Implicit quiescent optical solitons with complex Ginzburg–Landau equation having nonlinear chromatic dispersion

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This paper retrieves stationary (or quiescent) optical solitons to complex Ginzburg–Landau equation that comes with nine forms of nonlinear refractive index structures. The computer software package retrieves the soliton solutions to the reduced ordinary differential equation that emerges from Lie point symmetry. Both, linear temporal evolution as well as generalized temporal evolutions, are accounted for. All solutions that emerge from the software package are implicit.

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1. Introduction

A number of preposterous effects ensue when the chromatic dispersion (CD) is rendered to be nonlinear due to the mishandling and other physical abuses to an optical fiber. One such disastrous consequence is that the solitons stall while propagating for long distances through such a fiber. This leads to the so called stationary (also known as quiescent) solitons. These stationary solitons have been studied for a wide variety of models in the context of nonlinear fiber optics. They are nonlinear Schrödinger's equation (NLSE), Sasa–Satsuma equation, Lakshmanan–Porsezian–Daniel equation and others [1–4, 10, 11]. In this context, the stationary solitons with nonlinear CD for NLSE has been studied with several forms of nonlinearity. They are quadratic–cubic law, a couple Kudryashov's law and others [6, 10–20, 24–28, 30].

The current paper will study stationary solitons that emerge from a different model, namely the complex Ginzburg–Landau equation (CGLE) which is yet another model governing the propel of solitons through an optical fiber. This model has been studied in several different contexts [5, 7–9, 21, 22, 29]. The stationary soliton solutions were also studied in this context [8]. Now, the stationary soliton solutions that can be retrieved from any such nonlinear evolution equation, for nonlinear CD, are either implicit or explicit. The application of extended trial function approach, extended G'/G –expansion, extended Jacobi's elliptic function expansion reveals explicit stationary soliton solutions [15–19, 23–26]. However, the direct application of the software package gives implicit soliton solutions [1–4, 10, 11]. For CGLE, the explicit stationary soliton solutions have already been reported [26]. The current work will address CGLE with nonlinear CD with a direct access to the software package. Both, linear temporal evolution and generalized temporal evolution effects are studied. There are nine forms of nonlinear refractive index structures that are handled. The results are detailed and exhibited in the rest of the paper.

1.1. Governing model

The governing model is the unperturbed version of CGLE that comes with nonlinear CD and nine forms of nonlinear refractive index. The phase–amplitude decomposition of the solution hypothesis would lead to an ordinary differential equation (ODE) that would have a translational Lie symmetry. The reduced ODE would then be integrated directly using the software package for all nine nonlinear forms with linear as well as with generalized temporal evolutions. The detailed analysis along with the soliton solution methodology are all presented in subsequent sections.

2. Linear temporal evolution

The dimensionless form of CGLE with nonlinear CD is given by

$$iq_t + a(|q|^n q)_{xx} + G(|q|^2)q = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \quad (1)$$

In equation (1), the independent variables are x and t that represents the spatial and temporal co-ordinates respectively. The first term is the linear temporal evolution of solitons with its coefficient being $i = \sqrt{-1}$. The coefficient of a is the nonlinear CD. When $n = 0$, it would lead to linear CD in which case all soliton solutions are mobile [5, 7, 21, 29]. For non–zero n, the stationary soliton solutions that emerge are implicit, if a software package is implemented and explicit if an integration algorithm, such as extended trial function approach, is applied [26]. The functional G gives the generalized form of the intensity–dependent nonlinear refractive index. From the right hand side, α and β are general forms of nonlinearities while γ accounts for detuning effect. All of the coefficients a, α, β and γ are real–valued.

2.1. Kerr law

For Kerr law

$$G(|q|^2) = b|q|^2, \quad (2)$$

so that (1) transforms to:

$$iq_t + a(|q|^n q)_{xx} + b|q|^2 q = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \quad (3)$$

Here in (3), b is a real–valued constant. To look for stationary solitons, the starting substitution would be

$$q(x,t) = \phi(x)\mathrm{e}^{i\lambda t}. \quad (4)$$

After inserting the hypothesis (4) into (3) gives

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' - \alpha\phi^2(\phi')^2 + an(n+1)\phi^{n+2}(\phi')^2 + b\phi^6 - (\gamma+\lambda)\phi^4 = 0. \quad (5)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of Gauss' hypergeometric function

$$x = \pm \int \frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (6)$$

where

$$P = {}_2F_1(B_1, B_3; B_4; B_6)A_2 - {}_2F_1(B_2, B_3; B_5; B_6)A_3, \quad (7)$$

and

$$A_1 = \frac{\phi^4}{a(n+1)\phi^{n+2} - \beta\phi^2} \times \left\{\frac{a(n+1)\phi^n - \beta}{a(n+1)\phi^n}\right\}^{\frac{2\alpha-\beta n}{\beta n}},$$

$$A_2 = \frac{\gamma+\lambda}{n+2}, A_3 = \frac{b\phi^2}{n+4},$$

$$B_1 = -\frac{n+2}{n}, B_2 = -\frac{n+4}{n},$$

$$B_3 = \frac{2\alpha - \beta n}{\beta n}, B_4 = -\frac{2}{n},$$

$$B_5 = -\frac{4}{n},\ B_6 = \frac{\beta}{a(n+1)\phi^n}. \quad (8)$$

and Gauss' hypergeometric function in its generalized form is:

$${}_pF_q(a_1, \dots a_p; b_1, \dots b_q; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k \cdots (a_p)_k}{(b_1)_k \cdots (b_q)_k}\frac{z^k}{k!}, \quad (9)$$

with the Pochhammer symbol being

$$(p)_n = \begin{cases} 1 & n = 0, \\ p(p+1)\cdots(p+n-1) & n > 0. \end{cases} \quad (10)$$

The solution given by (6) will remain valid for

$$PA_1 > 0. \quad (11)$$

2.2. Power law

For power law

$$G(|q|^2) = b|q|^{2m}, \quad (12)$$

so that (1) changes to:

$$iq_t + a(|q|^n q)_{xx} + b|q|^{2m}q = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \quad (13)$$

