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Measuring elastic flexural properties on marble slabs by a LDV system

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Quality control of individual marble slab used for construction purposes prior to their application allows an effective reduction of security- and cost-relevant mechanical failures. Therefore, a non-destructive determination of flexural moduli of elasticity by consideration of marble's anisotropic behaviour is necessary. Today's relevant standardized measurement systems allow a reliable determination of such moduli exclusively on specimens with other geometries than slab shapes. A measurement system with a Laser Doppler Vibrometer (LDV) operating in accordance with an eigenfrequency method is introduced in this paper. This system is designed for automatic quality control environments and provides for fast and non-destructive determination of a reliable value range of flexural moduli with respect to a multitude of marble varieties. Results and advantages of the system are compared and discussed respectively to a frequently-used standardized ultrasonic measurement system.

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1. Introduction

An analysis of damages caused by mechanical failure on marble slabs [1] demonstrates that a comprehensive quality control of each slab used for construction purposes with safety requirements is indispensable, ideally immediately after fabrication. Moreover, a quality control of each individual slab effects also significant long-term cost reduction and increases the customer satisfaction concerning marble as construction material.

Considering the anisotropic structure of marble, mechanical failure occurs on marble slabs mainly by spreading of internal microcracks [2] caused by mechanical or thermal induced flexural load cycles [3]. Therefore, a reliable determination of flexural moduli of elasticity is necessary considering the anisotropic behaviour of marble. Furthermore a comprehensive quality control of each slab requires a non-destructive measurement procedure.

Current measurement systems applying standardized procedures [4] determine marble slab flexural moduli of elasticity by analysis of ultrasonic wave propagation properties through marbles structure. Because such methods are solely applicable on specimens with different geometries than slab shapes, a comprehensive measurement of each slab is not feasible, rather only statistic evaluation from a strongly limited number of specimens of a charge is possible.

The measurement system introduced in this paper, allows a fast and non-destructive determination of a reliable value range of flexural moduli inside of an automatic marble slab quality control context. To this end, a first order eigenfrequency is determined from a transient oscillation measured by a Laser Doppler Vibrometer (LDV) on the surface of a single-edge clamped marble slab, which has been previously excited by a force pulse. A reliable flexural modulus range is subsequently calculated for an orthogonal direction x or y of slabs larger surfaces by knowledge of an anisotropic parameter and reliability value. Both have been achieved through measurements within a marble slab charge composed by observance of textural and tonal similarity, which concerns in particular to charges intended for public sale, consisting from a few hundred to several thousand slabs. Today, such composition processes are profitably made on industrial scale by automatic systems [5].

The reliability value considers all relevant measurement deviations under worst case conditions and can be additionally optimized through the operating experience of the manufacturer.

To predetermine the anisotropic parameter a comparative aging procedure with a minimizing condition has been applied.

Measurement results and advantages of the introduced laser measurement system are compared and discussed respectively to a frequently used standardized method, based on the evaluation of quasi-longitudinal ultrasonic waves.

A fast execution of measurement sequences has been achieved by programming techniques and system integration concepts related to Graphics Processing Unit (GPU) taking consideration of industrial environment conditions [6].

2. Eigenfrequency method

Greubel et al. [7] have demonstrated that dynamic flexural moduli of elasticity E_{dyn} can be calculated with acceptable deviations on anisotropic slabs clamped at one edge along its width through following equation:

$$E_{dyn} = \frac{4 \cdot \omega_E{}^2 \cdot l^3 \cdot m}{b \cdot d^3} \cdot K \tag{1}$$

Equation (1) can be applied on a clamped anisotropic slab as shown in Fig. 1 if following conditions can be almost presumed:

- slab's thickness *d* is small compared to its own length *l* and width *b*
- slab's center plane lies in the xy-plane
- the slab is not deformable in its thickness direction;
- all slab points, which lie on a normal to the center plane before deformation, lie also on such a normal after deformation
- an influence of shear deformations can be neglected

Beyond slab's mass *m*, length *l*, width *b* and thickness *d* (see Fig.1), the solution of (1) demands a eigenfrequency ω_E of slab's elastic flexural mode and a multiplicative constant *K*, which considers the energetic distribution over entire slab's anisotropic structure, independently from a spatial direction. ω_E corresponds to the first order eigenfrequency of elastic flexural mode frequency range [7].

