Optical bistability in an optomechanical system assisted by quantum dot molecules

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Optical bistability (OB) in a hybrid optomechanical system with quantum dot molecules (QDMs) is investigated theoretically. The OB behavior between the optomechanical cavity photon number and the coupling-laser Rabi frequency can be controlled for different system parameters such as their frequency detuning, the coupling strength, the optical cavity decay rate and the tuning strength in the QDMs. The photon number of the optomechanical cavity vs. frequency detuning has been calculated numerically to explain the OB. It is revealed that the tunneling effect in the QDMs can be used to adjust the OB finely. Such a system may be used in the bistable optical switching and other quantum information technology.

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1. Introduction

Recently, a great deal of attention has been paid to the research of the hybrid optomechanical system, which examines the coupling among the optical cavity, the mechanical oscillator and other objects such as two-level [1-3] and three-level atoms [4], quibit [5-7], two level defects [8,9], quantum well [10], quantum dot [11], Kerr media [12,13,51] and so on, due to its potenticial application such as sensing [14-16,52,53], phonon laser [17,18], ground state cooling [6,19-21], photon blacade [9,22-25] et al. For example, Huang et al. [25] studied nonreciprocal photon blockade which can be used to generate nonclassical light based on photon blockade in a spinning Kerr resonator. Chen et al. [52] proposed one way to measure the Newtonian constant of gravitation in an optomechanical system with two cavities and two membrane resonators. Komori et al. [53] reported Attonewton-meter torque sensing up to 20 $aNm/\sqrt{Hz}$ with a macroscopic optomechanical torsion pendulum.

What is more, many interesting phenomena have also been researched for the optomechanical system such as the optomechanically induced transparency (OMIT) [26-29], quantum entanglement [30-32], high-order sidebands [3,33-36,54], Fano resonance [51,55] and so on. Xiong et al. [37] reviewed the theories and applications of the OMIT which showed different types of OMIT and potential utility in optical buffer, amplification, filter and so on. In [3], Liu et al. studied high-order sideband assisted by two-level atoms which could be tuned by different system parameters in a hybrid optomechanical system. In [51], Huang et al. reported the Fano resonance and amplification in a optomechanical system with a nonlinear Kerr medium. In [54], Liu et al. examined the high-order sidebands in a coupled double-cavity optomechanical system, which can be tuned for its range and the interval.

Recent researches also revealed the OB and optical multibility in the hybrid optomechanical system [13, 37-49,56,57]. For example, Sarma et al. [47] demonstrated OB in the atomic cavity could be controlled by the coupling laser and by altering the atom-cavity coupling strength. Kazemi et al. [48] examined OB in a two-mode optomechanical system assisted by a Bose–Einstein condensate which could be adjusted by the coupling field. Chen et al. [49] suggested OB in a three-mode optomechanical system assisted by two-level atoms and discussed the OB generation condition. In [56], Bhatt et al. reported polariton multistability in an optomechanical resonator with a quantum well and a $\chi^{(2)}$ second order nonlinear medium. In [57], Gao et al. investigated the OB in an optomechanical system with an N-type atomic ensemble.

In the present paper, motivated by the previous interesting researches of the hybrid optomechanical system, we theoretically investigate the OB behavior in an optomechanical system embedded with the QDMs which can be controlled finely by the system parameters and the tunneling effect in QDMs.

2. Model and theory

We consider a QDMs system confined in a cavity with resonance frequency ω_c. A strong coupling field with frequency ω_l (Rabi frequency Ω) drives the cavity, which couples the mechanical resonator through the radiation pressure and couples the QDMs through the Jayness-Cummings interaction with the coupling strength g [4]. Without optical excitation, the QDMs is in the state $|0\rangle$ for there is no excitons. With applying a laser field, the QDMs is in the state $|1\rangle$ for there are many direct excitons in one QD. After applying an external electric field, the electron can be tunneled from one QD to the other which forms indirect excitons, and QDM is in the state $|2\rangle$ [50,60]. Omitting the weak electron-electron interactions, the system Hamiltonian can be written as [4,50],