Upon substituting the hypothesis (4) into (13) leads to

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' - \alpha\phi^2(\phi')^2 + an(n+1)\phi^{n+2}(\phi')^2 + b\phi^{2m+4} - (\gamma+\lambda)\phi^4 = 0. \quad (14)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and ignoring the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm \int \frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (15)$$

where

$$P = {}_2F_1(B_1, B_2; B_3; B_4)A_2 - {}_2F_1\begin{pmatrix} B_2, -1+(1+m)B_3; \\ (1+m)B_3; B_4 \end{pmatrix} A_3, \quad (16)$$

and

$$A_1 = \frac{\phi^4}{a(n+1)\phi^{n+2} - \beta\phi^2} \times \left\{\frac{a(n+1)\phi^n - \beta}{a(n+1)\phi^n}\right\}^{\frac{2\alpha-\beta n}{\beta n}},$$

$$A_2 = \frac{\gamma+\lambda}{n+2}, A_3 = \frac{b\phi^{2m}}{2(m+1)+n},$$

$$B_1 = -\frac{n+2}{n}, B_2 = \frac{2\alpha - \beta n}{\beta n},$$

$$B_3 = -\frac{2}{n}, B_4 = \frac{\beta}{a(n+1)\phi^n}. \quad (17)$$

The solution given by (15) will remain valid for

$$PA_1 > 0. \quad (18)$$

2.3. Parabolic (Cubic–quintic) law

For this law

$$G(|q|^2) = k_1|q|^2 + k_2|q|^4, \quad (19)$$

so that (1) reduces to:

$$iq_t + a(|q|^n q)_{xx} + \begin{pmatrix} k_1|q|^2 \\ +k_2|q|^4 \end{pmatrix} q = \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \quad (20)$$

Then substituting the hypothesis (4) into (20) yields

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' - \alpha\phi^2(\phi')^2 + an(n+1)\phi^{n+2}(\phi')^2 + k_2\phi^8 + k_1\phi^6 - (\gamma+\lambda)\phi^4 = 0. \quad (21)$$

In (19)–(21), the constants k_1 and k_2 are real–valued. The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm \int \frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (22)$$

where

$$P = {}_2F_1(B_1, B_2; B_3; B_4)A_2 - {}_2F_1(B_2, B_5; B_6; B_4)A_3 - {}_2F_1(B_2, B_7; B_8; B_4)A_4, \quad (23)$$

and

$$A_1 = \frac{a(n+1)\phi^{n+2}}{(\beta - a(n+1)\phi^n)^2} \times \left\{\frac{a(n+1)\phi^n - \beta}{a(n+1)\phi^n}\right\}^{\frac{2\alpha}{\beta n}},$$

$$A_2 = \frac{\gamma+\lambda}{n+2}, A_3 = \frac{k_1\phi^2}{n+4}, B_6 = -\frac{4}{n},$$

$$A_4 = \frac{k_2\phi^4}{n+6}, B_1 = -\frac{n+2}{n},$$

$$B_2 = \frac{2\alpha - \beta n}{\beta n}, B_3 = -\frac{2}{n},$$

$$B_4 = \frac{\beta}{a(n+1)\phi^n}, B_5 = -\frac{n+4}{n},$$

$$B_7 = -\frac{n+6}{n}, B_8 = -\frac{6}{n}. \quad (24)$$

The solution given by (22) will remain valid for

$$PA_1 > 0. \quad (25)$$

2.4. Dual–power law

For dual-power law

$$G(|q|^2) = k_1|q|^{2m} + k_2|q|^{4m}, \tag{26}$$

so that (1) shapes up

$$iq_t + a(|q|^n q)_{xx} + \begin{pmatrix} k_1|q|^{2m} \\ +k_2|q|^{4m} \end{pmatrix} q = \alpha \frac{|q_x|^2}{q^*}$$

$$+\frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \tag{27}$$

After plugging (4) into (27) gives rise to

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' - \alpha\phi^2(\phi')^2$$

$$+an(n+1)\phi^{n+2}(\phi')^2$$

$$+k_2\phi^{4m+4} + k_1\phi^{2m+4} - (\gamma+\lambda)\phi^4 = 0. \tag{28}$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm\int \frac{d\phi}{\sqrt{2PA_1}}, \tag{29}$$

where

$$P = {}_2F_1(1, B_2; B_3; B_4)A_2$$

$$-{}_2F_1(1, B_5; (1+m)B_3; B_4)A_3$$

$$+\Gamma(B_1)\,{}_2\tilde{F}_1(1, B_6; (1+2m)B_3; B_4)A_4, \tag{30}$$

and

$$A_1 = \frac{\phi^{2-n}}{a(n+1)}, A_2 = \frac{\gamma+\lambda}{n+2},$$

$$A_3 = \frac{k_1\phi^{2m}}{2m+n+2}, A_4 = \frac{k_2\phi^{4m}}{n},$$

$$B_1 = -\frac{4m+n+2}{n},$$

$$B_2 = \frac{\beta(n-2)-2\alpha}{\beta n},$$

$$B_3 = -\frac{2}{n}, B_4 = \frac{\beta}{a(n+1)\phi^n},$$

$$B_5 = \frac{\beta(n-2m-2)-2\alpha}{\beta n},$$

$$B_6 = \frac{\beta(n-4m-2)-2\alpha}{\beta n}. \tag{31}$$

The solution given by (29) will remain valid for

$$PA_1 > 0. \tag{32}$$

2.5. Log law

For log law

$$G(|q|^2) = b\ln|q|^2, \tag{33}$$

so that (1) transforms to:

