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On revisited models of L-H transition for tokamak plasmas

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Models corresponding to L/H transition in tokamak plasmas are revisited. Values for the thickness of the meso-phase and for the range of the control parameter in the bifurcation region are obtained. We shown that in the collisional case the double hysteresis is absent when the control parameter is positive and there are critical values of the effective frequency and electron diffusivity for the existence of a simple bifurcation. The influence of the impurity flux on the radial electric field bifurcation and the time behaviour of the later on the basis of a tangent hyperbolic time-dependent ion temperature were also studied.

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1. Introduction

Since the discovery of the H mode, the transition from a low confinement to an improved confinement has been observed in many fusion devices. The study of the mechanisms that can lead to a bifurcation in the radial electric field is very important in order to obtain information related to the transport properties of the plasma edge. For such transition, a critical value for the control parameters (electron temperature, electron density and their gradients) must exist. The transition is analyzed through the change of the radial electric field to a more negative value or to a more positive value of its gradient. Among all the mechanisms able to generate the radial electric field, the collisional bulk viscosity loss ions was considered important. The model of Ref. [1] was improved by introducing in the ions loss cone flux a coefficient linear in the normalized radial electric field. The competition between the ion loss cone flux, the collisional bulk viscosity loss flux and the anomalous bipolar loss was examined in details. It is shown that in a given collisional regime the double hysteresis, which is a characteristic feature of a bifurcation model, is not observed when the control parameter is positive.

2. The general model

The general equation for the radial electric field dynamics including an impurity flux is [2]:

$$\frac{\partial E_r}{\partial t} = \frac{e}{\varepsilon_0 \varepsilon_\perp}\left[\Gamma_{e-i}^{anom} - \Gamma_i^{lc} - \Gamma_i^{bv} - \Gamma_{imp} + \text{"more"}\right] \tag{1}$$

where “more” represent other fluxes that may contribute to the radial electric field dynamics. Here Γ_{e-i}^{anom} is the anomalous bipolar loss in the constant-*l* approximation. The latter consists in: (a) the gradient of the electric field $\nabla E_r = E_r/l$, where *l* is some scale length and (b) the radial electric field is zero in the core region, i.e. in a region $r \le (a - l)$, where *a* is the minor radius (see Refs. [1,3-5]). The L-H transition as a bifurcation in the radial electric field was obtained as a zero-net radial current condition applied to edge non-ambipolar flows [1]. We introduce the dimensionless radial electric field X as:

$$X = \frac{e\rho_\theta E_r}{kT_i}, \tag{2}$$

where T_i is the temperature of ions, $\rho_\theta = m_i v_{Ti} / eB_p$ is the poloidal ion gyro-radius and E_r is the radial electric field. The ion loss cone flux $\Gamma_i^{lc}(X)$ in the region $|a - r| \le \rho_\theta$ (a is the minor radius) can represent the generation of particles from Coulomb collisions that escape into the scrape-off-layer (SOL) where they are absorbed by the limiter and/or by the divertor plates that balances the loss of trapped particles at the edge. A possible expression for the ion loss cone flux, denoted $\Gamma_i^{lc(1)}(X)$ is defined as [1]:

$$\Gamma_i^{lc(1)}(X) = c_1(X)\exp(-X^2) \tag{3}$$

The alternative expression for $\Gamma_i^{lc}(X)$, denoted by $\Gamma_i^{lc(2)}(X)$ is defined as (see Refs. [4, 6]):

$$\Gamma_i^{lc(2)}(X) = \frac{n_i \nu_{ii} \varepsilon^{1/2} \rho_\theta \nu_{*i}}{\left(\nu_{*i} + X^4\right)^{1/2}} \exp\left[-\left(\nu_{*i} + X^4\right)^{1/2}\right] \equiv \frac{2kn_i \varepsilon^2 (qR)^{-1} \nu_{*i}^2}{eB_p \left(\nu_{*i} + X^4\right)^{1/2}} T_i \exp\left[-\left(\nu_{*i} + X^4\right)^{1/2}\right] \tag{4}$$

The anomalous bipolar loss electron flux $\Gamma_{e-i}^{anom}(X)$ is due to a direct edge loss by turbulent diffusion and its expression is given as:

$$\Gamma_{e-i}^{anom}(X) = c_2 (\lambda - X), \quad c_2 = \frac{D_e n_e T_i}{\rho_\theta T_e} = \frac{D_e n_e}{T_e}\left(\frac{T_i}{2km_i}\right)^{1/2} \equiv D_{e-i}^{anom} \qquad (5)$$

