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Interaction of narrow solitons with point defects in nonlinear lattices

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We study the interaction of discrete (narrow) solitons with impurities in the integrable Ablowitz-Ladik lattice model. Analytical solutions are obtained for bright and dark solitons bound to the defect. On-site and inter-site (bond) defects are considered. A comparison with the standard discrete nonlinear Schrödinger equation is made. Scattering of the discrete solitons from point defects of different types is studied numerically. The model plays an important role in numerous physical systems, especially when the corresponding elementary excitations obey Pauli statistics.

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1. Introduction

Many problems in the nonlinear dynamics of spatially extended physical systems involve continuous media, so that nonlinear coherent excitations (solitons) are naturally described as solutions to partial differential equations, e.g. the nonlinear Schrödinger (NLS) equation. However, models describing microscopic phenomena in solid-state physics are inherently discrete, with the lattice spacing between the atomic sites being a fundamental physical parameter. For these systems, an accurate microscopic description involves a large set of coupled ordinary differential equations. Defects and discreteness effects may modify drastically the dynamics of the localized nonlinear excitations, even in the framework of the simplest models. Spatially localized modes of the discrete nonlinear lattice equations, called discrete solitons, have appeared in many areas of physics, such as biophysics, nonlinear optics, solid state physics, and more recently in the studies of Bose-Einstein condensates in optical lattices and photonic-crystal waveguides. Widely investigated are the standard discrete nonlinear Schrödinger (DNLS) equation, as well as the completely integrable discrete Ablowitz-Ladik (AL) equation [1-7]. Although the two equations have the same linear properties and yield the same NLS equation in the continuum limit, their nonlinear properties are different. This leads to differences in the dynamics of narrow discrete solitons for the two models.

A problem of continuing interest due to its theoretical and practical importance is the interaction of solitons with impurities. Extensively studied is the interaction of solitons with different point defects (linear, nonlinear and bond) for the DNLS equation [2,8,9]. Within the AL model, only the interaction of solitons with on-site defects has been considered [2,5]. This model is important for excitations which obey the Pauli statistics, like spin waves or electron excitons [10-12]. The soliton dynamics in coupled Ablowitz-Ladik chains was studied in [13].

In the present work, we investigate in detail the interaction of solitons with impurities in an AL chain. Analytical bound soliton-defect solutions are derived and their stability is analysed. The scattering of solitons from defects is studied numerically for different soliton velocities and impurity parameters.

2. Lattice NLS equations

We start with a generalized set of ordinary differential equations describing a hypothetical model of intramolecular excitations with the amplitude $\alpha_n(t)$ in a discrete chain with nearest-neighbour interactions:

$$\begin{aligned} i\frac{\partial \alpha_n}{\partial t} &= \frac{M}{2}(\alpha_{n+1}+\alpha_{n-1})(1+\gamma\,|\,\alpha_n\,|^2) \\ &\frac{J}{2}(|\,\alpha_{n+1}\,|^2+|\,\alpha_{n-1}\,|^2)\alpha_n + g\,|\,\alpha_n\,|^2\;\alpha_n \end{aligned} \tag{1}$$

For methodological reasons, we have included three different nonlinear terms which correspond to different physical systems. The lattice constant equals unity. For molecular crystals, the case g = 0, γ = -1 describes electronic excitations obeying Pauli statistics [12], where M is the intermolecular interaction and J the nonlinear dynamic interaction. There are two competing nonlinear terms – the dynamic ($\sim J$) and the kinematic ($\sim M$). If the dynamic interaction is small and can be neglected, then Eq. (1) turns into the AL equation which is completely integrable and has a bright soliton solution for γ = 1

$$\begin{aligned} \alpha_n(t) &= \sinh\frac{1}{L}\,\mathrm{sech}\,\frac{n-vt}{L}\,e^{i(kn-\omega t)} \\ v &= -ML\sin k\sinh\frac{1}{L},\quad \omega = M\cos k\cosh\frac{1}{L} \end{aligned} \tag{2}$$

and a dark soliton solution for $\gamma = -1$

$$\begin{aligned}\alpha_n(t) &= \tanh\frac{1}{L}\tanh\frac{n-vt}{L}e^{i(kn-\omega t)}\\ v &= -ML\sin k\tanh\frac{1}{L},\quad \omega = M\cos k\ \mathrm{sech}^2\frac{1}{L}\end{aligned} \tag{3}$$

L, v, k, and ω are the width, velocity, wavenumber and frequency of the soliton, respectively.

