Modeling and simulation of interaction of the ultrashort laser pulse with chirped mirror for structure design improvement

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The model for interaction of an ultrashort optical pulse with chirped mirror is developed through exact analysis of the pulse waveform. This allows to optimize chirped mirror design in terms of pulse parameters (duration, time-bandwidth product). Through waveform analysis we show existence of a range of layer numbers in the mirror, which provide simultaneously maximal pulse compression and minimal waveform distortion for the given incident chirped pulse. The model predicts possibility to obtain almost full pulse reconstruction under single reflection from the mirror. Using simulation we show that after reflection the pulse not only experiences compression but acquires waveform distortions as well. The pedestal on the leading edge of the pulse is due to group delay oscillation. Distortions of the trailing edge of the pulse are due to third-order dispersion in the mirror. Careful compensation of both dispersion parts results in full restoration of the pulse waveform. Magnitude and duration of the pulse can be restored via application of wider bandwidth chirped mirrors.

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1. Introduction

Femtosecond timescale becomes accessible due to progress in generation, amplification and measurement of ultrashort light pulses. Their applications range from optical communications to measurement of variety of physical processes in physics and biology. However, applications of ultrashort optical pulses require high-quality pulses. Dispersion becomes very important on this time scale. It is the reason of pulse broadening and transform-limited pulse turns to chirp. Dispersion arises in the active crystal rod and other optical components in the laser cavity. This is the cause for pulse broadening and distortion. Prism pairs are the standard device for dispersion compensation, but it suffers from considerable residual higher-order dispersion. Alternative solution for net dispersion control inside the laser cavity is chirped mirrors [1, 2] which provide strong reflection and the required phase properties over broad frequency range. The chirped mirrors (CMs) usually consist of alternate dielectric layers. Oxides SiO_2 and TiO_2 and are used mostly wide. In contrast to conventional Bragg mirrors, CMs have chirping of the Bragg wavelength. This produces the controlled group delay dispersion. Since the naturally occurring dispersion is usually positive (i.e., normal dispersion), the similar negative dispersion (anomalous dispersion) should be produced in the CMs. Along with dispersion the self-phase modulation can have considerable influence on the pulse formation in the laser cavity [3, 4].

State-of-the-art methods of CM design are based on the analysis of the mirror spectral response only. And the goal of design techniques is providing desired spectral response characteristics. Making certain assumptions about the desired reflectivity and phase properties one can optimize the mirror structure [5, 6]. Another approach is to calculate directly temporal profile of the pulse reflected from the CM in addition to the analysis of the reflectivity and the phase spectra of the CM [7, 8]. As we will show the analysis of the reflected pulse provides additional inside into physics of the phenomena and is capable to retrieve more detailed data for a design of the optimal mirror structure for dispersion compensation.

In this paper, we developed a model for interaction of an incident fs-pulse with the chirped mirrors. The model includes propagation effects of the pulse in dispersive media; spectral response of the chirped mirror; interaction of the pulse with the mirror. It is based on the Fourier approach instead of solution of a wave equation. The chirped mirror properties were calculated using transfer matrix method [9, 10].

The underlying physics provides the model with predictive capabilities. Therefore, we cannot only design the chirped mirror with the desired characteristics, but also predict pulse waveform, full width at half maximum (FWHM), time-bandwidth product. Additional information provided by the model developed allows correcting and improving chirped mirror constructions.

2. Formulation

During the propagation in the laser cavity the ultrashort pulse undergoes broadening and deformation due to the dispersion in the active medium. These effects can be well described using Sellmeier equation which approximates spectral dependence of the refractive index of a particular transparent medium. We chose Sapphire to model active medium. In this case Sellmeier equation is:

$$n(\lambda)=\sqrt{1+\frac{A_1\lambda^2}{\lambda^2-B_1}+\frac{A_2\lambda^2}{\lambda^2-B_2}+\frac{A_3\lambda^2}{\lambda^2-B_3}} \quad (1)$$

where the Sellmeier coefficients is defined for Sapphire (ordinary ray) as: $A_1 = 1.4313493$, $A_2 = 0.65054713$, $A_3 = 5.3414021$ and $B_1 = 0.00527993$, $B_2 = 0.01423827$, $B_3 = 325.0178$.