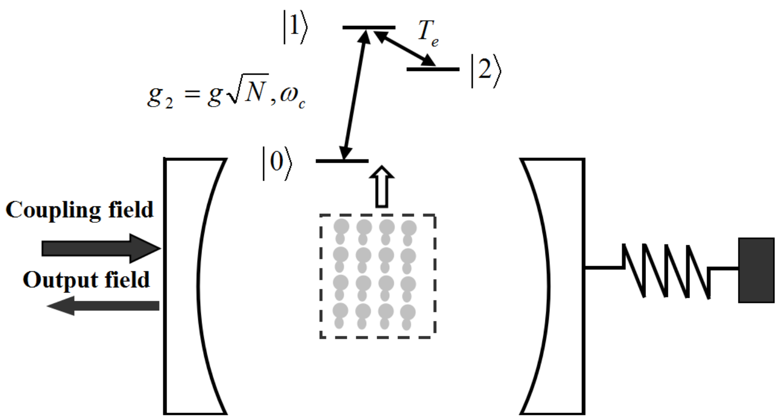


Fig. 1. Schematic of a hybrid optomechanical system with QDMs. Left (right) mirror is fixed (vibrating). The mechanical resonator couples the cavity field through the radiation pressure. The asymmetrical QDM system [50] with different band structure couples the cavity field with the collective strength g$_2$*. Interdot tunneling in the QDMs can be controlled by applying a gate electrode between them[58,59]*

$$\begin{aligned} H &= \hbar\omega_c c^+ c + \frac{1}{2}\hbar\omega_m (X^2 + P^2) + \hbar\sum_{i=1}^{N}(\omega_{10}\sigma_{11}^i + \omega_{20}\sigma_{22}^i) \\ &- \hbar g_1 c^+ c X + \hbar T_e \sum_{i=1}^{N}(\sigma_{12}^i + \sigma_{21}^i) + \hbar g \sum_{i=1}^{N}(c\sigma_{10}^i + c^+\sigma_{01}^i) + i\hbar\Omega(c^+ e^{-i\omega_l t} - c e^{i\omega_l t}) \end{aligned}, \tag{1}$$

where the first term is the energy of the optical cavity with c (c^+) being the annihilation (creation) operator. The second term is the energy of the mechanical resonator with ω_m , X and P being the frequency, position and momentum operator of the resonator respectively. The third term is the Hamiltonian of the N QDMs with $\sigma_{jj}^i = (|j\rangle\langle j|)_i$ being the operator of i-th QDM and ω_{jk} is the energy-level spacing between the state $|j\rangle$ and $|k\rangle$ in each QDM. The fourth term is the coupling between the optical cavity and the mechanical resonator with coupling strength g_1. The fifth term is the tunneling between the state $|1\rangle$ and $|2\rangle$ with tunneling strength T_e in each QDM [50,60]. The sixth term is the coupling between the optical cavity and the N QDMs with coupling strength g [11]. The last term is the interaction between the optical cavity and the coupling laser.

Using the Holstein-Primakoff transformation [4] and defining $A = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sigma_{01}^i$, $B = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\sigma_{02}^i$, we suppose the number of the QDM is large enough. So, $[A, A^+] = 1$ and $[B, B^+] = 1$ are satisfied. The Hamiltonian can be rewritten as [4],

$$\begin{aligned} H &= \hbar\omega_c c^+ c + \frac{1}{2}\hbar\omega_m (X^2 + P^2) + \hbar(\omega_{10}A^+A + \omega_{20}B^+B) \\ &- \hbar g_1 c^+ cX + \hbar T_e (A^+B + B^+A) + \hbar g\sqrt{N}(cA^+ + c^+A) + i\hbar\Omega(c^+e^{-i\omega_l t} - ce^{i\omega_l t}) \end{aligned} , \quad (2)$$

In the rotating frame with the coupling frequency ω_l , the hybrid system Hamiltonian can be expressed as,

$$\begin{aligned} H &= \hbar\Delta_c c^+ c + \frac{1}{2}\hbar\omega_m (X^2 + P^2) + \hbar(\Delta_1 A^+A + \Delta_2 B^+B) \\ &- \hbar g_1 c^+ cX + \hbar T_e (A^+B + B^+A) + \hbar g_2 (cA^+ + c^+A) + i\hbar\Omega(c^+ - c) \end{aligned} , \quad (3)$$

where, $g_2 = g\sqrt{N}$ is the collective coupling strength between the optical cavity and the QDMs. $\Delta_c = \omega_c - \omega_l$ is the detuning between the optical cavity and the coupling laser. $\Delta_1 = \omega_{10} - \omega_l$ ($\Delta_2 = \omega_{20} - \omega_l$)is the detuning between the transition frequency ω_{10} (ω_{20}) and the coupling laser.

