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Influence of magnetic shear and stochastic electrostatic field on the electron diffusion

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The electron diffusion induced by a two-dimensional electrostatic turbulence, in a sheared slab approximation of the toroidal magnetic geometry, is studied by direct numerical simulation. The transport properties of the electrons are obtained by numerical simulations assuming an isotropic spectrum of electrostatic drift type turbulence that is Gaussian for small wave-vectors and power-law k^{-3} for large wave-vectors. The 'radial' and the 'poloidal' running and asymptotic diffusion coefficients of electrons are obtained for physically relevant parameter values and the existence of an enhanced diffusion in the poloidal direction is observed in the presence of magnetic shear.

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1. Introduction

The understanding of anomalous transport in tokamak plasmas is still a major problem for the plasma physicists. Thus, if ε is the dimensionless electrostatic turbulence amplitude, the diffusion coefficient in the low frequency limit (wave frequency much smaller than the guiding-center rotation frequency) resulting from the Gaussian theory is: $D(\varepsilon) = \varepsilon^{\alpha}$, with $\alpha = 1$. This problem has been reconsidered in Ref. [1] where, on the basis of the percolation theory, it was found that the exponent should be α = 0.7. Other numerical simulations have given a critical exponent value between 0.7 and 0.8 (see e.g. Ottaviani [2] who obtained 0.8 with a set of 64 waves of random amplitudes and Reuss and Misguich [3] who obtained 0.7 using a turbulence spectrum defined on a limited wavelength range. This last result was recovered by B. Weyssow and M. De Leener (unpublished work). The turbulence spectrum defined in their work, on a larger wavelength range, was used in our paper. This type of diffusion, with non-integer exponent, was also observed in other numerical experiments [4-7]. Very recent works studied the same problems using a semi-analytical method, the decorrelation trajectory method (DCT) (see Refs. [8-12]). The later method can give information about the diffusion even for relatively large turbulence regime, i.e. Kubo numbers that are greater than 1. In our paper, the electron diffusion induced by a two-dimensional electrostatic turbulence (the electrostatic turbulence is related to low frequency and long wavelength drift modes) in a sheared slab approximation of the toroidal magnetic geometry, was studied by direct numerical simulation. The 'radial' and the 'poloidal' running and asymptotic diffusion coefficients of electrons are obtained for physically relevant Kubo numbers and an enhanced diffusion in the poloidal direction is observed in the presence of magnetic shear. The paper is organized as follow. In Section 2 the model equation and the parameters are established. In Section 3 the diffusion coefficient were calculated for a specific electrostatic spectrum by direct numerical simulation. The conclusions are presented in Section 4.

2. The model and the parameters

A two-dimensional sheared-slab system is considered with the main magnetic field $\mathbf{B}_0$ in the Oz -direction and the sheared term in the Oy -direction:

$$\mathbf{B}(X) = B_0\left[\mathbf{e}_z + XL_s^{-1}\mathbf{e}_y\right] \equiv B_0\left[\mathbf{e}_z + s(X)\mathbf{e}_y\right] \qquad (1)$$

The particular choice of the magnetic field (1), implies that all magnetic field gradients ($(\mathbf{b}\cdot\nabla)\mathbf{b}$, ∇B etc.) reduce to zero and that the guiding center position $\mathbf{X} \equiv (X,\ Y,\ Z)$ is given by (see e.g. [13]):

$$\dot{\mathbf{X}} = U\,\mathbf{b} + \frac{c}{B^2}\mathbf{E}\times\mathbf{B} \qquad (2)$$

In Eq. (2) $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the (total) magnetic field; U is the parallel guiding center velocity and $\mathbf{E}$ is the electrostatic field. In order to obtain the equations of motion for the guiding center to the first order in the drift parameter [13] (terms proportional to $s^2(X)$ are neglected) we will use the realistic assumption

$U = const \equiv V_{th}^{el}$, where V_{th}^{el} is the thermal electron velocity. In order to apply the DCT method the following dimensionless quantities $\mathbf{x} \equiv (x,\ y)$, z, τ and φ are defined in terms of the dimensional variables:

$$z = \frac{Z}{\lambda_{\|}}; \quad \tau \equiv \frac{t}{\tau_c}; \quad \mathbf{x} = \frac{\mathbf{X}}{\lambda_{\perp}}; \qquad (3)$$
$$\Phi(\mathbf{X}, Z, t) \rightarrow \varepsilon\varphi(\frac{\mathbf{X}}{\lambda_{\perp}}, \frac{Z}{\lambda_{\|}}, \frac{t}{\tau_c})$$