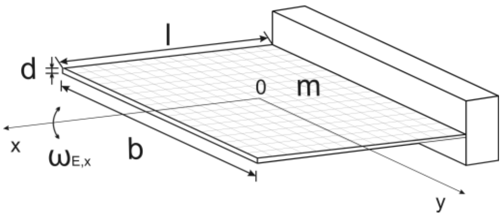

Fig. 1. Model of a cantilevered slab with flexural eigenfrequency.

A possibly pure flexural mode is induced on anisotropic slabs by a force pulse along:

$$-\frac{l}{2} \leq x \leq \frac{l}{2} \quad ; \quad y = 0 \tag{2}$$

Pronounced flexural oscillations can be measured in proximity of the free slab edge ($x \approx l/2,\ y=0$).

The isotropic approach of (1) is purposeful if the systematic deviation of ω_E and respectively E_{dyn} remains marginal. It should be particularly noted, that this deviation originates not only from measurement deviations but also from different reactions concerning the anisotropic behaviour of marble slab's structure in response to not absolutely reproducible force inductions.

Distinguishing the systematic deviation into known and unknown dues a reliable range of the flexural modulus of elasticity can be calculated as:

$$E = \frac{\omega_E^2 \cdot l^3 \cdot m}{b \cdot d^3} \cdot \zeta_E \pm \Delta_E \tag{3}$$

ζ_E is an anisotropic parameter which includes the energetic distribution over the slab, further multiplicative constants and systematic known deviations of *E*. The reliability value Δ_E expresses all systematic unknown deviations of *E* under certain conditions.

An applicability of (3) into automatic quality control systems requires a low Δ_E. Considering the anisotropic behaviour of marble slabs with respect to bending deformations, a reliable statement of the elastomechanic state can be seen as sufficiently reached, if *E* is calculated for each orthogonal direction *x* or *y* of slabs larger surfaces:

$$E_{x/y} = \frac{\omega_{E,x/y}^2 \cdot l^3 \cdot m}{b \cdot d^3} \cdot \zeta_{E,x/y} \pm \Delta_{E,x/y} \tag{4}$$

It has been experienced, that a low Δ_E is obtained, if $\zeta_{E,x/y}$ has been calculated from measurement values delimited to examinations within a marble slab charge, composed by criteria of aesthetic similarity. It has been shown, that aesthetic similarity of marble slabs can be accompanied with similarities of mechanic behaviour concerning entire slab reflections. [5].

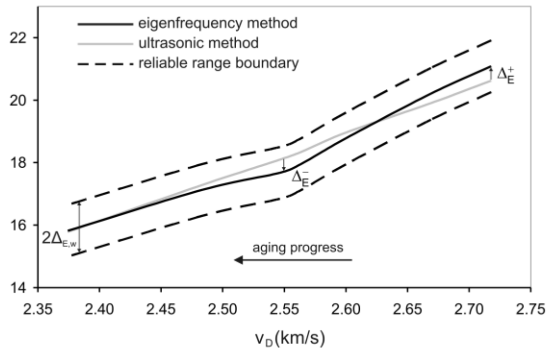

Fig. 2. Comparison of results with respect to quasi-longitudinal velocities v_D and a worst case reliability range $2\Delta_{E,w}$ obtained from examination in y-direction of Thassos Extra slabs and specimens.