In Eq. (5) D_e is the bipolar part of the electron effective diffusivity. The expression $c_1(X)$ is defined as:

$$c_1(X) = \varepsilon^{-1/2} n_i \nu_{ii} \rho_\theta F(X) \equiv \frac{2kn_i\varepsilon(qR)^{-1}\nu_{*i}}{eB_p} T_i F(X) \qquad (6)$$

F (X) is a function to be defined below, n_i represents the ion density while the parameter λ, which is related to the thermodynamic forces, is the control parameter, defined:

$$\lambda = -\rho_\theta \frac{T_e}{T_i}\left(\frac{n_e'}{n_e} + \alpha \frac{T_e'}{T_e}\right) \qquad (7)$$

In Eq. (7) prime denotes the radial gradient and α is a parameter close to unity. The flux $\Gamma_i^{bv}(X)$, which is the ion bulk viscosity loss, is obtained from the general neoclassical formula for the radial flux [2].
Particular expressions of $\Gamma_i^{bv}(X)$ and of the impurity flux $\Gamma_{imp}(X)$ will be defined later in specific subsections.

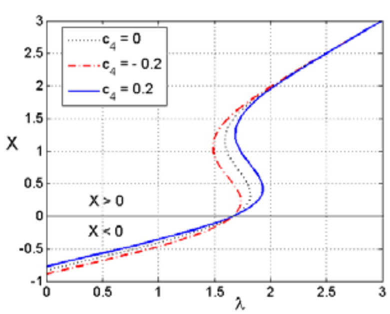

Fig. 1. The normalized electric field as function of the control parameter for different values of c_4 and for fixed value of $c_2 = 0.6$ and $c_3 = 1$.

3. The contribution of $\Gamma_i^{lc(1)}(X)$ and $\Gamma_{e-i}^{anom}(X)$

We analyze the stationarity condition $\left(\frac{\partial E_r}{\partial t} = 0\right)$, considering only the first two terms from Eq. (1) where $\Gamma_i^{lc(1)}(X)$ is given by Eq. (3). The radial electric field then results from the zero-net-current condition:

$$\Gamma_i^{lc(1)}(X) = \Gamma_{e-i}^{anom}(X) \equiv \gamma(X) \; . \qquad (8)$$

In the later equation $\gamma(X)$ is the dimensionless particle flux. We assume that that coefficient $c_1(X) = c_3 + c_4 X$ (or that $F(X)$ arising from a bounce averaging) is linear in the dimensionless radial electric field. Additional constraints $c_3 > 0$ (to insure an inward flux when $X = 0$) and $(c_4 / c_3) X \le 1$ when both c_4 and X are negative (to insure the inward flux for all values of X) are introduced.

The case $c_4 = 0$ was considered in [1]. The bifurcation equation is studied either by varying λ with c_2 / c_3 fixed or conversely. Obviously, no bifurcation appears when λ is kept fixed and $c_4 = 0$. The equation leads to a bifurcation only when λ is varied as considered in [1]. The behaviours of the normalized radial electric field and particle flux as functions of λ are represented in Fig. 1 and Fig. 2 respectively, for different values of c_4 and for fixed value of $c_2 = 0.6$ and $c_3 = 1$. The case $c_4 = 0$ corresponds to the model of Ref. [1] (dotted line) for which the meso-phase (see Ref. [5]) is $(\Delta\lambda)_{original} = 0.229$ with $\lambda \in [1.595, 1.824]$.

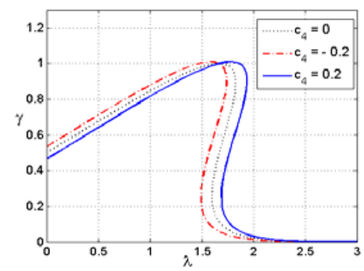

Fig. 2. The normalized particle flux as function of the control parameter for different values of c_4 and for fixed value of $c_2 = 0.6$ and $c_3 = 1$.