In Eq. (3) $\lambda_{\perp}, \lambda_{\|}$ are the perpendicular and the parallel correlation lengths, with respect to the main magnetic field, τ_c is the correlation time of the fluctuating electrostatic field. The dimensionless equations that can be analyzed by DCT are then:

$$\frac{dx(\tau)}{d\tau} = -K\frac{\partial\varphi(\mathbf{x}(\tau), \tau)}{\partial y}$$
$$\frac{dy(\tau)}{d\tau} = K_s\, x(\tau) + K\frac{\partial\varphi(\mathbf{x}(\tau), \tau)}{\partial x} \qquad (4)$$

In Eq. (4) the electrostatic Kubo number K and the shear Kubo number K_s are defined as:

$$K = \frac{\varepsilon c \tau_c}{B_0 \lambda_{\perp}^2},\ K_s = \frac{V_{th}^{el}\tau_c}{L_s}. \qquad (5)$$

The definitions from Eq. (5) are specific to the decorrelation trajectory method [12] and will be used in order to compare their values with those specific to the direct numerical simulations.

3. Diffusion coefficients

In our numerical simulations we used the capacity of the Universite Libre de Bruxelles (ULB) central parallel computer that accepts four matrices (4096×4096) for the discretized representation of the electrostatic stochastic field. The spectrum is constructed to be Gaussian for small wave-vectors and k^{-3} spectrum for large wave-vectors:

$$S(k) = \begin{cases} S_1(k) \equiv B\exp\left(-\frac{k^2}{\Delta}\right), & \text{for} \quad k \in [k_m,\ k_{tr}] \\ S_2(k) \equiv k^{-3}, & \text{for} \quad k \in [k_{tr},\ k_M] \end{cases} \qquad (6)$$

The continuity of the spectrum and of its first derivative in $k = k_{tr}$ lead to the following values for B and Δ:

$$B = k_{tr}^{-3}\exp\frac{k_{tr}^2}{\Delta},\ \ \Delta = \frac{2}{3}k_{tr}^2$$

and the extension of the range of the wave-vectors allows us to say that the spectrum is a "quasi-continuously" one [7]. The characteristics of the spectrum, i.e. the correlation length and the decorrelation time, is partly defined according to drift wave turbulence measurements and partly from the parameter used in the decorrelation trajectory method [12].

The spectrum given in Eq. (6) is obtained by building the electrostatic potential $\Phi(\mathbf{X}, t)$ as a sum of discrete plane waves of the form:

$$\Phi(\mathbf{X}, t) = \frac{\tilde{E}}{\sigma_S}\int d\mathbf{k}\, S_k(k)\cos(\mathbf{k}\cdot\mathbf{X} - \omega t + \phi_{\mathbf{k}}) \qquad (7)$$

The choice for the electrostatic stochastic potential field given in Eq. (7) has an explanation: if more than two waves are considered (such as in our case) the solutions of the equations of motion are chaotic and this provides a mechanism for anomalous diffusion (for large scale chaos). The quantity σ_S is defined as:

$$\sigma_S^2 = \frac{1}{2}\left[\int_{k_m}^{k_{tr}} dk S_1^2(k) k^2 + \int_{k_{tr}}^{k_M} dk S_2^2(k) k^2\right]$$

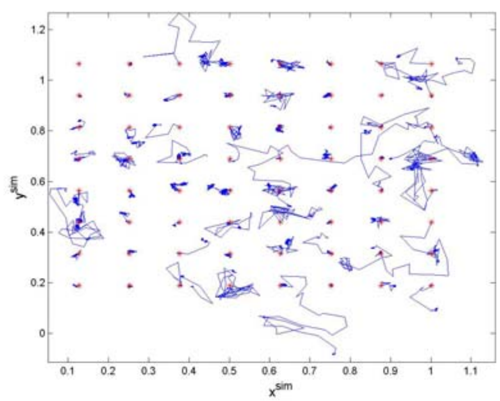

Fig. 1. A box with 64 trajectories for the shearless case and $K^{sim} = 2.5$.

