

# Modelling of the magnetic properties of amorphous soft magnetic materials for sensor applications

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Paper presents results of the modelling of the magnetic B(H) characteristics of amorphous magnetic materials with the following compositions in the as-quenched state:  $\text{Co}_{68}\text{Fe}_4\text{B}_{13.5}\text{Mo}_{1.5}$ ,  $\text{Fe}_{78}\text{Si}_{13}\text{B}_9$ ,  $\text{Fe}_{40}\text{Ni}_{38}\text{Mo}_4\text{B}_{18}$  and  $\text{Fe}_{73.5}\text{Nb}_3\text{Cu}_1\text{Si}_{13.5}\text{B}_9$ . On the base of experimental B(H) characteristics values of Jiles-Atherton-Sablik model's parameters were calculated. For this calculation evolutionary strategies together with gradient Hook-Jevis optimisation were applied. Presented results indicates the good agreement between experimental results and Jiles-Atherton-Sablik model. As a result this model can be utilized for modelling of magnetic characteristics of the soft amorphous magnetic materials for sensor applications as well as for quantitative determination of physical material parameters (such as anisotropy energy density) on the base of the magnetic hysteresis loop.

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## 1. Introduction

Magnetic alloys with amorphous structure are produced by rapid quenching techniques. Due to its extraordinary magnetic properties such as high permeability and low coercive force as well as high mechanical strength, these alloys are very promising materials for cores of magnetic [1] and magnetoelastic sensors [2, 3]. Especially in the case of magnetoelastic sensors, application of the soft amorphous alloys gives possibility of technological break-through in both compressive stress as well as torque measurements.

In spite of very significant possibilities of application of the amorphous magnetic materials, the physical description of the process of their magnetization seems to be still rather qualitative than quantitative. The model which gives possibilities to overcome this problem is Jiles-Atherton-Sablik (J-A-S) model [4]. This model creates not only the possibility of modelling of magnetic characteristics for technical applications, but also creates new chances for understanding of magnetization process of amorphous magnetic materials.

## 2. Energy-based model of magnetization

Jiles-Atherton-Sablik model of magnetization process [4] was developed for isotropic materials such as soft ferrites. On the other hand amorphous alloys, especially in the as-quenched state, exhibit significant anisotropic properties. This stress induced anisotropy is generated during rapid quenching process. To take into account anisotropic properties of the materials special extension of J-A-S model should be used [5]. In extended model anhysteretic magnetization  $M_{an}$  is calculated as a weighted sum of anisotropic magnetization  $M_{aniso}$  and isotropic magnetization  $M_{iso}$ :

$$M_{an} = t \cdot M_{aniso} + (1-t) \cdot M_{iso} \quad (1)$$

where  $t$  is the weight coefficient describing participation of anisotropic phase in the material [6]. Isotropic magnetization  $M_{iso}$  is given by the Langevin equation [4]:

$$M_{iso} = M_s \left[ \coth\left(\frac{H_{eff}}{a}\right) - \left(\frac{a}{H_{eff}}\right) \right] \quad (2)$$

where  $M_s$  is saturation magnetization,  $a$  quantifies domain walls density and  $H_{eff}=H+\alpha M$  is effective magnetizing field, where  $\alpha$  is representing inter domain coupling.

Anisotropic magnetization  $M_{aniso}$  is given by following equation [5]:

$$M_{aniso} = M_s \frac{\int_0^\pi e^{E(1)+E(2)} \sin \theta \cdot \cos \theta \cdot d\theta}{\int_0^\pi e^{E(1)+E(2)} \sin \theta \cdot d\theta} \quad (3)$$

where  $\theta$  is the integration parameter and  $E(i)$  is given by following dependence [5]:

$$E(i) = \frac{H_{eff}}{a} \cos \theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a} \sin^2 \phi_i \quad (4)$$

where  $K_{an}$  is anisotropic energy density, and  $\phi_1=\psi-\theta$  as well as  $\phi_2=\psi+\theta$ , where  $\psi$  is an angle between the easy axis of the material and the magnetizing field direction.