$$iq_t + a(|q|^n q)_{xx} + bq\ln|q|^2 = \alpha\frac{|q_x|^2}{q^*}$$

$$+\frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \tag{34}$$

Then inserting (4) into (34) leads to

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' - \alpha\phi^2(\phi')^2$$

$$+an(n+1)\phi^{n+2}(\phi')^2$$

$$+2b\phi^4\ln\phi - (\gamma+\lambda)\phi^4 = 0. \tag{35}$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm\int \frac{d\phi}{\sqrt{2}\sqrt{(X_3+\ln(\phi)X_4X_5)X_6 - X_1\left(\frac{\gamma X_2 + l\lambda X_2}{2l+n}\right)}}, \tag{36}$$

where

$$X_1 = \phi^{\frac{4\beta n - 2\alpha(n+2)}{\beta n}}$$

$$\times\left(\beta\phi^2 - a(n+1)\phi^{n+2}\right)^{\frac{2(\alpha-\beta n)}{\beta n}},$$

$$X_2 = P_2\phi^{\frac{2\alpha(n+2)}{\beta n}}\left(\frac{a(n+1)\phi^n - \beta}{a(n+1)\phi^n}\right)^{\frac{2\alpha-\beta n}{\beta n}}$$

$$\times\left(\beta\phi^2 - a(n+1)\phi^{n+2}\right)^{\frac{\beta n - 2\alpha}{\beta n}}$$

$$-P_1(\beta - a(n+1))^{\frac{\beta n - 2\alpha}{\beta n}}$$

$$\times\left(\frac{a(n+1)-\beta}{a(n+1)}\right)^{\frac{2\alpha-\beta n}{\beta n}},$$

$$X_3 = a(n+1)P_3\phi^{n+2}(\beta - a(n+1))$$

$$\times\left(\frac{a(n+1)-\beta}{a(n+1)}\right)^{\frac{2\alpha-\beta n}{\beta n}}$$

$$\times(\beta\phi^2-a(n+1)\phi^{n+2})^{\frac{2\alpha}{\beta n}}$$

$$-a(n+1)P_4(\beta-a(n+1))^{\frac{2\alpha}{\beta n}}$$

$$\times(\beta\phi^2-a(n+1)\phi^{n+2})\phi^{\frac{(n+2)(2\alpha+\beta n)}{\beta n}}$$

$$\times\left(\frac{a(n+1)\phi^n-\beta}{a(n+1)\phi^n}\right)^{\frac{2\alpha-\beta n}{\beta n}},$$

$$X_4=(n+2)P_5\phi^{-\frac{2(n+4)}{n}}(\beta-a(n+1))^{\frac{2\alpha}{\beta n}}$$

$$\times(\beta\phi^2-a(n+1)\phi^{n+2}),$$

$$X_5=a(n+1)\phi^{\frac{(n+2)(2\alpha+\beta(n+2))+4\beta}{\beta n}}$$

$$-\beta\phi^{\frac{2(n+2)(\alpha+2\beta)}{\beta n}},$$

$$X_6=2b\begin{Bmatrix}a(n+1)(n+2)^2\phi^{n+2}\\ \times(\beta-a(n+1))^{\frac{2\alpha}{\beta n}}\\ \times(\beta\phi^2-a(n+1)\phi^{n+2})^{\frac{2\alpha}{\beta n}}\end{Bmatrix}^{-1}, \quad (37)$$

and

$$P_1={}_2F_1(B_1,B_2;B_3;B_4),$$

$$P_2={}_2F_1(B_1,B_2;B_3;\phi^{-n}B_4),$$

$$P_3={}_3F_2\begin{pmatrix}B_3-1,B_3-1,B_5;\\ B_3,B_3;\dfrac{\beta}{a(n+1)}\end{pmatrix},$$

$$P_4={}_3F_2\begin{pmatrix}B_3-1,B_3-1,B_5;\\ B_3,B_3;B_7\end{pmatrix},$$

$$P_5={}_2F_1(1,B_6;B_3;B_7), \quad (38)$$

and also

$$B_1=-\frac{n+2}{n},B_2=\frac{2\alpha-\beta n}{\beta n},$$

$$B_3=-\frac{2}{n},B_4=\frac{\beta}{an+a},B_7=\frac{\beta}{a(n+1)\phi^n},$$

$$B_5=\frac{2\alpha-\beta n}{\beta n},B_6=\frac{\beta(n-2)-2\alpha}{\beta n}. \quad (39)$$

The solution given by (36) will remain valid for

$$(X_3+\ln(\phi)X_4X_5)X_6-X_1\left(\frac{\gamma X_2+l\lambda X_2}{2l+n}\right)>0. \quad (40)$$

2.6. Power law

For this law

$$G(|q|^2)=\frac{k_3}{|q|^4}+k_1|q|^2+k_2|q|^4, \quad (41)$$

so that (1) turns into:

$$iq_t+a(|q|^nq)_{xx}+\begin{pmatrix}\dfrac{k_3}{|q|^4}+k_1|q|^2\\ +k_2|q|^4\end{pmatrix}q$$

$$=\alpha\frac{|q_x|^2}{q^*}+\frac{\beta}{4|q|^2q^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\ -\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q. \quad (42)$$

And then putting the hypothesis (4) into (42) yields

$$a(n+1)\phi^{n+3}\phi''-\beta\phi^3\phi''+an(n+1)\phi^{n+2}(\phi')^2$$

$$-\alpha\phi^2(\phi')^2+k_2\phi^8+k_1\phi^6-(\gamma+\lambda)\phi^4+k_3=0. \quad (43)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2PA_1}}, \quad (44)$$

where

$$P={}_2F_1(B_1,B_2;B_3;B_4)A_2$$

$$-{}_2F_1(B_2,B_5;B_6;B_4)A_3$$

$$-{}_2F_1(B_2,B_7;B_8;B_4)A_4$$

$$-{}_2F_1(B_2,B_9;B_{10};B_4)A_5, \quad (45)$$

and

$$A_1=\frac{a(n+1)\phi^{n-2}}{\{\beta-a(n+1)\phi^n\}^2}$$

$$\times\left\{\frac{a(n+1)\phi^n-\beta}{a(n+1)\phi^n}\right\}^{\frac{2\alpha}{\beta n}},$$