Based on the Hamiltonian Eq. (3) and considering the communication relation $[c, c^+] = 1$ and $[X, P] = i$, the Heisenberg-Langevin equations can be written as [48].

$$\frac{dX}{dt} = \omega_m P , \quad (4a)$$

$$\frac{dP}{dt} = -\gamma P - \omega_m X + g_1 c^+ c , \quad (4b)$$

$$\frac{dc}{dt} = -[\kappa + i(\Delta_c + g_1 X)]c - ig_2 A + \Omega , \quad (4c)$$

$$\frac{dA}{dt} = -(\gamma_1 + i\Delta_1) A - iT_e B - ig_2 c , \quad (4d)$$

$$\frac{dB}{dt} = -(\gamma_2 + i\Delta_2) B - iT_e A , \quad (4e)$$

where κ is the decay rate of the optical cavity. γ_1 (γ_2)is the decay rate of the transition between $|0\rangle$ and $|1\rangle$ ($|2\rangle$) in the QDM.

In the steady state, we can obtain the steady solution X_s, A_s, B_s, c_s for the operators X, A, B, c respectively as follows,

$$X_s = \frac{g_1}{\omega_m}|c_s|^2 \quad , \tag{5a}$$

$$B_s = -\frac{iT_e}{\Gamma_2}A_s \quad , \tag{5b}$$

$$A_s = -\frac{ig_2\Gamma_2}{\Gamma_1\Gamma_2 + T_e^2}c_s \quad , \tag{5c}$$

$$c_s = \frac{\Omega}{\kappa + i(\Delta_c + g_1 X_s) + \dfrac{g_2^2\Gamma_2}{\Gamma_1\Gamma_2 + T_e^2}} \quad , \tag{5d}$$

where, $\Gamma_1 = \gamma_1 + i\Delta_1$ and $\Gamma_2 = \gamma_2 + i\Delta_2$.

From Eqs. (5), we can get the following equation for $|c_s|^2$ and the OB behaviors can be deduced from it for certain values in the next part,

$$\Omega^2 = \left|\kappa + i(\Delta_c + \frac{g_1^2}{\omega_m}|c_s|^2) + \frac{g_2^2\Gamma_2}{\Gamma_1\Gamma_2 + T_e^2}\right|^2 |c_s|^2 \quad , \tag{6}$$

To understand the condition to generate OB, one should also solve equation [48],

$$\frac{\partial\Omega^2}{\partial|c_s|^2} = 0 \tag{7}$$

when Eq. (7) is solved to have two different positive solutions of $|c_s|^2$, OB can be found. However, it is very cumbersome to derive the analytical expression of the OB condition. We will give the numerical results in the following.

3. Results and discussion

Due to the interaction between the optical cavity and the mechanical resonator, the optical bistability is generated showing in Eq. (6) which can also be influenced by the embedded QDMs in our hybrid system. To theoretically investigate the OB, we choose experimentally realizable parameters as follows: $L = (1-25)\times 10^{-3}m$, $m = 5-145ng$, $\omega_m = 1MHz$ [46]. Considering weak tunneling regime, the tunneling coupling is selected to be $1-100MHz$ in frequency ($0.004-0.4\mu eV$ in energy)[60], which depends the barrier characteristics and the external electric field. To be simple, all the following corresponding parameters are scaled by $\omega_m = 1MHz$.