The phase accumulated during propagation in the medium is:

$$\varphi(\omega)=\frac{\omega n(\omega)L_m}{c} \quad (2)$$

where L_m is the length of the material, $n(\omega)$ is the frequency dependence of material refractive index, described by (1), ω is the angular frequency and c is the light speed in vacuum. The group delay (GD) and group delay dispersion (GDD) are calculated as follows:

$$GD(\omega)=\frac{d\varphi(\omega)}{d\omega} \quad (3)$$

$$GDD(\omega)=\frac{d^2\varphi(\omega)}{d\omega^2} \quad (4)$$

The wavelength dependence GD and GDD in Sapphire crystal rod 2.3 mm long is shown in Fig. 1. GDD value at 800 nm is $133\,\text{fs}^2$. Owing to GDD pulse is broadened and is transformed to the chirp.

Let we have transform-limited Gaussian pulse:

$$E(t)=E_0\exp\left(-2\ln 2\frac{t^2}{{\tau_0}^2}\right)\exp(i\omega_0 t) \quad (5)$$

where E_0 is the pulse amplitude, τ_0 is FWHM, ω_0 is the central pulse frequency. We use the Fourier transformation of the initial pulse to describe the influence of the material dispersion.

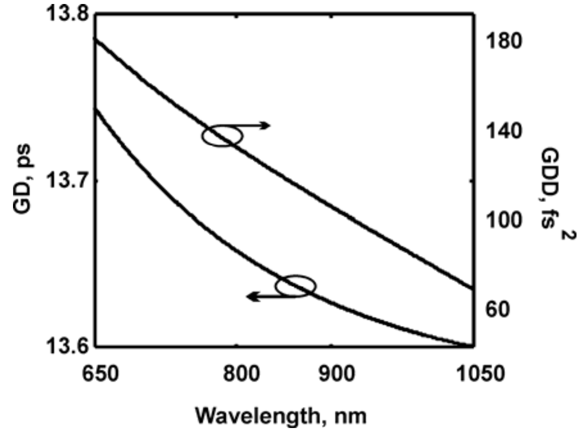

Fig. 1. Wavelength dependence of the group delay and group delay dispersion in Sapphire crystal with length 2.3 mm. GDD value at 800 nm is $133\ fs^2$.

It finally gives us the temporal profile of the pulse passed through the dispersive medium:

$$F_{pas}(\omega)=\int_{-\infty}^{\infty}E(t)\exp(-i\omega t)dt \quad (6)$$

$$E_{pas}(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F_{pas}(\omega)\exp(i\omega t)\exp[-i\varphi(\omega)]d\omega \quad (7)$$

where $E_{pas}(t)$ is the pulse passed through dispersive media and $\varphi(\omega)$ is defined by (2). Fig. 2 shows input transform-limited Gaussian pulse and the output broadened pulse which passed through Sapphire crystal of 2.3 mm in length.

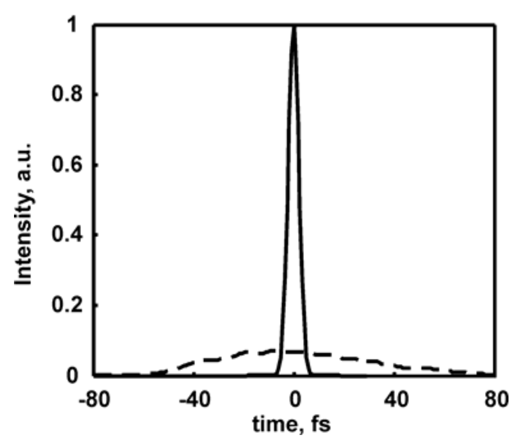

Fig. 2. Transform-limited pulse and broadened pulse. The solid line shows transform-limited pulse, FWHM 5 fs. The dashed line shows chirped pulse, FWHM 75 fs.

Next we consider a multilayer microstructure consisting of dielectric layer pairs with refractive indices n_h, n_l. Refractive-index profile is presented in Fig. 3.