A comparative aging procedure includes a cyclic aging process consisting of a standardized frost action cycle [8] followed by a standardized drying cycle [9], effected simultaneously on different marble slabs and specimens, which have been cut out along the edges of the same slabs. After each aging progress flexural moduli of elasticity $E_{x/y}$ are detected on those slabs with uncertainties $\Delta_{1,x/y}$ by means of (4), and on the belonging specimens, using a standardized method of the state of technology with uncertainties $\Delta_{2,x/y}$. Starting from a non-aged slab state, $\zeta_{Ex/y}$ is finally determined by a comparison of two result trends, obtained from a number of aging cycles necessary to reach a defective slab state, which occurs previously to mechanical failure, and belongs to a range of ultrasonic quasi-longitudinal velocities $\Delta v_{D,F}$. This defective state is reached if significant damages of the surface become visible and further aging cycles have only marginal influence on flexural moduli and at the same time damages have noticeably increased. An overall aging progress is expressed subsequently through the ultrasonic quasi-longitudinal velocity range $\Delta v_{D,F}$.

Considering the maximum deviation Δ^+_E and a minimum deviation Δ^-_E of the two overlapping result trends, $\zeta_{Ex/y}$ has to be chosen in such manner, that the result trends are overlapped according to following minimizing condition:

$$\left|\Delta_E^+\right| = \left|\Delta_E^-\right| =: \Delta_3 \qquad (5)$$

For example, two result trends from the examination of Thassos Extra slabs and related specimens in *y*-direction are shown in Fig. 2. To comply with (5), the anisotropic parameter in *y*-direction $\zeta_{E,y}$ has to be equal to 3.66 and $\zeta_{E,x} = 3.70$ for a correspondent examination in *x*-direction. Consistent with this experiences, it has been found, that the difference between $\zeta_{E,x}$ and $\zeta_{E,y}$ remains generally low within a marble charge with respect to all examined varieties, wherefore a mean anisotropic parameter:

$$\zeta_{E,D} = \frac{\zeta_{E,x} + \zeta_{E,y}}{2} \qquad (6)$$

can be used for both directions if a deviation range $\pm\Delta_4$ is additionally taken into account. A worst case reliability value $\Delta_{E,w}$ has to be calculated according to the following equation:

$$\Delta_{E,w} = \Delta_4 + \sum_{i=1}^{3} \max(\Delta_{i,x}, \Delta_{i,y}) \qquad (7)$$

Equation (7) considers a plausible elastomechanic state, in which the deviations with influence on $E_{x/y}$ have reached their maximum amounts simultaneously. In this case, maximum amounts of $\Delta_{1,x/y}$, $\Delta_{2,x/y}$ and $\Delta_{3,x/y}$ are related to $\Delta_{i,x/y}$ over entire result trends, which describe the entire amount of measurement results between the highest possible number of aging cycles. To obtain reliable values for $E_{x/y}$, it must be taken in account that the elastomechanic state during measurements is always unknown, thus $\Delta_{E,w}$ has to be considered for all measurements.

Further parameters of (4) can be measured with insignificant variances, thus their deviation influence on $E_{x/y}$ is negligible. Considering (4), (6) and (7), with respect to the examined marble slabs, (3) has to be rewritten as:

$$E_{x/y} = \frac{\omega^2_{E,x/y} \cdot l^3 \cdot m}{b \cdot d^3} \cdot \zeta_{E,D} \pm \Delta_{E,w} \qquad (8)$$

3. LDV system

Maximum uncertainties of $\Delta_{1,x/y}$ are reduced significantly if $\omega_{E,x/y}$ are derived from LDV-measure-ments. Fig. 3 shows a model of the introduced LDV system, which allows an automatic and non-destructive measurement of reliable ranges of flexural moduli on marble slabs according to (8). Marble slabs are clamped at one edge of their larger surface (*x,y*) and excited through a force pulse consisting in a peak force of around 10 N applied within a dozen milliseconds by a supporting device I.

Spectral analysis on marble surfaces of multiple varieties has clarified, that the wavelength range of He-Ne-lasers are particularly suitable considering reflected laser light intensities.

Therefore, laser light with a wavelength of λ=632.8 nm is emitted by a HeNe-laser situated into the LDV-unit III and further wavelength-stabilized against temperature- and humidity changes by subsystems of the same unit.