$$A_2=\frac{\phi^4(\gamma+\lambda)}{n+2},A_3=\frac{k_1\phi^6}{n+4},$$

$$A_4=\frac{k_2\phi^8}{n+6},A_5=\frac{k_3}{n-2},$$

$$B_1=-\frac{n+2}{n},B_2=\frac{2\alpha-\beta n}{\beta n},$$

$$B_3 = -\frac{2}{n}, B_4 = \frac{\beta}{a(n+1))\phi^n},$$

$$B_5 = -\frac{n+4}{n}, B_6 = -\frac{4}{n}, B_7 = -\frac{n+6}{n},$$

$$B_8 = -\frac{6}{n}, B_9 = \frac{2-n}{n}, B_{10} = \frac{2}{n}. \quad (46)$$

The solution given by (44) will remain valid for

$$PA_1 > 0. \quad (47)$$

2.7. Generalized anti–cubic law

For this law

$$G(|q|^2) = \frac{k_3}{|q|^{2(m+1)}} + k_1|q|^{2m} + k_2|q|^{2(m+1)}, \quad (48)$$

so that (1) becomes

$$iq_t + a(|q|^n q)_{xx} + \begin{Bmatrix} \frac{k_3}{|q|^{2(m+1)}} + k_1|q|^{2m} \\ +k_2|q|^{2(m+1)} \end{Bmatrix} q$$

$$= \alpha \frac{|q_x|^2}{q^*} + \frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \quad (49)$$

Plugging the hypothesis (4) into (49) brings about

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' + an(n+1)\phi^{n+2}(\phi')^2$$

$$-\alpha\phi^2(\phi')^2 + k_2\phi^{2m+6} + k_1\phi^{2m+4}$$

$$+k_3\phi^{2-2m} - (\gamma+\lambda)\phi^4 = 0. \quad (50)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm \int \frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (51)$$

where

$$PA_1 > 0,$$

$$P = P_1 A_2 - b(\phi^{2+4m}(P_2 A_3 + P_3 A_4) + P_4 A_5),$$

$$P_1 = {}_2F_1(B_1, B_2; B_3; B_4),$$

$$P_2 = {}_2F_1(B_2, (m+1)B_3 - 1; (m+1)B_3; B_4),$$
$$P_3 = {}_2F_1(B_2, (m+2)B_3 - 1; (m+2)B_3; B_4),$$

$$P_4 = {}_2F_1(B_2, B_5; B_6; B_4), \quad (52)$$

and

$$A_1 = \frac{a(n+1)\phi^{n-2m}}{\{\beta - a(n+1)\phi^n\}^2}$$

$$\times\left\{\frac{a(n+1)\phi^n - \beta}{a(n+1)\phi^n}\right\}^{\frac{2\alpha}{\beta n}},$$

$$A_2 = \frac{(\gamma+\lambda)\phi^{2m+2}}{n+2}, A_3 = \frac{k_1}{2m+n+2},$$
$$A_4 = \frac{k_2\phi^2}{2m+n+4}, A_5 = -\frac{k_3}{2m-n},$$

$$B_1 = -\frac{n+2}{n}, B_2 = \frac{2\alpha - \beta n}{\beta n},$$

$$B_3 = -\frac{2}{n}, B_4 = \frac{\beta}{a(n+1)\phi^n},$$

$$B_5 = \frac{2m-n}{n},\ B_6 = \frac{2m}{n}. \quad (53)$$

The solution given by (51) will remain valid for

$$PA_1 > 0. \quad (54)$$

2.8. Quadratic–cubic law

For this law

$$G(|q|^2) = k_1|q| + k_2|q|^2, \quad (55)$$

so that (1) changes to:

$$iq_t + a(|q|^n q)_{xx} + \begin{pmatrix} k_1|q| \\ +k_2|q|^2 \end{pmatrix} q = \alpha\frac{|q_x|^2}{q^*}$$

$$+\frac{\beta}{4|q|^2 q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q. \quad (56)$$

Then, inserting (4) into (56), one recovers

$$a(n+1)\phi^{n+3}\phi'' - \beta\phi^3\phi'' + an(n+1)\phi^{n+2}(\phi')^2$$

$$-\alpha\phi^2(\phi')^2 + k_2\phi^6 + k_1\phi^5 - (\gamma+\lambda)\phi^4 = 0. \quad (57)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm \int \frac{d\phi}{\sqrt{2A_1 P}}, \quad (58)$$

where

$$P = -\,{}_2F_1(B_1, B_2; B_3; B_4)A_2$$

$$+b\phi\begin{pmatrix} {}_2F_1(B_2,B_5;B_6;B_4)A_3 \\ +\,{}_2F_1(B_2,B_7;B_8;B_4)A_4 \end{pmatrix}, \quad (59)$$

and

$$A_1=-\frac{a(n+1)\phi^{n+2}\left(1-\frac{\beta\phi^{-n}}{an+a}\right)^{\frac{2\alpha}{\beta n}}}{\{\beta-a(n+1)\phi^n\}^2},$$

$$A_2=\frac{\gamma+\lambda}{n+2}, A_3=\frac{k_1}{n+3}, A_4=\frac{k_2\phi}{n+4},$$

$$B_1=-\frac{n+2}{n}, B_2=\frac{2\alpha}{\beta n}-1, B_3=-\frac{2}{n},$$

$$B_4=\frac{\beta\phi^{-n}}{an+a}, B_5=-\frac{n+3}{n}, B_6=-\frac{3}{n},$$

$$B_7=-\frac{n+4}{n}, B_8=-\frac{4}{n}. \quad (60)$$

The solution given by (58) will remain valid for

$$A_1P>0. \quad (61)$$

2.9. Quadratic–cubic law

For this law

$$G(|q|^2)=k_1|q|^2+k_2|q|^4+k_3(|q|^2)_{xx}, \quad (62)$$

so that (1) turns into:

$$iq_t+a(|q|^nq)_{xx}+\begin{Bmatrix} k_1|q|^2+k_2|q|^4 \\ +k_3(|q|^2)_{xx} \end{Bmatrix}q$$

$$=\alpha\frac{|q_x|^2}{q^*}+\frac{\beta}{4|q|^2q^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix}+\gamma q. \quad (63)$$