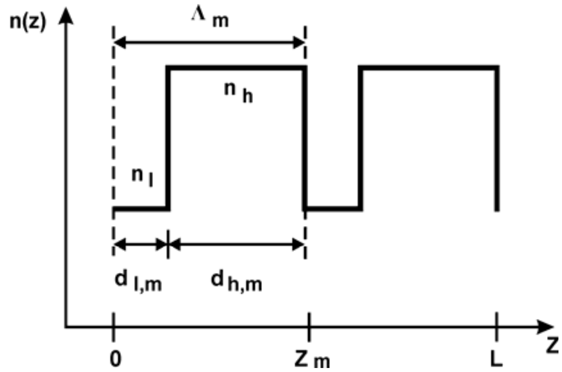

Fig. 3. Refractive-index profile of a chirped mirror.

Here the continuous variable z specifies the location in the structure and m is a discrete variable which counts the unit cells along the z-axis.

The unit cell is defined as combination of low-index layer and high-index layer. So the period of index profile is given as $\Lambda_m = d_{h,m} + d_{l,m}$, where $d_{h,m}$ and $d_{l,m}$ is the thickness of high- and low index layer, respectively. The Bragg wavelength is defined for a separate unit cell as:

$$\lambda_{Bm} = 2 \cdot (n_h d_{h,m} + n_l d_{l,m}) \quad (8)$$

We assume the layers are free of absorption that is justified for dielectric materials and light is normally incident on the layers. Spectral response of such structure is calculated by transfer matrix method [9, 10]. In contrast to the conventional quarterwave structure chirped mirror has the Bragg wavelength chirping given by (8) or Bragg wavenumber chirping defined as:

$$k_{Bm} = 2\pi / \lambda_{Bm} \quad (9)$$

The latter is more suitable owing to linear chirp produced in dispersion medium relates to the frequency. Thus, we use the linearly chirped Bragg wavenumber. Corresponding Bragg wavelength variation is shown in Fig. 4 over 650-1050 nm wavelength range, that coincides with the wavelength range of Ti:Sapphire fluorescence (incident pulse spectrum). Variation of layer thicknesses over 20 layer pairs is shown in Fig. 5.

The chirped mirror with the Bragg wavelength variation shown in Fig. 4 provides the deeper penetration for longer wavelengths resulting in the stronger delay of the longer wavelengths with respect to shorter ones. We use such structure as a benchmark one because of its simplicity and general essence. It allows us also to obtain mostly general results which will be representative for all CMs.

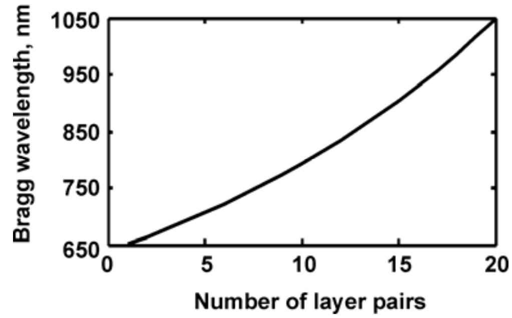

Fig. 4. The Bragg wavelength is varying along the structure from 650 until 1050 nm.

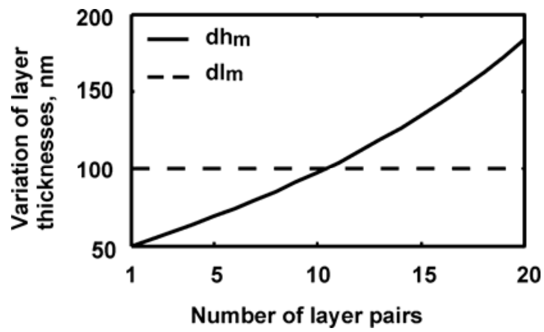

Fig. 5. The thicknesses of high-index layers (dashed line) and thicknesses of low-index layers (solid line) relates to Bragg wavelength shown in Fig. 4.

In Fig. 6 reflectivity and group delay for the chirped mirror are shown. Refractive indexes of layer materials are $n_h = 2.5$ (TiO_2), $n_l = 1.5$ (SiO_2).The number of layer pairs is 20.