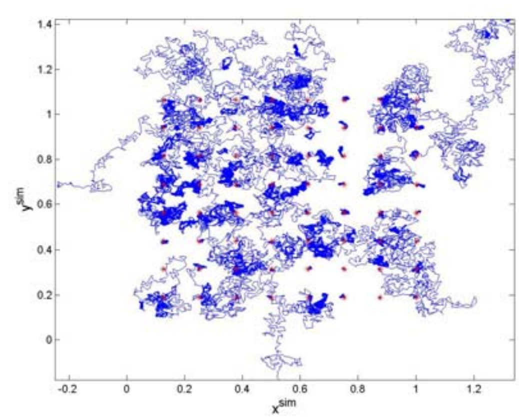

Fig. 2. Another box with 64 trajectories in the case of

$$K_s^{sim} = 1.5 \textit{ and } K^{sim} = 2.5$$

In order to get the required exponential decrease of the spatial autocorrelation of the potential (and consequently of the velocity correlation), in the numerical simulation we take:

$$k_m = 8\pi, \quad k_{tr} = 256\pi, \quad k_M = 512\pi$$

The components of the wave vectors are thus integer multiples of the elementary discretization wave vector $k_0 = 2\pi$. The perpendicular correlation length of the potential in the numerical simulation is $\lambda_\perp^{sim} \approx 0.015\,L$ m where L is the periodicity length (related to a characteristic wave-vector); the potential is defined on a square of unit size L, supposed to be periodic in the plane (x, y). The following dimensionless quantities are used in the numerical simulations:

$$\tau \equiv \frac{t\,\omega}{2\pi}; \quad \mathbf{x} = \frac{\mathbf{X}}{\lambda_{tr}} = \frac{\mathbf{X}\,k_{tr}}{2\pi} \tag{8}$$

In Eq. (7) λ_{tr} is the length wave corresponding to k_{tr} and ω is the single frequency. The dimensionless system for the numerical simulations has the same form as for the DCT method [see Eq. (4)] but with the following definitions for the corresponding electrostatic and shear Kubo numbers that are obtained from the system (2) using the expressions (7) and (8) as:

$$K^{sim} = \frac{k_{tr}}{\omega}\frac{2\pi\tilde{E}\,k_{tr}}{\sigma_S}\frac{c}{B_0}; \quad K_s^{sim} = \frac{2\pi}{\omega}V_{th}^{el}\frac{1}{L_s}$$

The quantity $\frac{\tilde{E}}{\sigma_S}$ is related to the amplitude of the electrostatic fluctuation ε.

The following ratios between the two sets of Kubo numbers specific to direct numerical simulation and decorrelation trajectory method [12] are obvious:

$$\frac{K^{sim}}{K} = \frac{(k_{tr})^2(\lambda_\perp)^2}{\omega\,\tau_c}\frac{2\pi\tilde{E}}{\sigma_S}\frac{1}{\varepsilon}$$

$$\frac{K_s^{sim}}{K_s} = \frac{2\pi}{\tau_c\,\omega}$$

For any pair of the Kubo numbers (K^{sim} and K_s^{sim}) we considered a (relatively) large number of trajectories (64×96×8) in order to ensure a good statistics.

In Figure 1 a single box with 64 trajectories are displayed, in shearless case and for a relatively large level of turbulence, $K^{sim} = 2.5$. In any similar box like that from Figure 1, we considered the equal-distributed starting points for trajectories. Red asterisks represent the initial conditions for trajectories (at $\tau = 0$). When the level of stochasticity and/or the shear are increased, the shape of any box with 64 trajectories could have a different look than that from Figure 1. In this spirit, we can see in Figure 2, a typical box, where some trajectories have very long journeys. The later characteristic will be reflected directly in the values of the mean squared displacements $\langle \Delta x^2(\tau) \rangle$ and $\langle \Delta y^2(\tau) \rangle$ and therefore in the diffusion coefficients. The running diffusion coefficient in x- direction is defined as:

$$D_{xx}^{sim}(\tau) = \frac{1}{2}\frac{d}{d\tau}\left\langle \Delta x^2(\tau) \right\rangle \tag{9}$$

A similar formula for $D_{yy}^{sim}(\tau)$ can be defined. We have calculated the mean squared displacement in x- and y- direction and also the running diagonal diffusion coefficients that are represented in Figures 3 and 4.

We can see from Figures 3 and 4 that the running diffusion coefficient in the x- direction is by an order of magnitude smaller than that in the y-direction; the diffusion coefficients tend to reach a final stationary value which is almost reached for $\tau_x \geq 0.25$ for the radial one. For the poloidal one, the existence of the magnetic shear, postponed the achievement of the final saturation value: in this case the time is $\tau_y \geq 0.45$. Thus an asymptotic diffusive regime in both directions is present. In the shearless case, for both weak and relatively strong electrostatic turbulence regimes, the numerical simulations results are in agreement with the results obtained by the DCT method [12]. For the shearless case, the diagonal running diffusion coefficients are approximately equals each other. This is a verification of the code used in the numerical evaluation.