Once substituting the hypothesis (4) into (63) leads to

$$a(n+1)\phi^{n+3}\phi''+2k_3\phi^5\phi''-\beta\phi^3\phi''$$

$$+an(n+1)\phi^{n+2}(\phi')^2+2k_3\phi^4(\phi')^2-\alpha\phi^2(\phi')^2$$

$$+k_2\phi^8+k_1\phi^6-(\gamma+\lambda)\phi^4=0. \quad (64)$$

In (62)-(64), the constants k_l for $l=1,2,3$ are all real–valued. The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\exp\left(\int_1^\phi Bd\xi\right)\sqrt{\int_1^\phi A\exp\left(-2\int_1^\tau Bd\xi\right)\tau^3d\tau}}, \quad (65)$$

where

$$A=\frac{-b\tau^2(k_2\tau^2+k_1)+\gamma+\lambda}{a(n+1)\tau^{n+2}+2k_3\tau^4-\beta\tau^2},$$

$$B=\frac{\xi^2(\alpha-2k_3\xi^2)-an(n+1)\xi^{n+2}}{a(n+1)\xi^{n+3}+2k_3\xi^5-\beta\xi^3}. \quad (66)$$

The solution given by (65) will remain valid for

$$\int_1^\phi A\exp\left(-2\int_1^\tau Bd\xi\right)\tau^3d\tau>0. \quad (67)$$

3. Generalized temporal evolution

In this case, the governing model is structured as follows:

$$i(q^l)_t+a(|q|^nq^l)_{xx}+G(|q|^2)q^l=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix}+\gamma q^l. \quad (68)$$

3.1. Kerr law

In the case of Kerr law, the model becomes

$$i(q^l)_t+a(|q|^nq^l)_{xx}+|q|^2q^l=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix}+\gamma q^l, \quad (69)$$

and then employing the hypothesis (4), one obtains

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''-\alpha\phi^2(\phi')^2$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$

$$+b\phi^{2l+4}-(\gamma+l\lambda)\phi^{2l+2}=0. \quad (70)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (71)$$

where

$$P={}_2F_1(B_1,B_3;B_4;B_6)A_2$$

$$-\,{}_2F_1(B_2,B_3;B_5;B_6)A_3, \quad (72)$$

and

$$A_1=\frac{\phi^{2l+2}}{a(l+n)\phi^{2l+n}-\beta\phi^2}$$

$$\times\left\{1-\frac{\beta\phi^{-2l-n+2}}{a(l+n)}\right\}^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$A_2=\frac{\gamma+\lambda l}{2l+n}, A_3=\frac{b\phi^2}{2l+n+2},$$

$$B_1=-\frac{2l+n}{2l+n-2}, B_2=-\frac{2l+n+2}{2l+n-2},$$

$$B_3=\frac{2\alpha-\beta n}{\beta(2l+n-2)}, B_4=-\frac{2}{2l+n-2},$$

$$B_5=-\frac{4}{2l+n-2}, B_6=\frac{\beta\phi^{-2l-n+2}}{a(l+n)}. \quad (73)$$

The solution given by (71) will remain valid for

$$PA_1>0. \quad (74)$$

3.2. Power law

In the case of power law, the model reads as

$$i(q^l)_t+a(|q|^n q^l)_{xx}+|q|^{2m}q^l=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \quad (75)$$

and then using the hypothesis (4), one gets

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$

$$-\alpha\phi^2(\phi')^2+b\phi^{2l+2m+2}-(\gamma+l\lambda)\phi^{2l+2}=0. \quad (76)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (77)$$

where

$$P={}_2F_1(B_1,B_2;B_3;B_4)A_2$$

$$-{}_2F_1(B_2,-1+(1+m)B_3;(1+m)B_3;B_4)A_3, \quad (78)$$

and

$$A_1=\frac{\phi^{2l+2}}{a(l+n)\phi^{2l+n}-\beta\phi^2}$$

$$\times\left\{1-\frac{\beta\phi^{-2l-n+2}}{a(l+n)}\right\}^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$A_2=\frac{\gamma+\lambda l}{2l+n}, A_3=\frac{b\phi^{2m}}{2(l+m)+n},$$

$$B_1=-\frac{2l+n}{2l+n-2}, B_2=\frac{2\alpha-\beta n}{\beta(2l+n-2)},$$

$$B_3=-\frac{2}{2l+n-2}, B_4=\frac{\beta\phi^{-2l-n+2}}{a(l+n)}. \quad (79)$$

The solution given by (77) will remain valid for

$$PA_1>0. \quad (80)$$

3.3. Parabolic (Cubic–quintic) law

In the case of this law, the model shapes up

$$i(q^l)_t+a(|q|^n q^l)_{xx}+\begin{pmatrix}k_1|q|^2\\+k_2|q|^4\end{pmatrix}q^l=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \quad (81)$$

and then employing the hypothesis (4), one recovers

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''-\alpha\phi^2(\phi')^2$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$

$$+k_2\phi^{2l+6}+k_1\phi^{2l+4}-(\gamma+l\lambda)\phi^{2l+2}=0. \quad (82)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (83)$$

where

$$P={}_2F_1(B_1,B_2;B_3;B_4)A_2$$

$$-{}_2F_1(B_2,B_5;B_6;B_4)A_3$$

$$-{}_2F_1(B_2,B_7;B_8;B_4)A_4, \quad (84)$$

and

$$A_1=\frac{\phi^{2l+2}}{a(l+n)\phi^{2l+n}-\beta\phi^2}$$

$$\times\left\{1-\frac{\beta\phi^{-2l-n+2}}{a(l+n)}\right\}^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$A_2=\frac{\gamma+\lambda l}{2l+n}, A_3=\frac{k_1\phi^2}{2l+n+2},$$