In J-A-S model the irreversible magnetization  $M_{irr}$  can be calculated from the following equation [4]:

$$\frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} \frac{dM_{irr}}{dM} \quad (5)$$

where parameter  $\delta = +1$  for  $\frac{dH}{dt} \geq 0$ ,  $\delta = -1$  for  $\frac{dH}{dt} < 0$  and  $k$  quantifies average energy required to break pinning site. Reversible magnetization  $M_{rev}$  can be calculated from equation [4]:

$$M_{rev} = c \cdot (M_{an} - M_{irr}) \quad (6)$$

where  $c$  describes magnetization reversibility. Total magnetization  $M$  is given as the sum of reversible magnetization  $M_{rev}$  and irreversible magnetization  $M_{irr}$ . Finally the flux density  $B$  in the material is equal  $M \cdot \mu_0$ .

Fig. 2 presents the influence of the anisotropy  $K_{an}$  on the shape of the hysteresis loop generated according to the J-A-S model. Calculations were made for following model's parameters:  $a = 19$  A/m,  $k = 2.5$  A/m,  $c = 0.68$ ,  $M_s = 3.24 \times 10^5$  A/m,  $\alpha = 4.8 \times 10^{-5}$ . The results of the modelling are in a good, qualitative agreement with experimental data achieved, when anisotropy was induced during the field annealing of the soft amorphous alloys [7]. Increasing of the anisotropy in the direction of the magnetizing field (as presented in Fig. 1b) causes squareness of the hysteresis loop. On the other hand anisotropy perpendicular to the magnetizing field (Fig. 1c) causes significant flattening of the hysteresis loop.

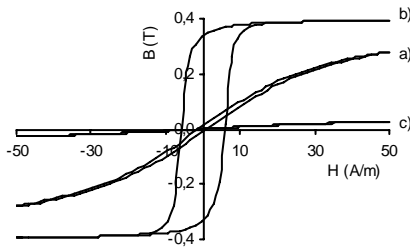


Fig. 1. The influence of the anisotropy  $K_{an}$  on the shape of the hysteresis loop calculated according to J-A-S model: a) isotropic model ( $K_{an} = 0$ ), b) anisotropy parallel to the magnetizing field ( $K_{an} = 2660$  J/m<sup>3</sup>,  $t = 0.9$ ,  $\psi = 0$ ), c) anisotropy perpendicular to the magnetizing field ( $K_{an} = 2660$  J/m<sup>3</sup>,  $t = 0.9$ ,  $\psi = \pi/2$ ).

### 3. Method of investigation

The experimental results are based on the measurements of quasi-static, magnetic hysteresis loops  $B(H)$  of four amorphous alloys compositions:  $\text{Fe}_{78}\text{Si}_{13}\text{B}_9$ ,  $\text{Fe}_{73.5}\text{Nb}_3\text{Cu}_1\text{Si}_{13.5}\text{B}_9$ ,  $\text{Fe}_{40}\text{Ni}_{38}\text{Mo}_4\text{B}_{18}$  and  $\text{Co}_{68}\text{Fe}_4\text{B}_{13.5}\text{Mo}_{1.5}$ . Investigated materials were in as-quenched state. The measurements of  $B(H)$  characteristics were carried out on HBPL hysteresograph in the room temperature.

On the base of experimental  $B(H)$  hysteresis loops values, of Jiles-Atherton-Sablik model's parameters were calculated during the optimization process. The target function was given by the following equation:

$$F = \sum_{i=1}^n (B_{J-A-S}(H_i) - B_{meas}(H_i))^2 \quad (7)$$

where  $B_{J-A-S}$  were results of the modelling, and  $B_{meas}$  were results of the experimental measurements. For minimisation of the target function the evolutionary strategies [8] together with gradient Hook-Jevis optimization were applied. Finally Pearson correlation coefficient  $r^2$  was calculated between the model  $B_{J-A-S}$  and experimental  $B_{meas}$  data. Pearson  $r^2$  coefficient describes the ratio of the experimental results variation which is described by the model. As a result this coefficient can be utilized to describe the quality of the modelling results.