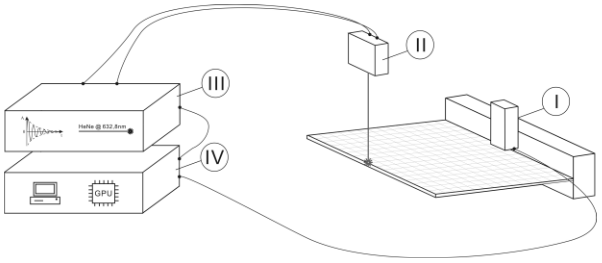

Fig. 3. Schema of the LDV system examining a slab in x-direction.

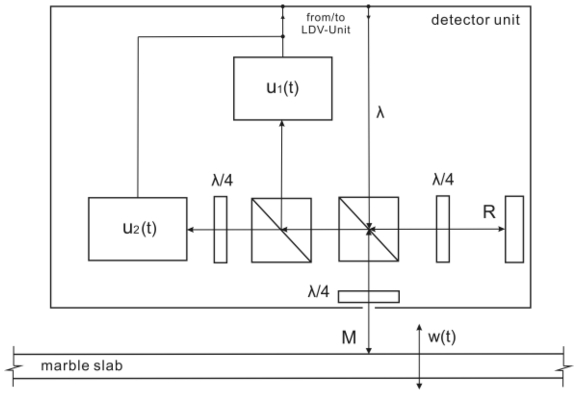

Fig. 4. Detector head model with detected voltages and laser-paths.

Laser light is guided along a fibre optic to a detector head II, shown in Fig. 4. According to the configuration of a Michelson interferometer, inside of the detector head, the laser light beam is divided by a beam splitter and forwarded to the reference surface of a mirror *R* and to a determined circular area *M* of the surface (*x,y*).

The interference beam obtained from the superposition of partial beams reflected from the surfaces *R* and *M*, contains information of the marble slabs oscillation transient $w_{x/y}(t)$ and is also converted inside of the detector head to voltages:

$$u_1(t) = U \cdot \cos\phi(t) \qquad (9)$$

and

$$u_2(t) = U \cdot \sin\phi(t) \qquad (10)$$

with a phase shift:

$$\phi(t) = \arctan\frac{u_2(t)}{u_1(t)} + k \cdot \pi \tag{11}$$

wherein U is a constant amplitude voltage and k a non-negative integer. The voltages (9) and (10) including k are submitted to the LDV-unit III, where subsequently an oscillation transient:

$$w_{x/y}(t) = \frac{\lambda \cdot \phi(t)}{4\pi} \tag{12}$$

is calculated with a maximum resolution of few nanometre.

After digitization, the discrete transient data is loaded into the computing unit with GPU (IV) and mapped in a frequency range by means of a Fast Fourier Transform (FFT).

The eigenfrequency value $\omega_{E,x/y}$ is extracted from the corresponding amplitude spectrum magnitude, which has been multiplied with 2π, by detection of its global maximum value.

To assure fast calculation of a reliable range of flexural moduli of elasticity, all computations are executed entirely on GPU, beginning at the FFT. To this end, the unit IV includes a common graphics card, connected to a common CPU motherboard system.

Time overheads have been avoided, by implementation of the related signal processing programs pursuant to a three-layer model [6], without signal processing operators from external software libraries.

Control tasks to carry out the measurements in the LDV system are performed contemporaneously to GPU processing by the CPU and their peripheries, reducing the overall execution times.

4. Results

Results and advantages of the introduced LDV system are discussed hereafter for charges of seven different marble varieties (see Table 1). Considering this charges, it has been experienced that the accuracy of $\zeta_{E,D}$ would not be significantly improved by examination of more than a half of dozen slabs per charge. Obviously, the amount of examined slabs has always to be adapted empirically to their anisotropic behaviour and to the used composition criteria of the charge.

To each examined slab, the flexural moduli of elasticity of nine related specimens per direction, have been measured by standardized evaluation [5] of quasi-longitudinal waves using a Geotron ® UKS-D ultrasonic system.

Table 1. Examined marble varieties and dimensions of selected slabs and specimens.