$$A_4=\frac{k_2\phi^4}{2l+n+4}, B_1=-\frac{2l+n}{2l+n-2},$$

$$B_2=\frac{2\alpha-\beta n}{\beta(2l+n-2)}, B_3=-\frac{2}{2l+n-2},$$

$$B_4=\frac{\beta\phi^{-2l-n+2}}{a(l+n)}, B_5=-\frac{2l+n+2}{2l+n-2},$$

$$B_6=-\frac{4}{2l+n-2}, B_7=-\frac{2l+n+4}{2l+n-2},$$

$$B_8=-\frac{6}{2l+n-2}. \tag{85}$$

The solution given by (83) will remain valid for

$$PA_1>0. \tag{86}$$

3.4. Dual–power law

In the case of the law, the model becomes

$$i(q^l)_t+a(|q|^n q^l)_{xx}+\begin{pmatrix}k_1|q|^{2m}\\+k_2|q|^{4m}\end{pmatrix}q^l=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \tag{87}$$

and then by the hypothesis (4), one has

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''-\alpha\phi^2(\phi')^2$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$

$$+k_2\phi^{2l+4m+2}+k_1\phi^{2l+2m+2}-(\gamma+l\lambda)\phi^{2l+2}=0. \tag{88}$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2A_1P}}, \tag{89}$$

where

$$P={}_2F_1(1,B_2;B_3;B_4)A_2$$

$$-{}_2F_1(1,B_5;(1+m)B_3;B_4)A_3$$

$$+\Gamma(B_1)\,{}_2\tilde{F}_1(1,B_6;(1+2m)B_3;B_4)A_4, \tag{90}$$

and

$$A_1=\frac{\phi^{2-n}}{a(l+n)}, A_2=\frac{\gamma+\lambda l}{2l+n},$$

$$A_3=\frac{k_1\phi^{2m}}{2l+2m+n}, A_4=\frac{k_2\phi^{4m}}{2l+n-2},$$

$$B_1=-\frac{2l+4m+n}{2l+n-2}, B_2=\frac{\beta(n-2)-2\alpha}{\beta(2l+n-2)},$$

$$B_3=-\frac{2}{2l+n-2}, B_4=\frac{\beta\phi^{-2l-n+2}}{a(l+n)},$$

$$B_5=\frac{\beta(-2m+n-2)-2\alpha}{\beta(2l+n-2)},$$

$$B_6=\frac{\beta(-4m+n-2)-2\alpha}{\beta(2l+n-2)}. \tag{91}$$

The solution given by (89) will remain valid for

$$A_1P>0. \tag{92}$$

3.5. Log law

In the case of log law, the model becomes

$$i(q^l)_t+a(|q|^n q^l)_{xx}+bq^l\ln|q|^2=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \tag{93}$$

and then utilizing the hypothesis given by (4), one achieves

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''-\alpha\phi^2(\phi')^2$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$

$$+2b\phi^{2l+2}\ln\phi-(\gamma+l\lambda)\phi^{2l+2}=0. \tag{94}$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\sqrt{X_1\left\{(X_3+\ln(\phi)X_4X_5)X_6-\frac{\gamma X_2}{2l+n}-\frac{l\lambda X_2}{2l+n}\right\}}}, \tag{95}$$

where

$$X_1=\phi^{\frac{4\beta(l+n-1)-2\alpha(2l+n)}{\beta(2l+n-2)}}$$

$$\times(\beta\phi^2-a(l+n)\phi^{2l+n})^{\frac{2(\alpha-\beta(l+n-1))}{\beta(2l+n-2)}},$$

$$X_2 = P_2 \phi^{\frac{2(2l+n)(\alpha+\beta(l-1))}{\beta(2l+n-2)}} \left(1 - \frac{\beta\phi^{-2l-n+2}}{al+an}\right)^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}}$$

$$\times (\beta\phi^2 - a(l+n)\phi^{2l+n})^{\frac{\beta n-2\alpha}{\beta(2l+n-2)}}$$

$$-P_1(\beta - a(l+n))^{\frac{\beta n-2\alpha}{\beta(2l+n-2)}} \left(1 - \frac{\beta}{al+an}\right)^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$X_3 = aP_3(l+n)\phi^{2l+n}(\beta - a(l+n))^{\frac{n}{2l+n-2}}$$

$$\times \left(1 - \frac{\beta}{al+an}\right)^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}}$$

$$\times (\beta\phi^2 - a(l+n)\phi^{2l+n})^{\frac{2\alpha}{\beta(2l+n-2)}}$$

$$-aP_4(l+n)(\beta - a(l+n))^{\frac{2\alpha}{\beta(2l+n-2)}}$$

$$\times (\beta\phi^2 - a(l+n)\phi^{2l+n})^{\frac{n}{2l+n-2}} \phi^{\frac{(2l+n)(2\alpha+\beta(4l+n-4))}{\beta(2l+n-2)}}$$

$$\times \left(1 - \frac{\beta\phi^{-2l-n+2}}{al+an}\right)^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$X_4 = P_5(2l+n)\phi^{-\frac{2(2l+n+2)}{2l+n-2}}$$

$$\times (\beta - a(l+n))^{\frac{2\alpha}{\beta(2l+n-2)}}$$

$$\times (\beta\phi^2 - a(l+n)\phi^{2l+n})^{\frac{n}{2l+n-2}},$$

$$X_5 = a(l+n)\phi^{\frac{\frac{(2l+n)(2\alpha+\beta(4l+n-2))}{\beta}+4}{2l+n-2}}$$

$$-\beta\phi^{\frac{2(2l+n)(\alpha+\beta+\beta l)}{\beta(2l+n-2)}},$$

$$X_6 = \frac{\begin{pmatrix} 2b\phi^{-2l-n}(\beta-a(l+n))^{-\frac{2\alpha}{\beta(2l+n-2)}} \\ \times(\beta\phi^2 - a(l+n)\phi^{2l+n})^{-\frac{2\alpha}{\beta(2l+n-2)}} \end{pmatrix}}{a(l+n)(2l+n)^2}, \tag{96}$$