### 4. Results

In Fig. 1 the results of modelling are presented, whereas Table 1 summarizes the values of J-A-S model's parameters for all four amorphous alloys compositions.

Table 1. Parameters of J-A-S model for different amorphous alloys compositions in as-quenched state.

		$\text{Fe}_{78}\text{B}_{13}\text{Si}_9$	$\text{Fe}_{73.5}\text{Si}_{13.5}\text{Nb}_3\text{Cu}_1\text{B}_9$	$\text{Fe}_{40}\text{Ni}_{38}\text{Mo}_4\text{B}_{18}$	$\text{Co}_{68}\text{Fe}_4\text{B}_{13.5}\text{Si}_{13.5}\text{Mo}_{1.5}$
a	A/m	19.3	54.8	21.4	37.0
k	A/m	2.50	5.93	2.45	4.97
c		0.68	0.46	0.59	0.62
$M_s$	A/m	$3.23 \times 10^5$	$3.67 \times 10^5$	$2.73 \times 10^5$	$2.99 \times 10^5$
$\alpha$		$4.80 \times 10^{-5}$	$2.82 \times 10^{-4}$	$9.69 \times 10^{-5}$	$4.43 \times 10^{-4}$
$K_{an}$	J/m <sup>3</sup>	2580	4716	2980	3076
t		0.54	0.24	0.50	0.001
$r^2$	%	99.8	99.9	99.9	99.8