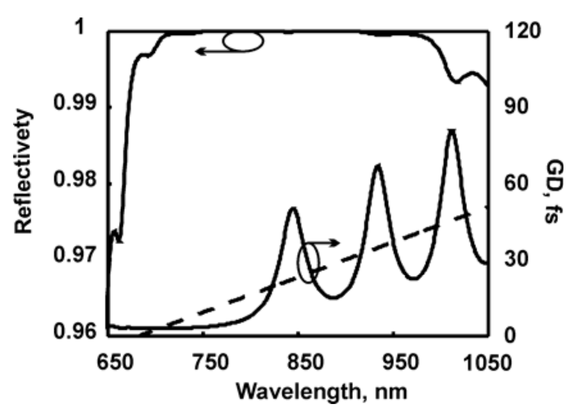

Fig. 6. Reflectivity and group delay for a chirped mirror which is chirped as it is shown in Fig.4. The solid lines show the results for the calculated chirped mirror and the dashed line represents the desired GD curve.

One can see the strong oscillations of GD in Fig. 6. This effect is the reason of pulse distortion after reflection and so it prevents to maximal broadened pulse compression. The goal of design techniques is to obtain as smooth GD curve as possible. The desired GD curve is shown in Fig. 6 as the dashed line. Positive slope of GD curve reveals the negative GDD. The GDD value at 800 nm is $-31\,\mathrm{fs}^2$.

Having a broadened pulse spectrum and a spectral response of chirped mirror we find the pulse reflected from CM:

$$E_{ref}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_{pas}(\omega) r_M(\omega)\exp(i\omega t)d\omega \qquad (10)$$

where F_{pas} - spectrum of pulse passed through dispersive medium (pulse incident on CM), r_M - spectral response of CM.

Fig. 7 shows an incident broadened pulse and the pulse reflected from the chirped mirror with characteristics shown in Fig. 6. From Fig. 7 we can see the reflected pulse has been compressed but it is far from initial transform-limited pulse as shown in Fig. 2. There are appreciable distortions of the pulse shape. It is obvious that negative GDD produced in the given chirped mirror isn't enough for full dispersion compensation. So it is necessary to use chirped mirror to produce stronger negative dispersion in the case of given dispersive medium.

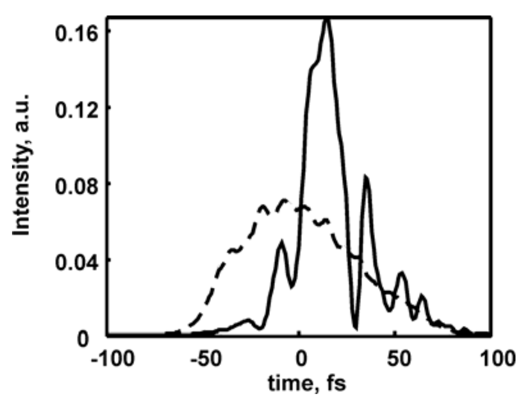

Fig. 7. An incident broadened pulse and compressed reflected pulse. The number of layer pairs is 20. The dashed line shows an incidence broadened pulse, FWHM 75 fs. The solid line shows the reflected compressed pulse, FWHM 22 fs.

3. Results

The easiest way to strengthen negative dispersion is to increase the number of layer pairs. Fig. 8 shows temporal profiles for the incident broadened pulse and the pulse reflected from the chirped mirror with extended number of layers. The Bragg wavenumber is also linearly chirped and thus the corresponding Bragg wavelength curve looks like that presented in Fig.4, but until 39 number of layer pairs. Thus GDD value at 800 nm is $-67\,\mathrm{fs}^2$.

From Fig. 8 we can see the reflected pulse is compressed stronger than that one shown in Fig. 7. The pulse shape distortions are weaker. However it is still not fully compressed. This is evident from Fig. 9, where the time dependence of instantaneous frequency for the input transform-limited pulse, broadened pulse and compressed pulse are shown. The instantaneous frequency defines as temporal derivative of the pulses phase. It is useful for illustration of the chirp magnitude. We see that the slope of the compressed pulse is less relative to that of broadened pulse, i.e. the chirp is weaker.