For $J = \gamma = 0$, Eq. (1) describes the dynamics of Bose-type excitations with a nonlinear constant g [9]. In this case, (1) has the form of the standard DNLS equation which is nonintegrable and can have stable soliton solutions only for wide solitons $(L >> 1)$. Important differences between the solutions of the AL and DNLS equations are that in the AL equation the velocity depends on L and that the type of soliton solution of the AL equation remains the same in the whole Brillouin zone ($0 \le k \ge \pi$), while the solution of the DNLS equation changes from a bright to a dark soliton or vice-versa at $k = \pi/2$.

In the continuum limit $\alpha_n(t) \to \alpha(x,t)$, (1) turns into the nonlinear Schrödinger equation

$$i\frac{\partial\alpha}{\partial t} = M\alpha + \frac{M}{2}\frac{\partial^2\alpha}{\partial x^2} + G|\alpha|^2\alpha \tag{4}$$

with a composite nonlinear constant $G = (\gamma M + J + g)$, which is completely integrable and, depending on the sign of M/G, has bright or dark soliton solutions.

Equations similar to (1), containing a number of nonlinear terms, have been derived for magnetic chains (see e.g. [10,11]).

In what follows, we shall consider a chain containing a guest molecule with different parameters at site $n = 0$. This will lead to additional terms in Eq. (1) corresponding to linear, nonlinear and bond defects. Our numerical results have shown that defects in the nonlinear terms J and g do not play an important role. Thus, we concentrate on the following equation ($J = 0$):

$$\begin{aligned} i\frac{\partial\alpha_n}{\partial t} &= \varepsilon\delta_{n,0}\alpha_n + \frac{1}{2}\{[M + \mu(\delta_{n,0} + \delta_{n+1,0})]\alpha_{n+1}\\ &+ [M + \mu(\delta_{n,0} + \delta_{n-1,0})]\alpha_{n-1}\}(1 + \gamma|\alpha_n|^2) + g|\alpha_n|^2\alpha_n\end{aligned} \tag{5}$$

where ε and μ characterize the impurity. For $g = 0, \gamma \ne 0$, Eq. (5) corresponds to the perturbed AL equation, and for $g \ne 0, \gamma = 0$ to the perturbed DNLS equation. Note that ε corresponds to a linear on-site defect, while μ introduces two neighbouring inter-site linear and nonlinear defects.

We shall investigate the interaction of solitons with localized defects with nonzero ε, μ. Positive values of the defect parameters correspond to repulsion, while negative values correspond to attraction.

3. Bound soliton-defect solutions

Now, we investigate the static case $k = v = 0$. For the Ablowitz-Ladik model with linear on-site defects ($\varepsilon \ne 0, \mu = 0$), Eq. (5) possesses the following exact bound soliton-defect solutions:

For $\gamma = 1, g = 0$

$$\begin{aligned}\alpha_n(t) &= \sinh\frac{1}{L}\mathrm{sech}\left(\frac{|n|}{L} + \Delta\right)e^{-i\omega t}\\ \omega &= M\cosh\frac{1}{L},\quad \tanh\Delta = \varepsilon/(M\sinh\frac{1}{L})\end{aligned} \tag{6}$$

For $\gamma = -1, g = 0$

$$\begin{aligned}\alpha_n(t) &= \tanh\frac{1}{L}\tanh\left(\frac{|n|}{L} + \Delta\right)e^{-i\omega t}\\ \omega &= M\,\mathrm{sech}^2\frac{1}{L},\ \sinh 2\Delta = -\frac{2M}{\varepsilon}\tanh\frac{1}{L}\end{aligned} \tag{7}$$

If $\Delta > 0$, the function $|\alpha_n(t)|$ has a single maximum for (6) (minimum for (7)) at $n = 0$, and if $\Delta < 0$ there are two maxima for (6) (two minima for (7)) at $n = \pm\Delta L$.