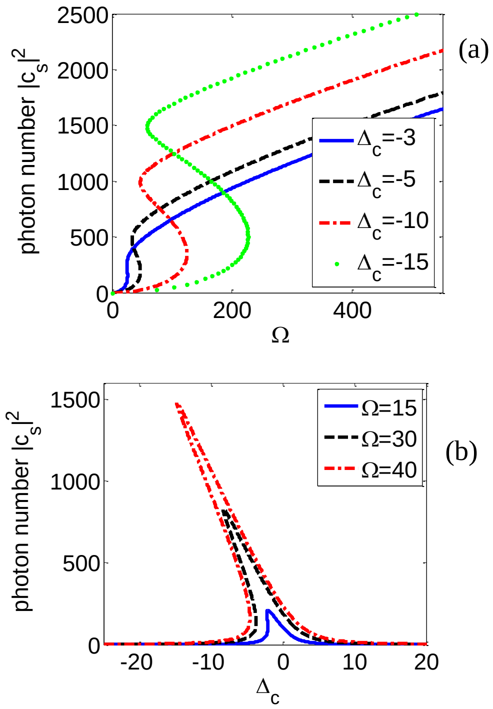


Fig. 2. (a) Photon number $|c_s|^2$ of the optomechanical cavity vs. the Rabi frequency Ω for different frequency detuning Δ_c =-3,-5,-10 and -15 of the coupling laser. (b) Photon number $|c_s|^2$ of the optomechanical cavity vs. frequency detuning Δ_c for different Rabi frequency Ω =15, 30 and 40 of the coupling laser. The other parameters are $\kappa = 1$ [46], $\gamma_1 = \gamma_2 = 1$ [50], $\Delta_1 = \Delta_2 = 0$, $T_e = 1$, $g_1 = 0.1$, $g_2 = 1$ [46] (color online)

First, Fig. 2(a) shows the mean photon number $|c_s|^2$ of the optomechanical cavity vs. the Rabi frequency

Ω of the coupling laser for different frequency detuning Δ_c. We can easily find with the increasing of the coupling laser frequency detuning $|\Delta_c|$, the hysteresis cycle becomes wider and the threshold becomes larger. When $|\Delta_c|<3$, the OB hysteresis cycle vanishes. So, we can control the OB by changing the frequency detuning of the coupling laser. Fig. 2(b) shows the mean photon number $|c_s|^2$ of the optomechanical cavity vs. frequency detuning Δ_c for different Rabi frequency. As we can see from the figure, only when the Rabi frequency is so powerful (i.e. $\Omega>15$) enough, the OB occurs.

Second, Fig.3 shows the mean photon number $|c_s|^2$ of the optomechanical cavity vs. the Rabi frequency Ω of the coupling laser for different tunneling strength. As we can see from the figure, with the increasing of the tunneling coupling, the OB hysteresis cycle becomes wider and the threshold does not change obviously. But when the tunneling strength is too large (i.e. $T_e > 5$), the OB curve will change slowly. So, we can adjust the OB finely by changing the tunneling strength.

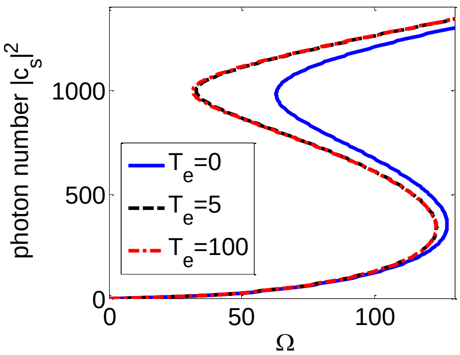
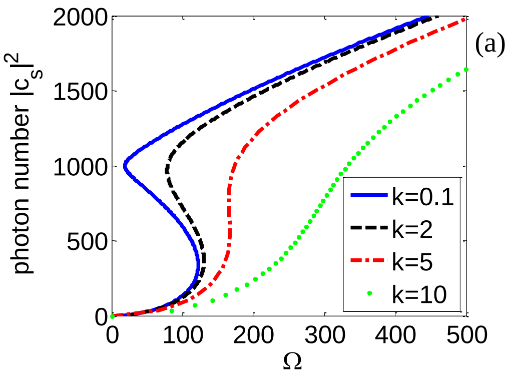


Fig. 3 Photon number $|c_s|^2$ *of the optomechanical cavity vs. the Rabi frequency Ω of the coupling laser for different tunneling strength* T_e*=0, 5 and 100. The other parameters are* $\kappa=1, \gamma_1=\gamma_2=1$, $\Delta_1=\Delta_2=0, \Delta_c=-10, g_1=0.1, g_2=1$ *(color online)*