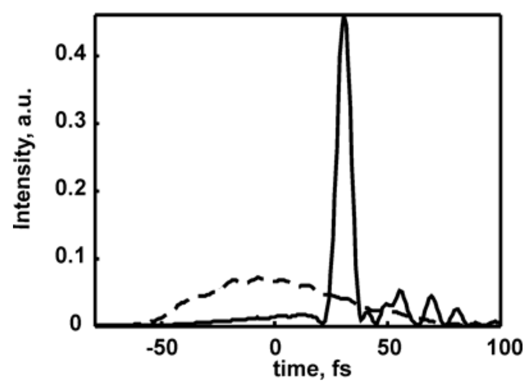

Fig. 8. An incident broadened pulse and compressed reflected pulse. The number of layer pairs is 39. The dashed line shows incident broadened pulse, FWHM 75 fs. The solid line shows reflected compressed pulse, FWHM 8.5 fs.

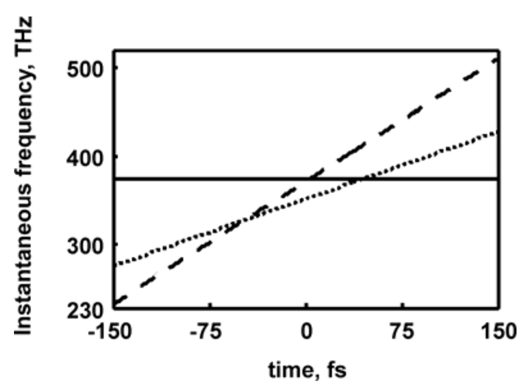

Fig. 9. The instantaneous frequency. The solid line shows the instantaneous frequency of transform-limited pulse, no chirp. The dashed line shows the instantaneous frequency of the broadened pulse, strong chirp (up-chirp). The dotted line shows instantaneous frequency of the compressed pulse.

Fig. 10 shows dependences of GDD on the number of structure layer pairs. Two cases of GDD are shown - for the structure having smooth phase properties and for CM structure with GD oscillations. In the last case GDD curve demonstrates an oscillate behavior.

One would expect the fully compressed pulse in the case of full dispersion compensation, i.e. the chirped mirror dispersion will be totally opposite to media dispersion. From Fig. 10 we can see that the negative dispersion value produced in the chirped mirror structure is close to the value of the positive dispersion in the medium ($133\,\text{fs}^2$) in region around 40-50 number of layer pairs (domain of the full dispersion compensation). Less number of layer pairs doesn't provide the required value of GDD (domain of incomplete dispersion compensation). And more number of layer pairs provides the excessive value of negative GDD (domain of overcompensation). To understand how various domains do impact on pulse behavior we calculate the dependence of the reflected pulse FWHM on the number of structure layer pairs. In Fig. 11 FWHM dependence for structure with GD oscillations and for that one with desired smooth phase properties are shown.

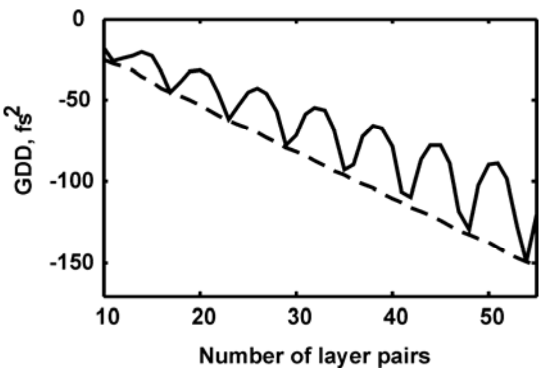

Fig. 10. The dependence of GDD value at 800 nm on the number of layer pairs. The solid line represents chirped mirror structure with GD oscillations. The dashed line represents the chirped mirror structure with desired phase properties.

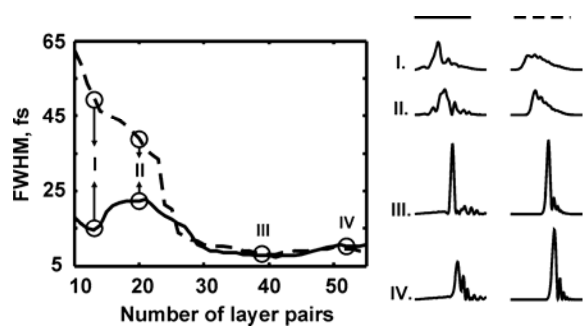

Fig. 11. Dependence of the reflected pulse FWHM on the number of layer pairs. The solid line represents CM with GD oscillations. The dashed line represents CM with desired smooth phase properties. The temporal pulse profiles reflected from CM with GD oscillations (first column) and without it (second column) are shown from the right.