For $c_4 = 0.2$ the later is $(\Delta\lambda)_{linear} = 0.247$ with $\lambda \in [1.688, 1.935]$ and for $c_4 = -0.2$ is $(\Delta\lambda)_{linear} = 0.249$ with $\lambda \in [1.488, 1.737]$. A difference in the thickness of the meso-phase in the range of the control parameter is clearly observed as the coefficient c_1 is slightly varied with the radial electric field.

4. The contribution of $\Gamma_i^{lc(2)}(X)$, $\Gamma_i^{bv}(X)$ and $\Gamma_{e-i}^{anom}(X)$

We now consider a different situation involving three mechanisms that are able to cause a bifurcation in the electric radial field: the ion collisional bulk viscosity flux $\Gamma_i^{bv}(X)$ [7], the ion loss cone flux in the form given by Eq. (4) and the anomalous bipolar loss $\Gamma_{e-i}^{anom}(X)$ given in Eq. (5). We consider the edge of the circular tokamak and a positive single-ion-species as component of the main plasma. In the collisional limit $(\nu_{*i} \ge 1)$, the ion bulk viscosity flux has the following expression [2]:

$$\Gamma_i^{bv}(X) = \frac{\varepsilon^{7/2} k n_i T_i}{reB} \frac{\nu_{*i}(X + X_0)}{X^2 + \nu_{*i}^2 \varepsilon^3} \qquad (9)$$

In Eq. (9) $X = -X_0$ is the ambipolar electric field, which gives a zero value to the radial ion particle flux. The following equation is obtained using the assumptions $n_i = n_e \equiv n$ and $T_i = T_e \equiv T$:

$$\frac{2\nu_{*i}^2}{\left(\nu_{*i} + X^4\right)^{1/2}} \exp\left[-\left(\nu_{*i} + X^4\right)^{1/2}\right] + \frac{\varepsilon^{3/2}\nu_{*i}(X + X_0)}{X^2 + \nu_{*i}^2\varepsilon^3} = 2d(\lambda - X) \equiv 2\gamma(X) \qquad (10)$$

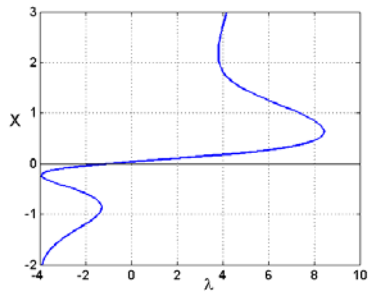

Fig. 3. The normalized radial electric field X *is represented as a function of the control parameter* λ *for:* $\nu_{*i} = 4$, $\varepsilon = 0.213$, $X_0 = -0.25$ *and* $d = 0.05$.

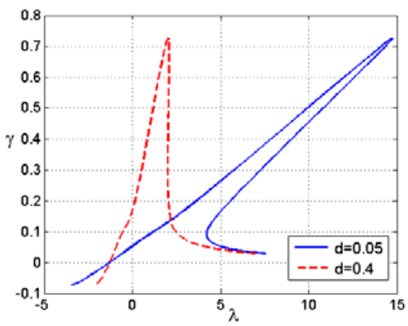

Fig. 4. Even the simple bifurcations disappear for $d \geq 0.4$ *and* $X_0 = 0.25$.

In Fig. 3 the normalized radial electric field X is represented as function of the control parameter λ; for this case we used the effective frequency $\nu_{*i} = 4$, the inverse aspect ratio $\varepsilon = 0.213$, $d = \frac{D_e}{\rho_0^2 \omega_b \varepsilon^{1/2}} = 0.05$ and $X_0 = -0.25$. In the collisional case $(\nu_{*i} \geq 3)$, the double hysteresis, which has the same characteristic feature as in the original model given in [3], disappears for positive values of λ. There is thus only one bifurcation for the normalized radial electric field and for the particle flux. For $d \geq 0.4$ and $X_0 = 0.25$, the simple bifurcations of the radial electric field and of the particle flux disappear (see Fig. 4).

