

Numerical modeling of coherent fluctuations of a laser beam using heat transfer in solids

M. OANE*, F. SCARLAT, I. N. MIHAILESCU

National Institute for Laser, Plasma and Radiation Physics, P.O. Box MG 36 Magurele, R-76900 Bucharest, Romania

The main goal of the present paper is to present a semi-classical approach regarding the quantum fluctuation in intensity of a laser beam. We suppose that we have a laser field and his fluctuations, which are suppose to be weak coherent electromagnetic fields at the same wave-length. We write down the heat equation [1-4] and analyze the temperature given by the "coupling" factor given by the interference between the two electromagnetic fields [4]. We consider a parallelepiped sample (semiconductor) with dimensions a, b, and c. The information given by this temperature gives us useful information about the fluctuations of the laser field (like for example the average number of photons produce by fluctuations). For solving the heat equation the integral transform technique was used. We also try to suggest in the present paper that with standard laser calorimetry it is possible to obtain useful information about laser intensity fluctuations.

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The problem of fluctuations of an electromagnetic field was for the first time studied in a coherent mode by Landau and Lifsit [5]. With the discovery of the laser the quantum noise theory and experiment has begun very important.

Let us suppose that we have a laser beam, with coherent quantum fluctuations (a weak electromagnetic field of the same wavelength).

If we use the standard notations from quantum electrodynamics, the weak coherent electromagnetic field can be write in the following form [6]:

$$|c\rangle = \exp(-\frac{1}{2}|c|^2) \sum_{n=0}^{\infty} \frac{c^n}{\sqrt{n!}} |c\rangle. \quad (1)$$

We have the following properties for this state: $\langle c|c\rangle = 1$, the mean number of photons:

$$\bar{N} = \langle c|N|c\rangle = |c|^2.$$

The expectation value of the electric field \vec{E} is given by:

$$\langle c|E|c\rangle = -\varepsilon_r(\vec{k}) \cdot 2 \cdot (\frac{\hbar\omega_k}{2V})^{1/2} |c| \sin(\vec{k} \cdot \vec{x} - \omega_k t + \delta) \quad (2)$$

From theory we know that the relative fluctuation in photon number is given by the formula $\frac{\Delta N}{N} = N^{-1/2}$.

When $\bar{N} \rightarrow \infty$ the fluctuations of $\Delta \vec{E}$ becomes negligible. The state $|c\rangle$ is called a coherent state and represents the closest quantum-mechanical approach to a classical electromagnetic field.

We will introduce this form of signal for $I \sim \langle \vec{E}^2 \rangle$ in our general theory, in order to obtain an analytical

expression for coupling between a strong field and a weak field.

We have for the weak coherent field:

$$\langle c|E|c\rangle = -\varepsilon_r(\vec{k}) \cdot 2 \cdot (\frac{\hbar\omega_k}{2V})^{1/2} \sqrt{N} \sin(\vec{k} \cdot \vec{x} - \omega_k t + \delta) \quad (3)$$

For the strong electromagnetic laser field (reference field) we have the following formula:

$$E_{mn} = -\varepsilon_r(\vec{k}) \cdot E_{0mn} [H_m(\frac{\sqrt{2} \cdot x}{w}) H_n(\frac{\sqrt{2} \cdot y}{w}) \exp[-(\frac{x^2+y^2}{w^2})]] \quad (4)$$

Here: $H_m(\frac{\sqrt{2} \cdot x}{w}), H_n(\frac{\sqrt{2} \cdot y}{w})$ are Hermite functions.

For simplicity we will take: $E_{mn} = E_{00}$ (Gaussian mode).

(5)

The different characteristics of dielectrics under one laser beam irradiation have been very well studied in literature. We will take the case of a ZnSe sample (all characteristics of the material can be found in reference 1).

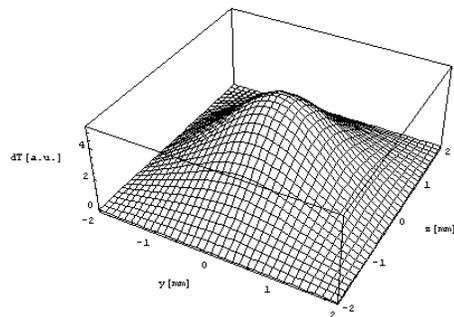


Fig. 1. The temperature field in the plane $x=0$, during a 100s irradiation with a 10 W CO_2 laser beam.