Variety	Slabs	Specimens
Arabescato Garfagnana (ARG)	l x b x d (cm) 25 x 25 x 1	l x b x d (cm) 5 x 1 x 1
Carrara D (CAD)		
Cremo Delicato (CRD)		
Estremoz (ETR)		
Statuario Venato (STV)		
Thassos (THA)		
Thassos Extra (THE)		

Measurements on slabs and specimens have been repeated at least a hundred times, to allow statistic statements on identified normally distributed variances.

Table 2 shows the calculated anisotropic parameters $\zeta_{E,D}$ related to the overall aging progress Δv_D and to the defective value range $\Delta v_{D,F}$ for the examined varieties. In reference to an isotropic slab state ($\zeta_{E,D} \approx 1$), anisotropic parameters belong to $\zeta_{E,D} > 1$.

It is further observed, that at examination with respect to different marble varieties the achieved aging progress Δv_D can be of very different extent while the range $\Delta v_{D,F}$, which belongs to a defective slab state, remains between 5 and 13 km/s.

This small range of $\Delta v_{D,F}$, is conducive to reliability concerning the automatic detection of defective slabs by the introduced LDV system.

Table 2. Anisotropic parameters $\zeta_{E,D}$ of different marble varieties and their related quasi-longitudinal velocity ranges Δv_D and $\Delta v_{D,F}$.

Variety	$\boldsymbol{\zeta_{E,D}}$	$\boldsymbol{\Delta v_D}$ (km/s)	$\boldsymbol{\Delta v_{D,F}}$ (km/s)
ARG	2.74	1.70 – 2.14	1.59 – 1.70
CAD	3.41	2.15 – 2.51	2.03 – 2.15
CRD	1.93	1.44 – 2.58	1.39 – 1.44
ETR	2.87	2.01 – 2.54	1.90 – 2.01
STV	2.29	1.69 – 2.42	1.56 – 1.69
THA	2.78	2.16 – 2.72	2.11 – 2.16
THE	3.68	2.47 – 2.74	2.37 – 2.47

A worst case comparison of deviations between eigenfrequency- and ultrasonic measurements (see Tab. 3) shows that normally distributed uncertainties of measurement points from eigenfrequency measurements are lower by a decade. Both systematic deviations can not be reduced by methodical techniques.

Table 3. Marble variety related maxima of normally distributed uncertainties from orthogonal (x,y) eigenfrequency measurements (Δ_{1x}, Δ_{1y}) and ultrasonic measurements (Δ_{2x}, Δ_{2y}) with respect to Δv_D.

Variety	**max($\boldsymbol{\Delta_{1x}, \Delta_{1y}}$)** (GPa)	**max($\boldsymbol{\Delta_{2x}, \Delta_{2y}}$)** (GPa)
ARG	0.076	0.98
CAD	0.078	0.54
CRD	0.013	0.28
ETR	0.026	0.40
STV	0.019	0.40
THA	0.022	0.50
THE	0.026	0.37

Quasi-longitudinal velocities have been obtained from measurements effected by a Geotron ® UKS-D ultrasonic system.

Reaching for optimization of the automatic quality control, according to values of Table 5, further reductions of $\Delta_{E,w}$ have to be reached in proximity of $\Delta v_{D,F}$ in particular for the varieties ARG, CRD, ETR and THA. This can be methodically achieved by decreasing $\Delta_{3,x/y}$ and Δ_4 (see Tab. 4) by additional calculation of anisotropic parameters $\zeta_{E,crit}$, belonging to a reduced range of quasi-longitudinal velocities $\Delta v_{D,crit}$ adjacent to $\Delta v_{D,F}$.

It has been found, that an additional $\zeta_{E,crit}$=1.64 applied to a critical range $\Delta v_{D,crit}$, which extends from 1.44 to 1.64 km/s, reduces max(Δ_{3x}, Δ_{3y}) from 1.48 to 0.30 and Δ_4 from 0.37 to 0.10. On the whole, $\Delta_{E,w}$ has been reduced from 2.14 to 0.69, reaching the same scale as the worst case deviations from ultrasonic measurements (max(Δ_{2x}, Δ_{2y}) = 0.28).