From Fig. 11 we can see minima of FWHM for both curves close to 40 number of the layer pairs. This minima is nearly coincides with region of full dispersion compensation which is found in Fig. 10. Therefore, such a number of layer pairs provide the best reflected pulse as shown in Fig. 11 from the right. The third pulse profile has the lowest FWHM and distortions as well as the highest amplitude. The behavior of dashed line is quite smooth in the whole range of layer pairs. And this reveals a gradual pulse compression process. FWHM of the reflected pulse continually decreases to the minima around 40 layer pairs. At the same time the solid line diverges from the dashed line and doesn't demonstrate similar gradual pulse compression in the domain of incomplete dispersion compensation. This is due to GD oscillations which prevent continuous chirp compensation process, i.e. continuous decreasing of the dotted line slope in Fig. 9. That is why there are strong pulse distortions in this domain and small pulse amplitude as we can see from pulses profiles shown in Fig. 11 on the right. Due to strong pulse distortions we can not consider pulse to be well compressed as it would be expected from the FWHM value only. Finally, both curves reveal that the reflected pulse has worse parameters than into domain of full dispersion compensation.

After 50 layer pairs FWHM of the reflected pulse increases again and pulse distortion also becomes stronger due to the excessive value of negative GDD. This behavior sketches domain of overcompensation (see Fig. 10). Excessive negative dispersion acts in this case as the uncompressed positive medium dispersion and pulse acquires a down-chirp. This situation would be corresponding to the negative slope of the dotted line in Fig. 9. Thus, in this domain the reflected pulse has also worse parameters than those in the domain of the full dispersion compensation for both curves with GD oscillations and with desired phase properties.

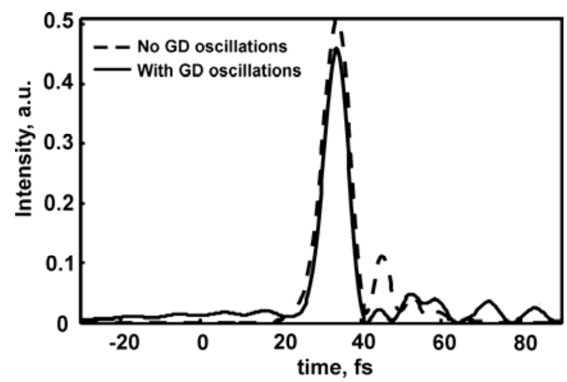

Fig. 12. The reflected pulses in the case of oscillations GD and for the desired smooth phase properties. The number of structure layer pairs is 42. The solid line shows reflected pulse in the case of GD oscillations, FWHM 7.4 fs, time-bandwidth product 0.64. The dashed line shows the reflected pulse in the case of the desired phase properties, FWHM 8.2 fs, time-bandwidth product 0.71.

It is interesting to compare the temporal pulse profile in the case of GD oscillations with that one when the phase curve is smooth. We have calculated both profiles for 42 layer pairs. This corresponds to the global minimum of the both curves in Fig. 11. Pulse profiles are presented in Fig. 12.

From Fig. 12 we see there is a pedestal on the leading front of the pulse in the case of GD oscillations. In addition to that, the pulse magnitude is smaller in that case. The reason lies in the parasitic interference of the long wavelength components with the rest part of the pulse spectrum. Longer wavelengths pass through the first section of the mirror structure, which acts as a transmission grating for these wavelengths. However, due to weak Fresnel reflection at the interface between the first layer and ambient medium some longer wavelength of the pulse spectrum reflects from the first section of the mirror instead of the back one. And therefore they are concentrated in the leading edge of the pulse resulting in the appearance of the pedestal. Previously this effect has been discussed by many authors [11] and similarity to Gires–Tournois interferometer was identified. On the contrary, the pedestal is absent in the case if there is no GD oscillations.