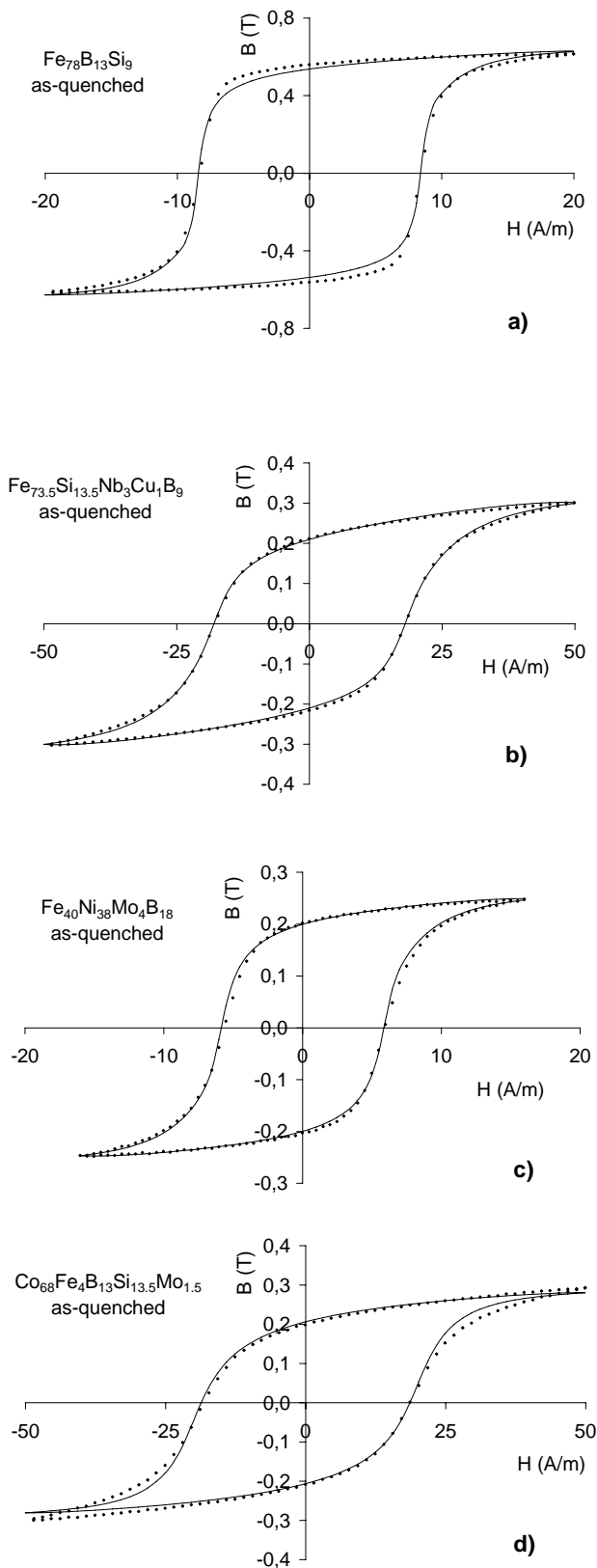


Fig. 2. The results of the modelling of the  $B(H)$  hysteresis loop (dots "•" experimental data, solid line "—" results of the simulation) for different alloys compositions in as-quenched state: a)  $Fe_{78}Si_{13}B_9$ , b)  $Fe_{73.5}Nb_3Cu_1Si_{13.5}B_9$ , c)  $Fe_{40}Ni_{38}Mo_4B_{18}$ , d)  $Co_{68}Fe_4B_{13.5}Mo_{1.5}$ .

It should be indicated, that for all four alloys compositions very good agreement between experimental data and results of the modelling was achieved. For all alloys compositions Pearson  $r^2$  coefficient exceeds 99.8%. Such good agreement between experimental data and model create possibilities of technical application of J-A-S model for modelling of the functional properties of the cores of magnetic sensors and other inductive components.

On the other hand values of saturation magnetization  $M_s$  achieved for all alloys compositions seems to be significantly lower than experimental values, determined for investigated amorphous alloys subjected to high magnetizing fields [9]. This phenomenon is probably caused by the fact, that in the J-A-S model, saturation magnetization  $M_s$  determines the saturation during the domain wall movement, whereas experimental saturation for high magnetizing field is connected with domain walls rotation.

It should be highlighted, that for  $Co_{68}Fe_4B_{13.5}Mo_{1.5}$  alloy composition, information about very low participation of anisotropic phase ( $t \approx 0$ ) was achieved from simulation. This result can be explained by the fact, that  $Co_{68}Fe_4B_{13.5}Mo_{1.5}$  alloy is nearly zero magnetostrictive [10]. As a result it exhibit low stress sensitivity and there is no anisotropy  $K_{an}$  induced during the rapid quenching process. This result confirms, that J-A-S model gives not only the possibility of simulation of the magnetic  $B(H)$  hysteresis loops for technical purposes, but also creates novel possibilities of interpretation of experimental results. In this case, the lack of the anisotropy was determined quantitatively on the base of hysteresis loop.

## 5. Conclusions

Presented results indicates that Jiles-Atherton-Sablik model with anisotropic extension can be utilized for the modelling of the magnetic characteristics of soft amorphous magnetic materials for sensors applications. For all four alloys compositions, high values of Pearson  $r^2$  coefficient (over 99.8 %) were achieved between experimental data and the results of simulation.

Due to the fact, that in the J-A-S model, saturation magnetization  $M_s$  determines the saturation during the domain wall movement, whereas experimental saturation for high magnetic field is connected with domain walls rotation, values of saturation magnetization  $M_s$  achieved from the modelling are lower than experimental data presented in the literature. On the other hand results of modelling of  $B(H)$  characteristics for  $Co_{68}Fe_4B_{13.5}Mo_{1.5}$  alloy confirms it isotropic character. This results is in excellent agreement with information from the literature, about nearly-zero magnetostrictive, isotropic character of  $Co_{68}Fe_4B_{13.5}Mo_{1.5}$  alloy. Moreover it confirms the possibility of application of the Jiles-Atherton-Sablik model for the quantitative determination of the physical parameters of the materials (such as anisotropy energy density  $K_{an}$ ) on the base of magnetic hysteresis loop.

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