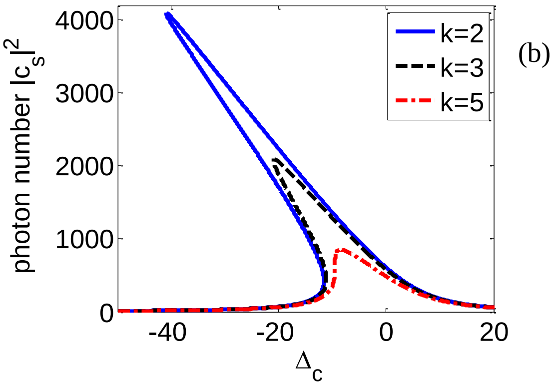


Fig. 4 (a) Photon number $|c_s|^2$ *of the optomechanical cavity vs. the Rabi frequency Ω of the coupling laser for different decay rate K=0.1, 2, 5 and 10 with* Δ_c *= = -10. (b) Photon number* $|c_s|^2$ *of the optomechanical cavity vs. frequency detuning* Δ_c *for different decay rate K = 2, 3 and 5 with Ω = 160. The other parameters are* $T_e=1, \gamma_1=\gamma_2=1, \Delta_1=\Delta_2=0$, $g_1=0.1, g_2=1$ *(color online)*

Third, Fig.4 (a) shows the mean photon number $|c_s|^2$ of the optomechanical cavity vs. the coupling laser Rabi frequency Ω for different decay rate κ of the optical cavity. We can find that, with the increasing of the optical cavity decay rate, the OB hysteresis cycle becomes narrower. The OB vanishes when $\kappa>5$. Such also can be seen in Fig.4 (b), which shows the mean photon number of the optomechanical cavity vs. frequency detuning Δ_c for different κ. For $\Omega=160$, the OB occurs only when $\kappa<5$.

Fourth, Fig. 5(a) shows the mean photon number

$|c_s|^2$ of the optomechanical cavity vs. the coupling strength g_1 between optical cavity and the mechanical resonator. We can find that, with the decreasing of the coupling strength g_1, the OB hysteresis cycle becomes wider and the threshold becomes larger. But when $g_1 < 0.01$, the OB vanishes. Such also can be found in Fig. 5(b), which shows the mean photon number $|c_s|^2$ of the mean optomechanical cavity vs. frequency detuning Δ_c for different coupling strength g_1. Only when the coupling strength $g_1 > 0.01$, we can find OB in the hybrid system.

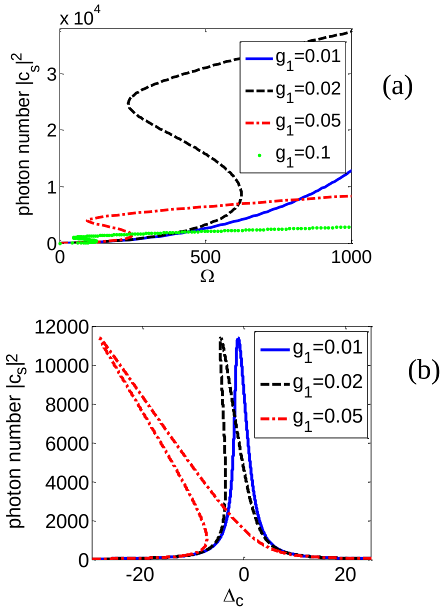


Fig. 5 (a) Photon number $|c_s|^2$ *of the optomechanical cavity vs. the coupling strength* g_1 *=0.01, 0.02, 0.05 and 0.1 with* $\Delta_c = -10$. *(b) Photon number* $|c_s|^2$ *of the optomechanical cavity vs. frequency detuning* Δ_c *for different coupling strength* g_1 *=0.01, 0.02, 0.05 with* $\Omega = 160$. *The other parameters are* $T_e = 1, \gamma_1 = \gamma_2 = 1, \Delta_1 = \Delta_2 = 0, g_2 = 1$ *(color online)*