Thus, applying the chirped mirror with desired smooth phase properties allows to eliminate a long wavelength raising part and to increase the peak amplitude. However, as it follows from Fig. 12 it is insufficient for stronger pulse compression. In addition the reflected pulse has the tail in the trailing edge. In order to analyze this distortion we modify (10) as follows:

$$E_{ref}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_{pas}(\omega)\exp(-i\phi(\omega))\exp(i\omega t)d\omega \qquad (11)$$

where the phase shift $\phi(\omega)$ is expanded in a series around the central frequency of the pulse ω_0:

$$\phi(\omega) = \phi(\omega_0) + D_1\cdot(\omega-\omega_0) + \sum_{n=2}^{\infty}\frac{1}{n!}D_n(\omega-\omega_0)^n \qquad (12)$$

with dispersion coefficients:

$$D_n = \left.\frac{d^n\phi}{d\omega^n}\right|_{\omega=\omega_0} \qquad (13)$$

D_1, D_2, D_3 defines the value of GD, GDD, and third-order dispersion (TOD) at ω_0 respectively. To set a phase shift (22) we used dispersion coefficients with the opposite sign calculated for the media as described in section 2.1.
The temporal pulse profiles calculated using (11) are shown in Fig. 13. Pulses with different values of TOD (D_3) are presented. They are compared to the pulse reflected from the structure with smooth GDD curve presented in Fig. 12.

As it follows from Fig. 13 the tail in the trailing edge are due to the uncompensated TOD. Thus in order to obtain the fullest dispersion compensation it is necessary to compensate not only GDD but TOD as well. It means that GDD characteristics of the chirped mirror should have the required curve slope.

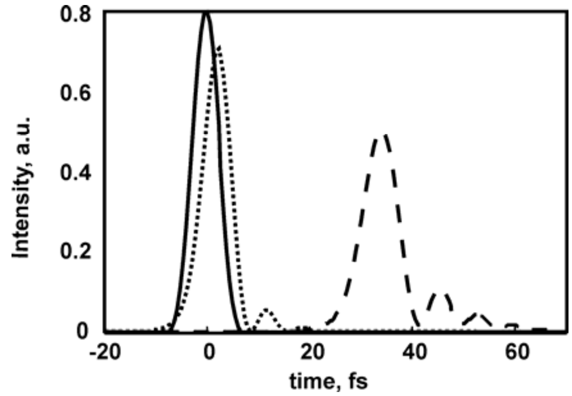

Fig. 13. The reflected pulses for desired smooth phase properties and in case of phase shift as series (12). The dashed line shows the reflected pulse in the case of the desired phase properties, FWHM 8.2 fs, time-bandwidth product 0.71. The dotted line shows the reflected pulse in the case of the formed phase shift as series ($D_1 = -13.6$ ps, $D_2 = -133$ fs^2, $D_3 = 0$), FWHM 7.4 fs, time-bandwidth product 0.64. The solid line shows the reflected pulse in the case of the formed phase shift as series ($D_1 = -13.6$ ps, $D_2 = -133$ fs^2, $D_3 = -96$ fs^3), FWHM 6.6 fs, time-bandwidth product 0.57.

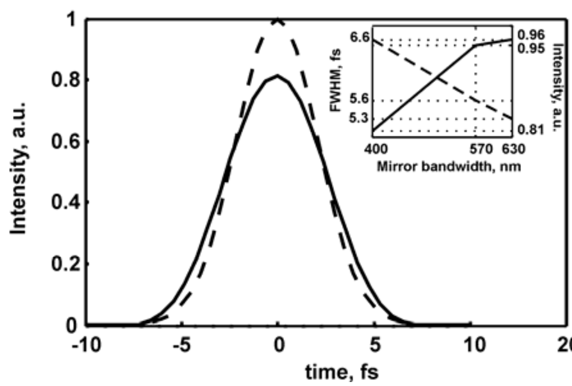

Fig. 14. The initial transform-limited pulse and reflected pulse cleaned from profile drawbacks. The dashed line shows initial transform-limited pulse FWHM 5 fs, time-bandwidth product 0.44. The solid line shows the reflected pulse as that one presented in Fig. 13 by the solid line, FWHM 6.6 fs, time-bandwidth product 0.57. Inset: FWHM of the "clear" reflected pulse (dashed line) and its peak intensity (solid line) depending on the mirror bandwidth.