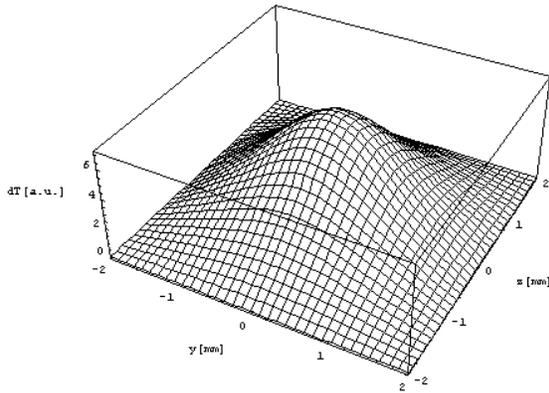


Fig. 2. The temperature field in the plane $x=0$, during a 100s irradiation with a 10 W CO₂ laser beam, plus the interference contribution.

We supposed that the sample is parallelepiped and the dimensions are: $a = 10$ mm, $b = 4$ mm and $c = 4$ mm. We present the thermal field at the surface of the sample, without (Fig. 1) and with (Fig. 2) fluctuations. We take the case of a cw CO₂ laser beam, with total power 10 W, operating in the TEM₀₀. In Fig. 2 is presented the temperature from Fig. 1 plus the interference temperature.

We have plotted the Figs. 1 and 2 using formulas developed in reference 4 (with the notations therein):

$$\Delta T_{\text{int}}(x, y, z, t) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{o=1}^{\infty} a_{\text{int}}(\alpha_i, \beta_j, \chi_o) b(\alpha_i, \beta_j, \chi_o, t) K_x(\alpha_i, x) K_y(\beta_j, y) K_z(\chi_o, z) \tag{6}$$

where:

$$a_{\text{int}}(\alpha_i, \beta_j, \chi_o) = \frac{\alpha^2 \sqrt{I_1 I_2} \cos \theta}{K C C_j C_o} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-\alpha x} K_x(\alpha_i, x) dx \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{c}{2}}^{\frac{c}{2}} e^{-2(\beta^2 + \chi^2) / w_m^2} K_y(\beta_j, y) K_z(\chi_o, z) dy dz \tag{7}$$

We can notice a difference of about 1 a.u., which is due to the small coherent electromagnetic field. If one combine formulas (4) and (5), we obtain (we take $\cos \theta = 1$):

$$I \sim \langle \bar{E}^2 \rangle = \langle (\bar{E}_1 + \Delta \bar{E})^2 \rangle = I_1 + I_2 + 2 \cdot E \cdot \Delta E = I_1 + I_2 + 2 \cdot E_{\text{mn}} \langle c | E | c \rangle \tag{8}$$

From (7) and (8) we obtain:

$$a_{\text{int}}(\alpha_i, \beta_j, \chi_o) = \frac{\alpha^2 \langle c | E | c \rangle E_{\text{mn}} \cos \theta}{K C C_j C_o} \int_{-\frac{a}{2}}^{\frac{a}{2}} e^{-\alpha x} K_x(\alpha_i, x) dx \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{c}{2}}^{\frac{c}{2}} e^{-2(\beta^2 + \chi^2) / w_m^2} K_y(\beta_j, y) K_z(\chi_o, z) dy dz \tag{9}$$

If we introduce in equation (9) equation (3) we can relate (via equation 6) the temperature with the number of photons: $\Delta T_{\text{int}} = \bar{N}^{1/2} \cdot f_{\text{system}}$ (10)

We obtain from above equation the mean number of photons given by the fluctuations of the laser beam:

$$\bar{N} = (\Delta T_{\text{int}} / f_{\text{system}})^2 \tag{11}$$

Here f_{system} depends only by the sample and the laser beam, and can be calculated easily with the formulas from this paper (one has to combine equations: (3), (9) and (6)).

Our model should be regarded like a semi-classical one, because we use classical electrodynamics formulas for the interference between classical field (laser-field) and his fluctuations (which are of quantum nature). Anyway our theory gives us a first approximation regarding the number of photons produced by laser intensity variations. The natural development of our model (from theoretical point of view), is to treat the laser field variation like a quantum perturbation [7,8].

The advantage of our model is that from practical point of view, we believe that the set-up of the experimental device can be easily done. Our computer simulations suggest that an extreme vacuum around the sample gives us better information regarding the temperature variation.

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*Corresponding author: mihai.oane@inflpr.ro