and

$$P_1 = {}_2F_1(B_1, B_2; B_3; B_4),$$

$$P_2 = {}_2F_1(B_1, B_2; B_3; \phi^{-2l-n+2}B_4),$$

$$P_3 = {}_3F_2\begin{pmatrix} lB_3 - \frac{n}{2l+n-2}, \\ lB_3 - \frac{n}{2l+n-2}, \\ B_5; B_3, B_3; \frac{\beta}{a(l+n)} \end{pmatrix},$$

$$P_4 = {}_3F_2\begin{pmatrix} lB_3 - \frac{n}{2l+n-2}, \\ lB_3 - \frac{n}{2l+n-2}, \\ B_5; B_3, B_3; B_7 \end{pmatrix},$$

$$P_5 = {}_2F_1(1, B_6; B_3; B_7), \tag{97}$$

and also

$$B_1 = -\frac{2l+n}{2l+n-2}, B_2 = \frac{2\alpha - \beta n}{\beta(2l+n-2)},$$

$$B_3 = -\frac{2}{2l+n-2}, B_4 = \frac{\beta}{a(l+n)},$$

$$B_5 = \frac{2\alpha - \beta n}{\beta(2l+n-2)}, B_6 = \frac{\beta(n-2) - 2\alpha}{\beta(2l+n-2)},$$

$$B_7 = \frac{\beta\phi^{-2l-n+2}}{a(l+n)}. \tag{98}$$

The solution given by (95) will remain valid for

$$X_1\left\{(X_3 + \ln(\phi)X_4X_5)X_6 - \frac{\gamma X_2}{2l+n} - \frac{l\lambda X_2}{2l+n}\right\} > 0. \tag{99}$$

3.6. Anti–cubic law

In the case of anti–cubic law, the model reads as

$$i(q^l)_t + a(|q|^n q^l)_{xx} + \begin{pmatrix} \frac{k_3}{|q|^4} + k_1|q|^2 \\ +k_2|q|^4 \end{pmatrix} q^l = \alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+ \frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix} 2|q|^2(|q|^2)_{xx} \\ -\{(|q|^2)_x\}^2 \end{bmatrix} + \gamma q^l, \tag{100}$$

and then from the hypothesis (4), one reveals

$$a(l+n)\phi^{2l+n+1}\phi'' - \beta\phi^3\phi'' - \alpha\phi^2(\phi')^2$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2 + k_2\phi^{2l+6}$$

$$+k_1\phi^{2l+4} - (\gamma + l\lambda)\phi^{2l+2} + k_3\phi^{2l-2} = 0. \tag{101}$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x = \pm\int \frac{d\phi}{\sqrt{2A_1P}}, \tag{102}$$

where

$$P = {}_2F_1(B_1, B_2; B_3; B_4)A_2$$

$$- {}_2F_1(B_2, B_5; B_6; B_4)A_3$$

$$-\,{}_2F_1(B_2,B_7;B_8;B_4)A_4$$

$$-\,{}_2F_1(B_2,B_9;B_{10};B_4)A_5, \qquad (103)$$

and

$$A_1=\frac{\phi^{2l-2}}{a(l+n)\phi^{2l+n}-\beta\phi^2}$$

$$\times\left\{1-\frac{\beta\phi^{-2l-n+2}}{a(l+n)}\right\}^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$A_2=\frac{\phi^4(\gamma+\lambda l)}{2l+n},A_3=\frac{k_1\phi^6}{2l+n+2},$$

$$A_4=\frac{k_2\phi^8}{2l+n+4},A_5=\frac{k_3}{2l+n-4},$$

$$B_1=-\frac{2l+n}{2l+n-2},B_2=\frac{2\alpha-\beta n}{\beta(2l+n-2)},$$

$$B_3=-\frac{2}{2l+n-2},B_4=\frac{\beta\phi^{-2l-n+2}}{a(l+n)},$$

$$B_5=-\frac{2l+n+2}{2l+n-2},B_6=-\frac{4}{2l+n-2},$$

$$B_7=-\frac{2l+n+4}{2l+n-2},B_8=-\frac{6}{2l+n-2},$$

$$B_9=-\frac{2l+n-4}{2l+n-2},\ B_{10}=\frac{2}{2l+n-2}. \qquad (104)$$

The solution given by (102) will remain valid for

$$A_1P>0. \qquad (105)$$

3.7. Generalized anti–cubic law

In the case of this law, the model is given by

$$i(q^l)_t+a(|q|^nq^l)_{xx}+\left\{\begin{matrix}\frac{k_3}{|q|^{2(m+1)}}\\+k_1|q|^{2m}\\+k_2|q|^{2(m+1)}\end{matrix}\right\}q^l$$

$$=\alpha\frac{|q_x|^2}{(q^l)^*}+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \qquad (106)$$

and then employing the hypothesis (4), one has

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''-\alpha\phi^2(\phi')^2$$

$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2+k_2\phi^{2l+2m+4}$$

$$+k_1\phi^{2l+2m+2}-(\gamma+l\lambda)\phi^{2l+2}+k_3\phi^{2l-2m}=0. \qquad (107)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \qquad (108)$$

where

$$P=P_1A_2-b(\phi^{2+4m}(P_2A_3+P_3A_4)+P_4A_5), \qquad (109)$$

and

$$P_1=\ {}_2F_1(B_1,B_2;B_3;B_4),$$

$$P_2=\ {}_2F_1(B_2,(m+1)B_3-1;(m+1)B_3;B_4),$$

$$P_3=\ {}_2F_1(B_2,(m+2)B_3-1;(m+2)B_3;B_4),$$

$$P_4=\ {}_2F_1(B_2,B_5;B_6;B_4), \qquad (110)$$

and

$$A_1=\frac{\phi^{2(l-m)}}{a(l+n)\phi^{2l+n}-\beta\phi^2}$$

$$\times\left\{1-\frac{\beta\phi^{-2l-n+2}}{a(l+n)}\right\}^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}},$$