Similar bound soliton-defect solutions hold for the perturbed DNLS equation with linear on-site defects in the wide soliton limit $(L >> 1)$.

For $\gamma = 0, g = M$

$$\begin{aligned}\alpha_n(t) &= \frac{1}{L}\mathrm{sech}\left(\frac{|n|}{L} + \Delta\right)e^{-i\omega t}\\ \omega &= M + \frac{M}{2L^2},\quad \tanh\Delta = \varepsilon L/M\end{aligned} \tag{8}$$

For $\gamma = 0, g = -M$

$$\begin{aligned}\alpha_n(t) &= \frac{1}{L}\tanh\left(\frac{|n|}{L} + \Delta\right)e^{-i\omega t}\\ \omega &= M - \frac{M}{L^2},\ \sinh 2\Delta = -\frac{2M}{\varepsilon L}\end{aligned} \tag{9}$$

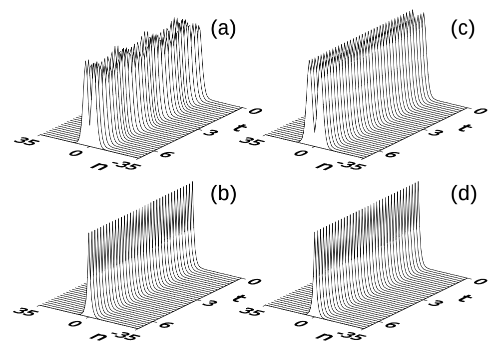

Fig. 1. Evolution of narrow bound soliton-defect solutions of the DNLS equation (8) [(a), (b)] and of the AL equation (8) [(c), (d)]. ε = 0.75 correspond to repulsion [(a), (c)] and ε = -0.75 – to attraction [(b), (d)]. M = -2, L = 2.5. The time is in units of 10^3.

In what follows, we present results of the numerical solution of Eq. (5) for narrow solitons with the initial form (6) or (8). Fig. 1 shows the results for bright solitons for the two types of discrete equation with a linear impurity. As can be expected, the solutions (6) for the AL model are stable [Fig.1(c), (d)], while the two-peak solution for the DNLS model (8) exhibits oscillations due to the violation of the wide-soliton limit [Fig. 1(a)]. The single-peak solutions for $\varepsilon < 0, \Delta > 0$ [Fig. 1(b), (d)] correspond to soliton-defect attraction, while the double-peak solutions with $\varepsilon > 0, \Delta < 0$ [Fig. 1(a), (c)] correspond to repulsion. However, deviations in the initial form or position of the input solution from (6) or (8) have different effects on their stability. Our numerical simulations showed that the single-peak solution is stable against perturbations, while the double-peak solution is unstable and easily destroyed. Similar results have been obtained for dark solitons of the form (7) or (9).

Now, we study the influence of bond defects ($\varepsilon = 0, \mu \neq 0$) on the soliton solutions. For the standard DNLS model and wide solitons, the problem was considered in [14] in detail. Here we concentrate on narrow solitons within the AL model. The input solution is chosen in the form (6) with $\varepsilon = 2\mu$ (the guest molecule alters the interaction energy with the two neighbours). Results for different values of μ are presented in Fig. 2. When the bond defect is repulsive, the soliton maxima oscillate [Fig. 2(a) and (b)], the oscillation period increases with the defect strength, and for great enough values the bound state splits into two solitons which propagate with opposite velocities [Fig. 2(c)]. For attractive defects, the soliton bound state is more stable and the amplitude exhibits small oscillations [Fig. 2(d)].