5. Non-stationary case: the contribution of $\Gamma_i^{lc(2)}(X)$, $\Gamma_i^{bv}(X)$, $\Gamma_{e-i}^{anom}(X)$ and $\Gamma_{imp}(X)$

A time-behaviour of the radial electric field is considered assuming a given time-dependency for the ion temperature:

$$T_i(t) = T_{0i}\left[1 + \eta \tanh \delta t\right] \equiv T_{0i} G(t). \qquad (11)$$

In Eq. (11) $\delta \in [5,\ 200]\, s^{-1}$. It is also assumed that the electron temperature does not vary in time and is given by $T_e = T_{0i} = 10^6$ K. The poloidal ion gyro-radius becomes:

$$\rho_0(t) = \frac{(2m_i k T_{0i})^{1/2}}{eB_P} G^{1/2}(t) \equiv \rho_{00} G^{1/2}(t), \qquad (12)$$

with $\rho_{00} = 1.33 \times 10^{-2}$ m.

The following parameters are used in this model: $D_e = 5 \cdot 10^{-2}\, m^2 s^{-1}$, $n_e = 10^{19}\, m^{-3}$, $F(X) \approx 1$, $\nu_{*i} = 2$, $B_P = 10^{-1}$ T, $\varepsilon_0 \varepsilon_\perp = 10^{-8}\, C^2 m^{-2} N^{-1}$, $R = 3$ m, $\varepsilon = 1/3$, $q = 3$. The perpendicular dielectric constant related to the poloidal flow is $\varepsilon_\perp = 1 + c^2\left(1 + 2q^2\right)/v_A^2$ while q is the safety factor profile, c is the velocity of light, $v_A = B/\sqrt{m_i n_i \mu_0}$ is the Alfven speed (B is the magnitude of the magnetic field).

We consider also the following spatial profiles for the temperature and for the number density for electrons [8]:

$$T_e(r/a) = T_{0e}\left(1 - \left(\frac{r}{a}\right)^2\right)^2, \quad n_e(r/a) = n_{0e}\left(1 - \left(\frac{r}{a}\right)^2\right) \qquad (13)$$

Choosing $\alpha \approx 1$ and $r/a = 0.98$ and considering that the electron temperature does not vary in time, the complete expression for λ is:

$$\lambda(t) = \frac{6\frac{r}{a}\left(1 - \left(\frac{r}{a}\right)^2\right) T_{0e}}{aeB_P}\left(\frac{2m_i k}{T_{0i}}\right)^{1/2} G^{1/2}(t) \equiv \lambda_0 G^{1/2}(t) \qquad (14)$$

The dimensionless radial electric field X becomes:

$$X(t) = bE_r(t) G^{-1/2}(t), \quad b = \frac{1}{B_P}\left(\frac{2m_i}{kT_{0i}}\right)^{1/2} \qquad (15)$$

For the specific parameters used in our paper we take: $b \approx 1.55 \times 10^{-4}\, mV^{-1}$ and $\lambda_0 = 3 \times 10^{-3}$.

In this section the time behaviour of the radial electric field is considered assuming the time-dependency for the ion temperature and using the following fluxes: $\Gamma_i^{bv}(X)$ in the collisional case, the anomalous bipolar loss $\Gamma_{e-i}^{anom}(X)$, the ion loss flux and the impurity flux. The later has the expression [9]:

$$\Gamma_{imp}(X) = \Gamma_{imp}^0\left[Xe^{-\frac{X^2}{4\varepsilon}} + \varepsilon^{-1/2}X^2\left(p - 1 + p\varepsilon^{-1/2}e^{-\frac{X^2}{4\varepsilon}}\right)\right],$$

$$\Gamma_{imp}^0 \equiv \frac{f\beta k n_i m_i Z \nu_{ii} \varepsilon^{1/2} T_i}{e^2 B^2 \rho_\theta} \text{ and } p = \frac{M}{Zm_i}. \qquad (16)$$

M is the mass number of an impurity ion, m_i is the mass number of the main ions, Z is the charge number of impurities, f is the percent of the expelled ions of impurity from plasma, $\beta = \frac{n_{imp}}{n_i}$ and the other parameters are already defined. The first term denotes an inward pinch due to collisions with bulk ions, and the second term shows centrifugal force; the latter can be outward. The equation of evolution for the radial electric field becomes:

$$\begin{aligned}\frac{\partial E_r(t)}{\partial t} &= D_1 G(t) - D_2 E_r(t) - \\ &- D_3 G(t)\frac{\nu_{*i}[bE_r(t)G^{-1/2}(t) + X_0]}{b^2 E_r^2(t)G^{-1}(t) + \nu_{*i}^2 \varepsilon^3} - \\ &- D_4 G(t)\frac{\nu_{*i}^2}{(\nu_{*i} + [bE_r(t)G^{-1/2}(t)]^4)^{1/2}} e^{[-(\nu_{*i} + [bE_r(t)G^{-1/2}(t)]^4)^{1/2}]} - \\ &- D_5 f\beta Z\nu_{*i} G(t)\left\{ bE_r(t)G^{-1/2}(t) e^{-\frac{[bE_r(t)G^{-1/2}(t)]^2}{4\varepsilon}} + \right. \\ &\left. + [bE_r(t)G^{-1/2}(t)]^2 \varepsilon^{-1/2}\left[p - 1 + p\varepsilon^{-1/2} e^{-\frac{[bE_r(t)G^{-1/2}(t)]^2}{4\varepsilon}} \right] \right\}\end{aligned} \quad (17)$$

The coefficients D_k (k=1-5) are given as:

$$D_1 = \frac{e}{\varepsilon_0 \varepsilon_\perp}\frac{D_e n_e}{T_e}\frac{T_{0i}}{\rho_{0\theta}}\lambda_0 \equiv D_0 \frac{T_{0i}}{\rho_{0\theta}}\lambda_0 \approx 1.8045\times 10^6 \frac{V}{m\,s}$$

$$D_2 = D_0 \frac{T_{0i}}{\rho_{0\theta}} b \approx 1.24\times 10^3 s^{-1}$$

$$D_3 = \frac{k n_i T_{0i}}{\varepsilon_0 \varepsilon_\perp r B}\varepsilon^{7/2} \approx 3.3\times 10^8 \frac{V}{m\,s}$$

$$D_4 = \frac{n_i}{\varepsilon_0 \varepsilon_\perp}\varepsilon^2 (qR)^{-1}\frac{2kT_{0i}}{B_P} \approx 3.4\times 10^9 \frac{V}{m\,s}$$

$$D_5 = \frac{k n_i T_{0i} q^{-3} R^{-1}}{\varepsilon_0 \varepsilon_\perp B_P}\varepsilon^4 \approx 2.103\times 10^7 \frac{V}{m\,s} \quad (18)$$

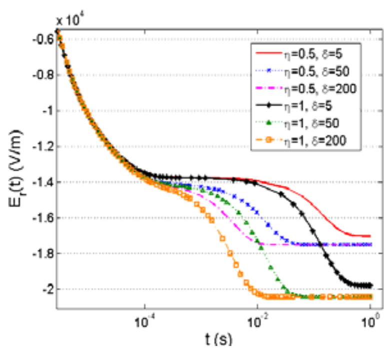

Fig. 5. The evolution of the radial electric field

In Fig. 5 it is shown the evolution in time of the solution of Eq. (17) for different parameters η and values of δ. The initial condition is: $t_0 = 0$ and $X(t_0) = 1$. The other parameters are: $\beta = 3/10$, $\nu_{*i} = 2$, $\varepsilon = 1/3$, $p = 1$.

It can be seen in Fig. 5, that the first decrease of the radial electric field is followed by an intermediate decreasing region before the reaching of the stationary regime in a region of negative values. It is obvious that the type of the heating dictates the stationary value for the radial electric field: the absolute stationary value of the radial electric field is increasing when η is increasing. The transitory regime behaves only in the time interval $t \in [10^{-4}, 1]$ sec. It was also observed that the time behaviour of the radial electric field is practically determined mainly by the second and the last term from Eq. (17). By the other hand the expelled impurity is not very important in the determination of the saturation value of the radial electric field.

6. Conclusions

The bifurcations of the normalized radial electric field and the particle flux are obtained as a balance of a linearly modulated ion loss cone flux and the anomalous electron loss. Small differences in the thickness of the meso-phase and in the range of values of the control parameter for the normalized radial electric field and the particle flux in the region of bifurcation were observed. The range of values of the control parameter leading to a bifurcation is obtained. The linear dependence of F on the radial electric field maintains the existence of a critical value λ_c of the control parameter. At this value, a transition from the branch of large flux (L-confinement) to that of small flux (H-confinement) occurs. The disappearance of the double hysteresis for an effective frequency $\nu_{*i} \geq 3$ and $d = 0.05$ is observed in comparison with the model studied in [3]; a simple bifurcation, however, is present. The time behaviour of the electric field was studied for different heating regimes and transitory regimes are found for different heating types.

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