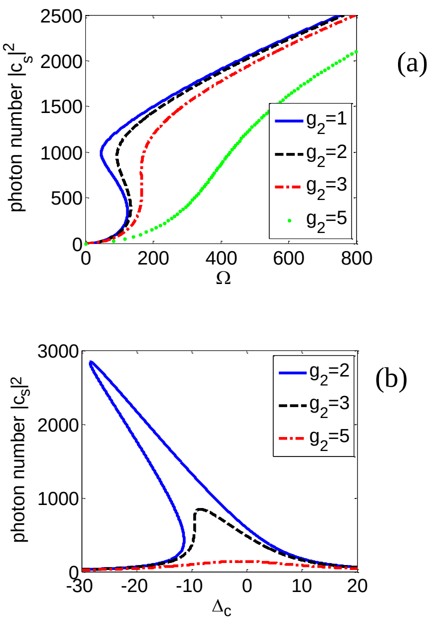
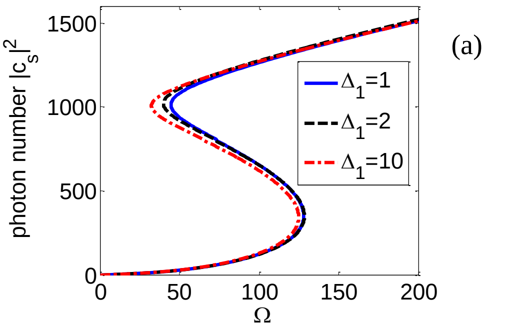


Fig. 6. (a)Photon number $|c_s|^2$ *of the optomechanical cavity vs. the coupling strength* g_2 *=1, 2, 3 and 5 with* $\Delta_c = -10$. *(b) Photon number* $|c_s|^2$ *of the optomechanical cavity vs. frequency detuning* Δ_c *for different coupling strength* g_2 *=2, 3 and 5 with* $\Omega = 160$. *The other parameters are* $T_e = 1, \gamma_1 = \gamma_2 = 1$, $\Delta_1 = \Delta_2 = 0, g_1 = 0.1$ *(color online)*

Fifth, Fig. 6(a) shows the mean photon number $|c_s|^2$ of the optomechanical cavity vs. the coupling strength g_2 between optical cavity and the QDMs. It is clear that, with the increasing of the coupling strength g_2, the OB hysteresis cycle becomes narrower and the threshold becomes larger. But when $g_2 > 3$, the OB vanishes. Such also can be found in Fig. 6(b), which shows the mean photon number $|c_s|^2$ of the mean optomechanical cavity vs. frequency detuning Δ_c for different coupling strength g_2. We can find OB in our system when $g_2 < 3$.

Finally, Fig. 7 shows the mean photon number $|c_s|^2$ of the optomechanical cavity vs. the frequency detuning (a) Δ_1 and (b) Δ_2. We can find with the increasing of the frequency detuning Δ_1 (Δ_2), the OB hysteresis cycle becomes wider (narrower) and the threshold becomes smaller (larger). But all the changes are slow. So the dependence of the mean photon number on the frequency detuning Δ_1 and Δ_2 is weak.



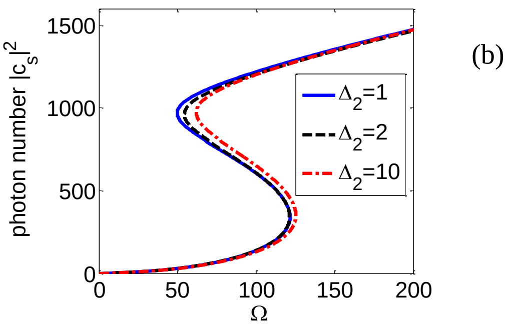


Fig. 7. Photon number $|c_s|^2$ of the optomechanical cavity vs. the frequency detuning (a) Δ_1 =1, 2, 10 with $\Delta_2 = 0$, and (b) Δ_2 =1, 2, 10 with $\Delta_1 = 0$, The other parameters are $T_e = 1, \gamma_1 = \gamma_2 = 1$, $\kappa = 1, \Delta_c = -10$, $g_1 = 0.1, g_2 = 1$ (color online)

4. Conclusion

To summarize, we have theoretically studied the OB behavior in an optomechanical cavity embedded with the QDMs. The OB can be controlled by tuning the coupling laser detuning Δ_c, tunneling strength T_e, optical cavity decay rate κ, coupling strength g_1 for mechanical resonator, coupling strength g_2 for QDMs, frequency detuning Δ_1 and Δ_2 in QDMs. The OB conditions also are discussed numerically by discussing the relation between the hoton number $|c_s|^2$ of the optomechanical cavity vs. frequency detuning Δ_c. Such a system may be applied in some quantum information science.

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