JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 9, No. 8, August 2007, p. 2339 - 2341

Optical bright solitons in inhomogeneous fiber core media

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We consider an optical fiber with inhomogeneous core and modify the nonlinear Schrödinger (NLS) equation by adding terms for phase modulation and fiber power gain/ loss, in order to include some inhomogeneous physical effects. The modified NLS equations in the anomalous dispersive regime are Hirota bilinearized, and exact bright soliton solutions are obtained. The results show that the area of the pulse envelope is preserved during propagation in the fiber, demonstrating that solitary wave propagation is maintained in the core medium we considered.

(Received February 23, 2007; accepted June 27, 2007)

Keywords: Optical fiber, Bright soliton, Nonlinear Schrödinger equation, Horota transformation, Self-phase modulation, Group velocity dispersion

1. Introduction

There has been increased interest in nonlinear systems that give rise to energy-localization effects with a relatively long lifetime due to their potential in applications, such as optical communications [1-5]. A typical example for such an effect is called "optical solitons" [6]. Solitons are a special breed of optical pulses that can propagate through an optical fiber undistorted over very long distances. The key to soliton formation in optical fiber is the counterbalance of the opposing forces of chromatic dispersion (group velocity dispersion, GVD) and self-phase modulation (SPM). The nonlinear Schrödinger (NLS) equation [7,8] which governs this soliton pulse propagation in fiber is given by

$$i\frac{\partial A}{\partial z} - s\frac{\partial^2 A}{\partial t^2} + 2\left|A\right|^2 A = 0 \qquad (1)$$

where $A(z,t)$ is the (slowly varying) envelope of the axial electric field and $s = \text{sign}(\beta) = \pm 1$ (β, the GVD parameter related to the frequency dependence of group velocity). Considering homogeneous fiber core medium, equation (1) has widely been studied by numerous authors [6-8]. However, in an actual fiber, the core medium is, in general, not uniform or homogeneous [9]. This inhomogeneity arises mainly due to imperfection or defects in fiber core medium and fluctuations in core/ cladding radius [9]. Therefore, the study of nonlinear wave propagation in inhomogeneous media is of great interest in the area of fiber optics communications.

Recently, depending on the physical situations, several modifications to equation (1) have been suggested and the pulse propagation have been discussed [8,10-12]. Of them, one of the important modifications, for example, was given by Flach [2], who discussed resonant light scattering by solitary waves, by considering the process of light scattering by optical solitons in a planar waveguide with homogeneous and inhomogeneous refractive index cores. In case of inhomogeneities in core medium, the NLS equation (Eq. 1) can be modified and be written in the general form

$$i\frac{\partial A}{\partial z} - s\frac{\partial^2 A}{\partial t^2} + 2\left|A\right|^2 A = \varphi t^2 A \pm igA \qquad (2)$$

where $\varphi(z)$ is phase modulation and $g(z)$ the fiber power gain $g(z)>0$/ loss $g(z)<0$. Since $s = \text{sign}(\beta)$ the solution of equation (2), either gain or loss case, depends on whether β is positive or negative. In all cases, the NLS equation can be solved by the inverse scattering transform method. The pulse-like solutions are found to occur only in the case of anomalous dispersion ($\beta < 0$), and are called "bright solitons". In the case of normal dispersion ($\beta > 0$), the solitary wave solutions of equation (2) appear as a dip in a constant background, and are called "dark solitons". Equation (2) contains arbitrary functions of z, so we need to identify the integrability conditions of the equation through linear eigenvalue problem. The Lax pair ensures the complete integrability condition of a nonlinear system of equations and is used to achieve soliton solutions. Here we exactly solve the NLS equations with additional terms for phase modulation and fiber gain/ loss in anomalous dispersive regime. In solving the modified NLS equations, the Hirota transformation method has been used.