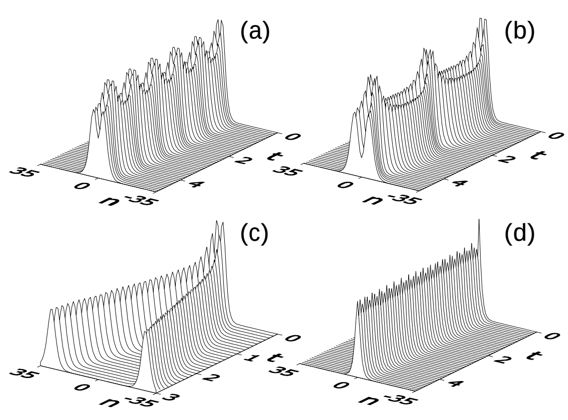

Fig. 2. Evolution of a bound soliton-defect solution (6) (ε = 2μ) for the AL model with two neighbouring bond defects: (a) μ = 0.2; (b) μ = 0.22; (c) μ = 0.24; (d) μ = -0.24. All other parameters are the same as in Fig. 1.

In the static case, for wide solitons ($L >> 1$) the two modified intermolecular bonds are equivalent to a linear point defect with strength 2μ.

4. Scattering of Ablowitz-Ladik solitons from defects

Of considerable interest is the scattering of solitons from defects. The scattering of slow and fast wide solitons from point defects was investigated in [9]. Here we consider the interaction of AL solitons of the form (2) with defects. This allows us to study narrow solitons with an arbitrary wavenumber k in the Brillouin zone ($0 \leq k \geq \pi$). The evolution depends strongly on the initial soliton velocity which is related to k through (2) and on the sign of the defect. The more interesting case is when the defect is attractive ($\varepsilon, \mu < 0$) (Fig. 3).

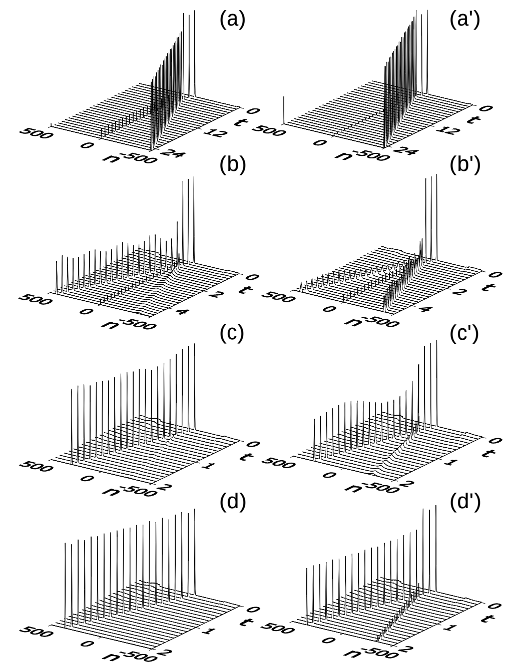

Fig. 3. Scattering of AL solitons (2) with different velocities from attractive bond defects with μ= -0.5 [(a) - (d)] and from a comparable on-site defect with ε =2μ= -1 [(a') - (d')]. (a), (a') correspond to v = 0.21; (b), (b') to v = 0.99; (c), (c') to v = 1.73; (d), (d') to v = 2.05.

For a given value of the defect and small initial velocities ($v < 0.2$) the soliton is completely reflected. With the increase in the velocity, a part of the soliton is trapped [Fig. 3(a), (a')]. A further increase in the velocity yields transmission + trapping + reflection [Fig. 3(b), (b')]. For larger values of the velocity, the soliton is almost totally transmitted [Fig. 3 (c), (c')]. Approaching the maximum value of the velocity $v = 2.05$ which corresponds to wavenumbers from the centre of the Brillouin zone ($k = \pi/2$), the soliton passes unchanged through the bond defect [Fig. 3 (d)], but for the linear defect a part of it is reflected [Fig. 3(d')]. So, in the whole range of initial velocities, the scattering from bond defects differs from the scattering from on-site defects with comparable strength. As a whole, linear on-site defects with comparable strength to bond defects induce stronger perturbations. This is due to the different velocity dependence of the corresponding perturbing terms. The difference in the scattering pattern of narrow slow solitons from on-site and bond defects [Fig. 3(a), (a')] vanishes for wide and slow solitons [9].

Acknowledgements

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