period is approximate to 250nm less than half of incident wavelength. This indicates that evanescent waves with super diffraction limited spatial frequency have dominant advantage by the excitation of SPR and further dramatically rule by taking advantage of the electromagnetic field in the inner ring of vortex beam is nearly equal to zero to eliminate the contributions of directly transmitted lights which traditional Gaussian beam can't realize it.

(a)   *(b)*
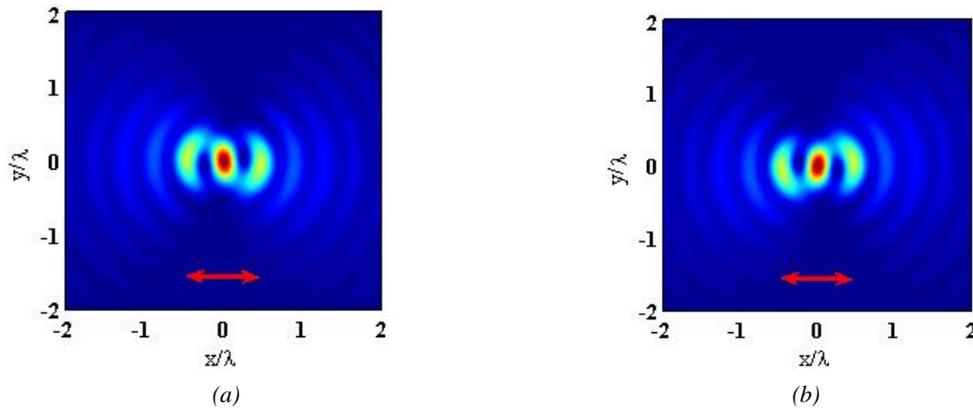
Fig. 2. The numerical result of SPs intensity on the an Au/air interface under normal illumination; (a) l =1; (b) l = −1.
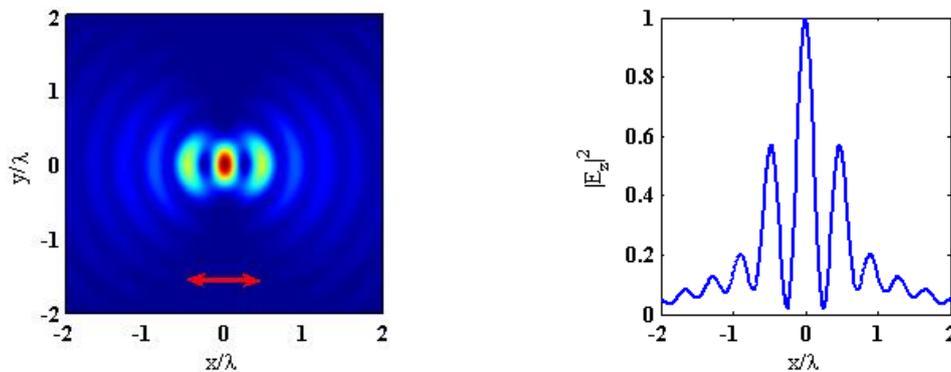
Fig. 3.(a) the numerical result of SPs intensity under normal illumination and $l = \pm 1$; (b) the lateral distribution of normalized field intensity.
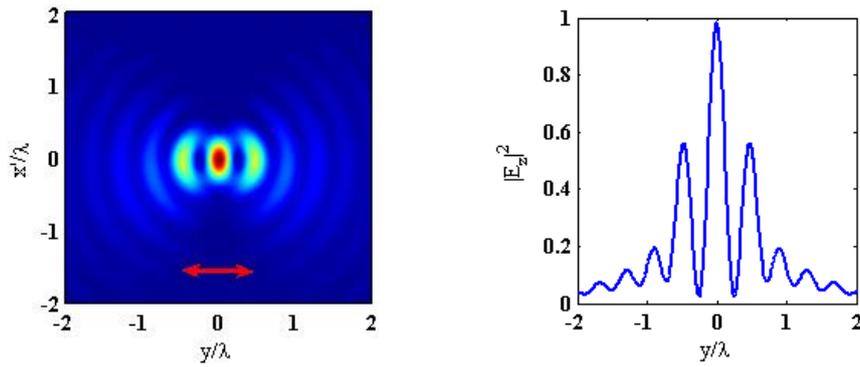
Fig. 4.(a) the numerical result of SPs intensity when α is 0.2rad with polarized direction along the y axis; (b) the lateral distribution of normalized field intensity along the y direction relative to maximum SPs intensity value undernormal illumination.

(a)   (b)
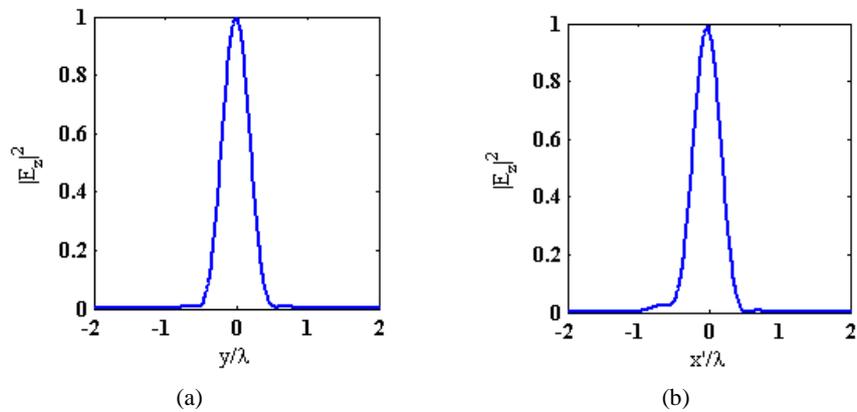
Fig. 5. (a)the distribution of field intensity along the y axis with normal illumination; (b) the longitudinal distribution of field intensity along the x' axis when α is 0.2rad with polarized direction along the y axis.