2. Soliton solutions and discussion

The modified NLS (Equation (2)) with phase modulation and fiber gain reduces to

$$i\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial t^2} + 2\left|A\right|^2 A = \varphi t^2 A + igA \qquad (3)$$

The Lax pair associated with equation (3) is derived as

$$\frac{\partial\psi}{\partial t}=P\psi \qquad \frac{\partial\psi}{\partial z}=Q\psi \qquad \psi=(\psi_1\psi_2)^T \tag{4}$$

$$P=\begin{pmatrix}-i\Delta & a\\ -a & i\Delta\end{pmatrix}$$

$$Q=2i\Delta^2\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}+2\Delta\begin{pmatrix}-igt & a\\ -a^* & igt\end{pmatrix}+i\begin{pmatrix}|a|^2 & a_t^*-2igta^*\\ a_t^*+2igta^* & -|a|^2\end{pmatrix}$$

where φ_2, a variable transformation, and Δ is the variable spectral parameter given by $\Delta=\Delta_0\exp(2\int_0^z gdz)$. Using the above variable transformation in equation (3), we obtain

$$i\frac{\partial a}{\partial z}+\frac{dg}{dz}t^2\frac{a}{2}+\frac{\partial^2 a}{\partial t^2}+2|a|^2a-2igt\frac{\partial a}{\partial t}-2iga-(g^2+\varphi)at^2=0 \tag{5}$$

The compatibility condition $\frac{\partial P}{\partial z}-\frac{\partial a}{\partial t}+[P,a]=0$ gives

$$i\frac{\partial a}{\partial z}+\frac{\partial^2 a}{\partial t^2}+2|a|^2a-2igt\frac{\partial a}{\partial t}-2iga=0 \tag{6}$$

By comparing equations (5) and (6), we find that equation (2) is completely integrable and it thus gives exact solutions, and the integrability condition is

$$\frac{dg}{dz}-2(\varphi+g^2)=0.$$

Now we introduce the Hirota derivative operators. A symbol D_x is called the Hirota derivative with respect to variable *x* and defined to act on a pair of functions *f*(x) and *p*(x) as follows:

$$D_x f(x).p(x)=(\partial_x-\partial_{x'})f(x)p(x')\big|_{x'=x},$$

where ∂_x denotes partial derivative with respect to *x*. Hirota derivatives are bilinear operators. We define the bilinear operators for our system as

$$D_z^m D_t^n b(z,t).c(z,t)=(\partial_x-\partial_{x'})^m(\partial_t-\partial_{t'})^n b(z,t)c(z',t')\big|_{x'=x,t'=t} \tag{7}$$

with the transformation in the form (for more information about Hirota's transformation, see [6])

$$a=\frac{b}{c}, \tag{8}$$

where *b*(z,t) and *c*(z,t) are complex and real functions respectively. Using the above transformation in equation (6), we obtain

$$(iD_z+D_t^2-2igtD_t-2ig)(b.c)=0, \quad D_t^2(c.c)=2b^2. \tag{9}$$

The multi-soliton solutions can be obtained by the following perturbation expansions of *b* and *c* :

$$b=\sigma b_1+\sigma^3 b_3+\sigma^5 b_5+\sigma^7 b_7+\ldots\ldots \tag{10}$$

$$c=1+\sigma^2 c_2+\sigma^4 c_4+\sigma^6 c_6+\ldots\ldots, \tag{11}$$

where σ is an expansion coefficient. For one-soliton solution (1SS) only the first term and the first two terms in equations (10) and (11) respectively are needed. As we want 1SS, we will consider here those terms only. Pluging $b=\sigma b_1$ and $c=1+\sigma^2 c_2$ in equation (9) and collecting the terms (coefficients of σ, σ^2, σ^3 and σ^4) with same power of σ, we obtain:

$$\left.\begin{aligned}&(iD_z+D_t^2-2igtD_t-2ig)(b_1.1)=0\\ &D_t^2(1.c_2+c_2.1)=2|b_1|^2\\ &(iD_z+D_t^2-2igtD_t-2ig)(b_1.c_2)=0\\ &D_t^2(c_2.c_2)=0\end{aligned}\right\} \tag{12}$$

We solve the set of equations (12). In deriving the solutions for b_1 and c_2, we conveniently assume $b_1=e^{\nu+\mu}$ and $c_2=e^{\nu+\mu+(\nu+\mu)^*}$, where $(\nu+\mu)^*$ is the complex conjugate of $(\nu+\mu)$. After some algebra, equation (6) yields

$$a(z,t)=2\varphi_2 e^{\nu}\sec h(\mu) \tag{13}$$

where $\nu=-2i\{\varphi_1 t+2\int_0^z(\varphi_1^2-\varphi_2^2)dz\}$ and $\mu=2(\varphi_2 t+4\int_0^z\varphi_1\varphi_2 dz)+\Omega$, where Ω is an integration constant, with $\varphi_1=\varphi_{10}\exp(2\int_0^z gdz)$ and $\varphi_2=\varphi_{20}\exp(2\int_0^z gdz)$. Using variable transformation, defined earlier ($a\rightarrow A$), we get

$$A(z,t)=2\varphi_2 e^{\nu-igt^2/2}\sec h(\mu) \tag{14}$$

Equation (14) is the exact bright 1SS of equation (3), i.e. with fiber gain and phase modulation, which with parameter values $\varphi_{20}=\varphi_{10}/2=1/2$, $g=0.2/z$ and Ω=0 is plotted in Fig. 1.