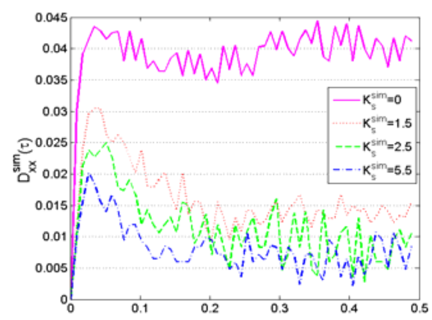

Fig 3. Running radial diffusion coefficients for $K^{sim} = 2.5$ *and four different values for magnetic shear.*

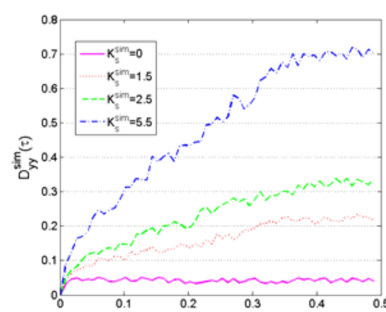

Fig 4. Running poloidal diffusion coefficients for $K^{sim} = 2.5$ *and four different values for magnetic shear.*

In Fig. 5 the ratio between the asymptotic poloidal and radial diffusion coefficients are shown as functions of the shear Kubo number for three levels of electrostatic turbulence: $K^{sim} = \{0.05, 0.5, 2.5\}$. An increase of the slope of this ratio when K^{sim} is increasing is observed. The effect of the magnetic shear is obvious: a reduction of the diffusion on the radial direction and an enhanced diffusion on the poloidal one.

We used in our paper values for the Kubo numbers that are compatible with the values specific to the DCT method. Then, for the shear Kubo number the following relation holds: $K_s^{sim} \approx 1.2/6K_s = 0.2K_s$ and therefore a relation between τ_c and ω: $\frac{K_s^{sim}}{K_s} = \frac{2\pi}{\tau_c \omega} \approx 0.2$ holds.

The following condition between the electrostatic Kubo numbers (for both numerical simulation and DCT) was used:

$$K^{sim} < (0.056/1.2)K .$$

Then, considering $\frac{2\pi\tilde{E}}{\sigma_S}\frac{1}{\varepsilon} = 1$, the following inequality is also valid: $\frac{K^{sim}}{K} = \frac{(k_{tr}\lambda_\perp)^2}{\omega\tau_c} \leq 0.05$.

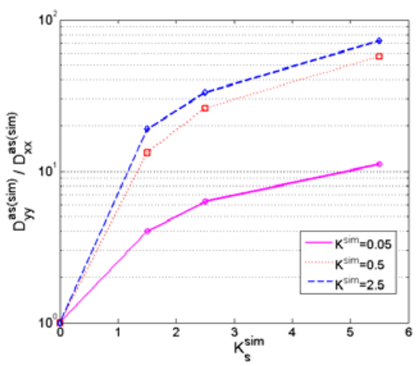

Fig. 5. Ratio between the poloidal and radial asymptotic diffusion coefficients as function of the shear Kubo number and three different values of the electrostatic Kubo number

This kind of direct numerical simulation allows us to state that the DCT method can be correctly applied in the case of inhomogeneous Langevin system of equations.

4. Conclusions

The global effects of K^{sim} and K_s^{sim} on the running and asymptotic diagonal diffusion tensor components are exhibited using direct numerical simulation for a guiding center system in a first order of drift approximation.

We presented here a severe selection of the results; a large numbers of runs were performed for this study, using mainly the computing facilities of ULB-VUB Belgium.

The radial running diffusion coefficient starts with a linear part characteristic to a ballistic regime, $D_{xx}(\tau) \sim \tau$. In all of the cases a trapping effect appears for large enough values of K^{sim} and/or K_s^{sim}.

The trapping regime does not appear for the poloidal diffusion coefficient in the same conditions.

We can conclude that an enhancing of the diffusion on poloidal direction and a relatively reduction on the radial one is caused by the presence of the magnetic shear for the same level of electrostatic turbulence. This behavior is expected because the shear term is oriented along the y-axis and is similar to a zonal flow generation.

In our paper we have calculated the diffusion coefficients for the electrons by direct numerical simulation and we have found a very good qualitative agreement with the results obtained by the decorrelation trajectory method [12]. This conclusion gives a relatively certitude in order to apply DCT to other problems of interest where a Langevin treatment can be done.

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