By comparing the "clear" reflected and initial transform-limited pulses on Fig. 14 we see that the reflected pulse is still different from the initial transform-limited one even when there are no distortions. The found reason is that usually the utilized mirror bandwidth 650-

1050 nm is actually narrower than the pulse spectrum. We can compensate this effect using a wider bandwidth of the mirror as it is shown in the inset of Fig.14.

4. Conclusions

We developed a model for interaction of an ultrashort optical pulse with the chirped mirror via exact analysis of the pulse waveform. The model includes propagation effects of the pulse in dispersive medium; the spectral response of the chirped mirror; the interaction of the pulse with the mirror.

Through the analysis of the pulse waveform we found a range of layer numbers in CM, which provide optimal pulse compression, i.e. the balance between the maximal pulse compression and minimal waveform distortion for the given incident chirped pulse. In the case of our benchmark structure and Sapphire crystal 2.3 mm long the best pulse compression is shown to be around 40 number of layer pairs. Less number of layers produces much wider pulses or causes strong distortions resulted from GD oscillations. The same holds for the case of a larger number of layers as compared to the optimal one.

The waveform analysis has also shown exactly that the pedestal on the leading edge of the pulse is due to group delay oscillations. Distortions on the trailing edge of the pulse are caused by uncompensated third-order dispersion in the mirror. Therefore, taking into account these facts in the chirped mirror design will prevent those distortions of the pulse.

We found also that one should use the chirped mirror with expanded bandwidth in order to reach the theoretical limit for magnitude of the reflected pulse. In the case of 5 fs pulse this bandwidth should be as large as 630 nm, instead of 400 nm as it is usually used in experiments and calculations.

References

[1] R. Szipöcs, K. Ferencz, C. Spielmann, F. Krausz Opt. Lett. **19**, 201 (1994).

[2] R. Szipöcs, A. Kohazi-Kis Appl. Phys. B **65**, 115 (1997).

[3] S. De Silvestri, P. Laporta O. Svelto IEEE Journal of Quantum Electronics **20**, 533 (1984).

[4] C. Spielmann, P. F. Curley, T. Brabec, F Krausz Ultrabroadband femtosecond lasers, IEEE Journal of Quantum Electronics **30**, 1100 (1994).

[5] G. Tempea, F. Krausz, C. Spielmann, K. Ferencz Dispersion control over 150 THz with chirped dielectric mirrors, IEEE Journal of Selected Topics in Quantum Electronics 4 193, 1998.

[6] N. Matuschek, L. Gallmann, D. H. Sutter, G. Steinmeyer, U. Keller, Appl. Phys. B **71**, 509 (2000).

[7] S. O. Y akushev, I. A. Sukhoivanov, O. V. Shulika, V. V. Lysak, S. I. Petrov, Simulation of interaction of the femtosecond laser pulse with chirped mirror, Numerical Simulation of Optoelectronic Devices, Nanyang Technological University, Singapore, 11 – 14 September 2006, Paper WB5, 2006a

[8] S. O. Yakushev, O. V. Shulika, V. V. Lysak, S. I. Petrov, I. A. Sukhoivanov, Improvement of Chirped Mirror Design for Femtosecond Pulse Compression, Frontiers in Optics, 2006 October 8-12, 2006, Rochester, New York, USA, Pres. Number JWD38, 2006b.

[9] N. Matuschek, F. X. Kaertner, U. Keller Exact coupled-mode theories for multilayer interference coatings with arbitrary strong index modulations, IEEE J. Quantum Electron 33 295, 1997.

[10] M. Bass, HANDBOOK OF OPTICS: Volume I Fundamentals, Techniques, and Design. 2nd ed., McGRAW-HILL, INC., 1995.

[11] Matuschek N., F. X. Kaertner, U. Keller Theory of double-chirping mirrors, IEEE Journal of Selected Topics in Quantum Electronics 4 197, 1998.

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