$$A_2=\frac{\phi^{2m+2}(\gamma+\lambda l)}{2l+n},A_3=\frac{k_1}{2l+2m+n},$$

$$A_4=\frac{k_2\phi^2}{2l+2m+n+2},A_5=\frac{k_3}{2l-2m+n-2},$$

$$B_1=-\frac{2l+n}{2l+n-2},B_2=\frac{2\alpha-\beta n}{\beta(2l+n-2)},$$

$$B_3=-\frac{2}{2l+n-2},B_4=\frac{\beta\phi^{-2l-n+2}}{a(l+n)},$$

$$B_5=\frac{-2l+2m-n+2}{2l+n-2},B_6=\frac{2m}{2l+n-2}. \qquad (111)$$

The solution given by (108) will remain valid for

$$PA_1>0. \qquad (112)$$

3.8. Quadratic–cubic law

In the case of this law, the model is structured as

$$i(q^l)_t+a(|q|^nq^l)_{xx}+\begin{pmatrix}k_1|q|\\+k_2|q|^2\end{pmatrix}q^l=\alpha\frac{|q_x|^2}{(q^l)^*}$$

$$+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \qquad (113)$$

and then utilizing the hypothesis (4), one obtains

$$a(l+n)\phi^{2l+n+1}\phi''-\beta\phi^3\phi''-\alpha\phi^2(\phi')^2$$
$$+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$
$$+k_2\phi^{2l+4}+k_1\phi^{2l+3}-(\gamma+l\lambda)\phi^{2l+2}=0. \quad (114)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\sqrt{PA_1}}, \quad (115)$$

where

$$P=-{}_2F_1(B_1,B_2;B_3;B_4)A_2$$
$$+b\phi\begin{pmatrix}{}_2F_1(B_2,B_5;B_6;B_4)A_3\\+{}_2F_1(B_2,B_7;B_8;B_4)A_4\end{pmatrix}, \quad (116)$$

and

$$A_1=-\frac{\phi^{2l+2}\left\{1-\frac{\beta\phi^{-2l-n+2}}{a(l+n)}\right\}^{\frac{2\alpha-\beta n}{\beta(2l+n-2)}}}{a(l+n)\phi^{2l+n}-\beta\phi^2},$$

$$A_2=\frac{\gamma+\lambda l}{2l+n}, A_3=\frac{k_1}{2l+n+1},$$

$$A_4=\frac{k_2\phi}{2l+n+2}, B_1=-\frac{2l+n}{2l+n-2},$$

$$B_2=\frac{2\alpha-\beta n}{\beta(2l+n-2)}, B_3=-\frac{2}{2l+n-2},$$

$$B_4=\frac{\beta\phi^{-2l-n+2}}{a(l+n)}, B_5=-\frac{2l+n+1}{2l+n-2},$$

$$B_6=-\frac{3}{2l+n-2}, B_7=-\frac{2l+n+2}{2l+n-2},$$

$$B_8=-\frac{4}{2l+n-2}. \quad (117)$$

The solution given by (115) will remain valid for

$$PA_1>0. \quad (118)$$

3.9. Parabolic–nonlocal law

In the case of parabolic–nonlocal law, the model shapes up

$$i(q^l)_t+a(|q|^n q^l)_{xx}+\begin{Bmatrix}k_1|q|^2+k_2|q|^4\\+k_3(|q|^2)_{xx}\end{Bmatrix}q^l$$

$$=\alpha\frac{|q_x|^2}{(q^l)^*}+\frac{\beta}{4|q|^2(q^l)^*}\begin{bmatrix}2|q|^2(|q|^2)_{xx}\\-\{(|q|^2)_x\}^2\end{bmatrix}+\gamma q^l, \quad (119)$$

and then by virtue of the hypothesis (4), one gets

$$a(l+n)\phi^{2l+n+1}\phi''+2k_3\phi^{2l+3}\phi''$$
$$-\beta\phi^3\phi''+a(l+n)(l+n-1)\phi^{2l+n}(\phi')^2$$
$$+2k_3\phi^{2l+2}(\phi')^2-\alpha\phi^2(\phi')^2$$
$$+bk_2\phi^{2l+6}+k_1\phi^{2l+4}-(\gamma+l\lambda)\phi^{2l+2}=0. \quad (120)$$

The above equation admits a single Lie point symmetry, namely $\frac{\partial}{\partial x}$. This symmetry will be used the integration process. Integrating and discarding the constants of integration, we have the following solution in terms of the hypergeometric function

$$x=\pm\int\frac{d\phi}{\sqrt{2}\exp(\int\phi B d\xi)\sqrt{\int\phi A\exp(-2\int\tau B d\xi)\tau^{1+2l}d\tau}}, \quad (121)$$

where

$$A=\frac{-b\tau^2(k_2\tau^2+k_1)+\gamma+\lambda l}{a(l+n)\tau^{2l+n}+2k_3\tau^{2l+2}-\beta\tau^2},$$

$$B=\frac{\xi^2(\alpha-2k_3\xi^{2l})-a(l+n-1)(l+n)\xi^{2l+n}}{a(l+n)\xi^{2l+n+1}+2k_3\xi^{2l+3}-\beta\xi^3}. \quad (122)$$

The solution given by (121) will remain valid for

$$\int\phi A\exp(-2\int\tau B d\xi)\tau^{1+2l}d\tau>0. \quad (123)$$

4. Conclusions

This paper recovered implicit stationary optical soliton solutions to CGLE with nonlinear CD and having nine forms of nonlinear refractive index change. The temporal evolutions were taken to be both linear as well as generalized. This software yielded overwhelming results that are displayed in this work. The results of the paper serves a warning to the telecommunications community that under no circumstances, should CD be rendered to be nonlinear to avoid the stalling of solitons and Gaussons during its propagation. These results would be later extended to birefringent fibers for CGLE having some of the nonlinearities. The results for birefringent fibers would be extended versions of the current results.

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