We continue to discuss inclined incidence for the superposition of two vortex beams with opposite topological charge $l = \pm 1$ which SPs field excited by linearly polarized incident beams is both x and y axis-symmetrical and is beneficial to analyze inclining situation. When vortex beam incident obliquely in the (x', y, z') coordinate with polarized direction along the y axis ($E_{inc} \sin\phi$)which inclining direction is perpendicular to polarization direction, it is predictable that SPs field can still keep x' axis-symmetrical but get brightened in the opposite inclining orientation as the Fig. 4(a) shown. The Fig. 4(b) shows the distribution of normalized field intensity along the axis x=0 relative to the maximum field intensity value without inclining is familiar with Fig. 3(b). Whereas we note that zero field peak appears a longitudinal offset and second field peak get increased in the opposite direction while vortex beams incline to positive x'-axis as shown in Fig.5.

(a)   (b)
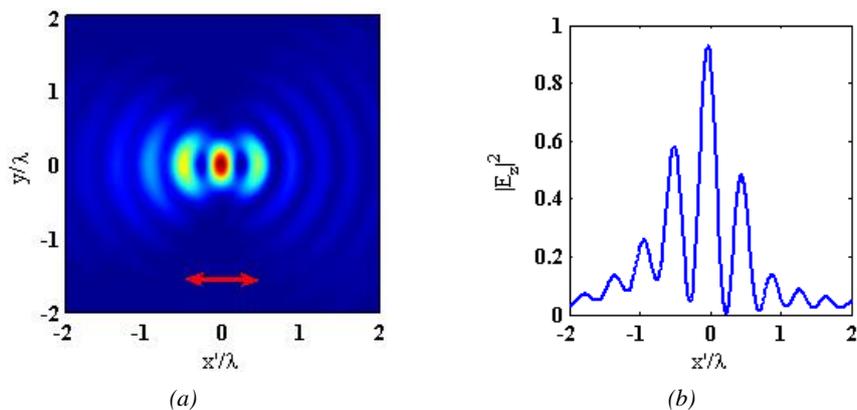
Fig. 6.(a) The numerical result of SPs intensity when α is 0.2rad with polarized direction along the x axis; (b) the lateral distribution of normalized field intensity along the x' direction.
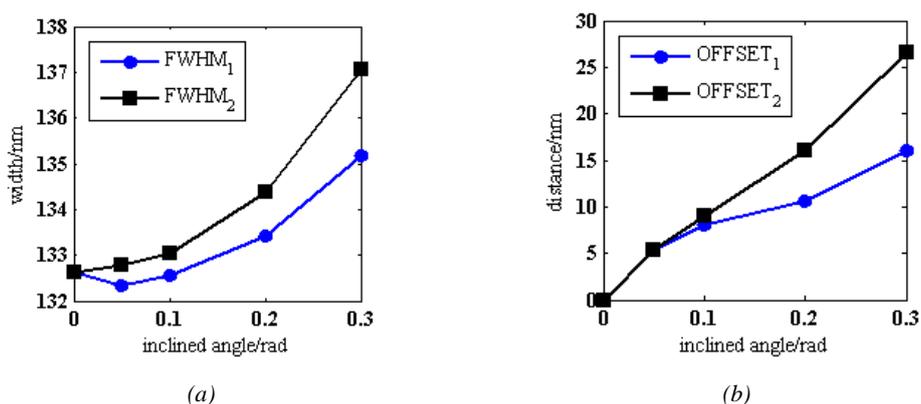
Fig. 7. The curve of (a)FWHM ; (b) offset; of zero order intensity with inclined angle (1: polarized direction along the   y axis, longitudinal offset; 2: polarized   direction along the x axis, lateral offset).

When inclining direction of vortex beam is coplanar with polarization direction($E_{inc} \cos\phi$), the fig.6 shows that zero order field peak has a lateral offset and one side of filed peaks in opposite inclining direction get increased relative to the other side and even exceed the same order peaks for normal incidence, which inclining maybe make general SPR effect for one side of opposite wave-vectors strengthened. Thus, it is of great significance that the amplitude and pattern of field peak is modulated by inclining vortex beam in micro-nano optical device. However, inclining maybe reduce transverse resolution potentially in the imaging system. The fig.7(a) indicates that when inclined angle get increasing, lateral FWHM for inclining direction coplanar with polarization direction are increasing, but for inclining direction inclining direction perpendicular to polarization direction, its lateral FWHM decreases resulting from longitudinal offset when inclined angle is small but then increases on the whole. The Fig. 7(b) shown zero order filed peak has an increasing longitudinal or lateral offset for two inclining pattern; and because center angular spectrum energy can just compensate for the loss of general SPR effect in small angle level.

4. Conclusion

In summary, with different polarization direction and a focused vortex beam, inclining will give rise to different SPs field, which the amplitudes of SPs field peaks and its standing wave pattern are modulated by inclined angle and given parameters, and the deterioration of lateral FWHM and field peaks resulted from inclining reduce imaging resolution. It should be pointed out that all changes for SPs field are essentially the compromise result of angular spectrum energy restriction and SPR effect besides polarization. for two opposite tilted way, deterioration effect for inclining coplanar with polarization direction is more and more worsen.

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