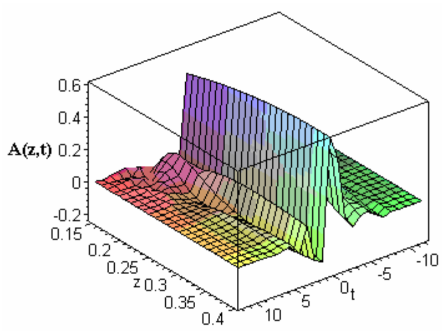

Fig. 1. Amplitude profile for the bright 1SS (with g>0): /A(z,t)/ as functions of z and t (a 3D plot of equation (14)).

Now we consider equation (2) with phase modulation and fiber loss. Proceeding as before, it is found that equation below is integrable with condition $\frac{dg}{dz} + 2(\varphi + g^2) = 0$:

$$i\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial t^2} + 2|A|^2 A = \varphi t^2 A - igA \qquad (15)$$

The 1SS of equation (15) is obtained by choosing the variable transformation: $A = a(z,t)e^{igt^2/2}$. Following the same procedure, we derive the following soliton solution

$$A(z,t) = 2\varphi_2 e^{\nu + igt^2/2} \sec h(\mu), \qquad (16)$$

Thus we have obtained the exact bright 1SS for the wave propagation in the inhomogeneous optical fiber core with phase modulation and fiber loss. Equation (16) is plotted ($\varphi_{20} = \varphi_{10}/2 = 1/2$, $g = 0.1/z$ and Ω=0) in Fig. 2.

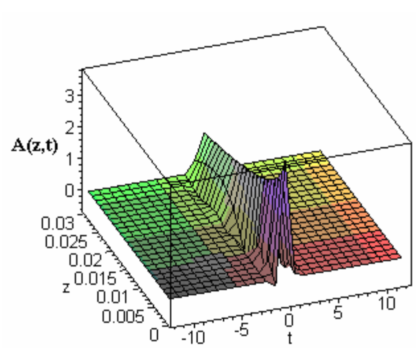

Fig. 2. Amplitude profile of /A(z,t)/ for the bright 1SS (with g<0): /A(z,t)/ vs. z and t (equation (16)).

Fig. 1 shows that the pulse amplitude increases as *z* increases. This is due to the admittance of $\varphi_2(z)$ in the amplitude (equation (14)), which shows that the soliton amplitude grows at the same rate as the respective power amplitude (P_A) with the inclusion of phase modulation and gain. The same modulating term, as in equation (14) but with opposite sign, is also appeared in the amplitude of soliton solution with fiber loss, i.e. in the amplitude of equation (16), which shows that the amplitude decays at the same rate as the respective P_A. This can be seen in Fig. 2, where soliton amplitude decreases as *z* increases. As a result of chirping, the pulse broadens as it propagates along the fiber, and as a result of damping, the amplitude of the pulse reduces. The amplitude of the pulse is found to increase or decrease, depending on the sign of *g*(*z*) (gain/ loss), with the same amount of broadening in the pulse width during its propagation such that the area of the pulse remains constant. With the inclusion of phase modulation and fiber loss, Fig. 2 clearly depicts the damping effect in solitary pulse propagation in a fiber with inhomogeneous core.

3. Conclusion

The more general case of inhomogeneities in fiber core medium is taken into consideration in this study. We have modified the NLS equation in order to include phase modulation and fiber loss/ damping effects. The complete integrability conditions are derived for all cases and the modified NLS equations in the anomalous dispersive regime, which govern the propagation of an optical pulse in a fiber with inhomogeneous core medium, are exactly solved to construct bright soliton solutions. We observe that the amplitude of the pulse is increased for gain and is decreased for damping as it propagates along the fiber. The pulse width is also found to broaden in propagation such that the area of the pulse envelope remains conserved.

Acknowledgement

The author thanks to Professor S. John, Department of Physics, University of Toronto, Canada for useful comments.

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