Table 4. All Δv_D related maximum orthogonal deviations (Δ_{3x}, Δ_{3y}) and Δ_4 deriving respectively from application of the minimizing condition (5) and from calculations with mean anisotropic parameters $\zeta_{E,D}$ applied on different varieties.

Variety	**max(Δ_{3x}, Δ_{3y})** (GPa)	**Δ_4** (GPa)
ARG	1.95	0.92
CAD	1.11	0.22
CRD	1.48	0.37
ETR	1.82	1.70
STV	1.30	0.12
THA	1.95	0.43
THE	0.79	0.45

Table 5. Marble variety dependent worst case reliability values $\Delta_{E,w}$ and ranges of flexural moduli $E_{x/y}(\Delta v_D)$ measured along Δv_D.

Variety	**$\Delta_{E,w}$** (GPa)	**$E_{x/y}(\Delta v_D)$** (GPa)
ARG	3.21	8.56 – 12.03
CAD	1.95	11.19 – 16.07
CRD	2.14	6.36 – 10.40
ETR	3.95	10.44 – 15.50
STV	1.84	8.44 – 13.82
THA	2.90	14.01 – 17.81
THE	1.63	15.82 – 21.55

With regard to a determined marble variety, an optimized $\zeta_{E,crit}$ and a corresponding $\Delta v_{D,crit}$ has to be set by the manufacturer, ideally by applying knowledge from long-term experiences on the LDV system.

However, the previous example has shown, that the assignment of appropriate anisotropic parameters to partial ranges of the eigenfrequency result trends contributes significantly to lower worst case reliability values. Profitably, a multiplicity of anisotropic parameters is assigned to ranges of with respect to the entire eigenfrequency result trends. An optimum of $\Delta_{E,w}$, can be reached if characteristic curves are applied to the entire eigenfrequency result trend.

The time expense for determination of aniso- tropic parameters and reliability values corresponds approximately to that of a common legally binding examination on marble slab charges, which today is profitably achieved by standardized ultrasonic methods.

It has been experienced, that the $\zeta_{E,D}$ and $\Delta_{E,w}$ listed in Table 2 and Table 5 have been fitted for different charges of CAD, STV and THE which makes clear, that $\zeta_{E,D}$ and $\Delta_{E,w}$ have to be not necessarily recalculated for each charge of a defined marble variety. The results of Tab5. suggest, that this advantageous property can be attributed in principle to varieties with low $\Delta_{E,w}$

By knowledge of anisotropic parameters and reliability values, each slab of a related marble charge can be examined within a few seconds per orthogonal direction.

A significant contribution to this short examination time has been reached as well through separate allocation of control and signal processing tasks to different processors.

Starting from transient acquisition times between 0.8 s and 1.2 s, the largest time share to achieve ranges of flexural moduli is spent on mapping oscillation transients into the frequency range by a FFT.

During the last decade, it has been experienced that outgoing from comparable CPUs, the execution of a FFT can be sped to two decades by GPUs of consumer graphics cards.

Time evaluations on the LDV system have shown that FFT execution times demand around a second. The application of a latest GPU should lead to further time savings.

Further control- and signal processing tasks have been achieved within half a second.

Overall, by knowledge of related anisotropic parameters and reliability values the introduced LDV system allows an automatic detection of the reliable range of flexural moduli of elasticity on both orthogonal directions of several hundred marble slabs belonging to a charge of a certain variety within a few hours.

5. Conclusions

A fast and non-destructive determination of a reliable value range of flexural moduli of elasticity with respect to a multitude of marble varieties is now possible for each marble slab of a determined charge by use of the introduced LDV System. An effective reduction of the reliable value range allows detection of defective marble slabs with acceptable deviations inside of a automatic quality control context, thus probability of cost- and security relevant damages on marble constructions can be significantly reduced. By knowledge of anisotropic parameters and reliability values examination times have been reduced, in particular through GPU processing, to few seconds per slab.

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