Optical filter and monochromator based on periodic structures with different refractive index materials

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An optical filter based on periodic structure *i.e.* photonic band gap (PBG) materials has been suggested. The theory and working principle of this filter is analogous to Kronig and Penny model in solid state physics. We study the ZnS/Ge (refractive indices for ZnS and Ge are 2.2 and 4.2 respectively) periodic multilayer structure taking the controlling parameters as Yablonovite structure. Emphasis of the present investigation is given to the study of the structure in the ultraviolet region of electromagnetic radiation. In this study we present the dependence of various properties of the structure considered on angle of incidence. As we increase the angle of incidence, the total number of allowed bands decreases but the band width of all bands increases. By cascading such filters we can design the optical monochromator.

1. Introduction

Since last two decades, many researchers paid considerable much attention to photonic band-gap (PBG) materials, the artificial material which consists of periodic dielectric components in the nano- and micro-metre scale regions [1-5]. It is well known that periodic structure of materials of different refractive indices can give rise to the photonic band gaps. Such materials now-a-days are known as photonic band-gap materials (PBGs). Such materials have been investigated intensively by many investigators as to their properties like their ability of controlling the propagation of light, suppression of spontaneous emission, etc. [6-8]. Tunable optical filters have received much attention due to their application in fibre-optic communications and other fields of optical technology [9-12].

Fabrication of filters in the near- and far- infrared region was suggested by Ojha et al. [13]. This model was based on weak guidance approximation such that $\left(\dfrac{n_1 - n_2}{n_1}\right) \ll 1$ and working principle was based on Kronig-Penny model in the band theory of solids. Chen et al. [14] suggested the design of optical filters using photonic band-gap air bridges and calculated important results regarding filtering properties. Recently D'Orazio et al. [15] have fabricated the photonic band-gap filter for wavelength division multiplexing.

In this paper, an optical filter based on periodic structure with different refractive index materials has been suggested. The theory and working principle of this filter is analogous to Kronig and Penny model in solid state physics.

2. Theoretical analysis

Consider optical radiation transmitted through the periodic structure of different refractive index materials analogues to the transmission of electron through a periodic lattice. For electron wave, there are some allowed and forbidden energy bands. If the electron waves are replaced by optical waves and the periodic atomic or molecular lattice is replaced by periodic refractive index pattern, we can expect allowed and forbidden bands of frequencies instead of energies. By choosing a linearly periodic refractive index profile in the filter material one obtains a given set of wavelength ranges that are allowed or forbidden to pass through the structure considered here. Selecting a particular *x*-axis through the material, we shall assume a periodic step function for

$$n(x) = \begin{cases} n_1, & 0 \le x \le a; \\ n_2, & -b \le x \le 0; \end{cases} \tag{1}$$

where $n(x) = n(x + md)$ and *m* is the translation factor, with $m = 0, \pm1, \pm2, \pm3, \ldots$, and $d = a + b$ is the lattice period with *a* and *b* being the width of the two regions having refractive indices (n_1) and (n_2) respectively. The refractive index profile of the materials in the form of rectangular symmetry is shown in the Fig. 1.
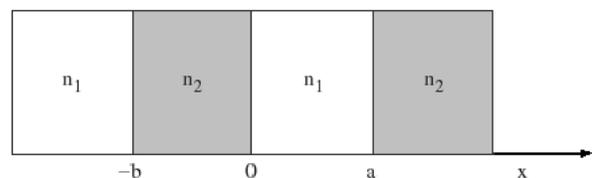

Fig. 2. Periodic refractive index profile of PBG.

If θ is the angle of incidence on this periodic structure, the one-dimensional wave equation for the spatial part of the electromagnetic eigen mode $\psi_k(x)$ is given by

$$\frac{d^2\psi_k(x)}{dx^2} + \frac{n^2(x) \cdot \cos^2\theta \cdot \omega_k^2}{c^2}\psi_k(x) = 0, \qquad (2)$$

where $n(x)$ is given by Equation (1). Therefore, Equation (2) may be written as a pair of equations as

$$\frac{d^2\psi_k(x)}{dx^2} + \frac{n_1^2 \cdot \cos^2\theta_1 \cdot \omega_k^2}{c^2}\psi_k(x) = 0; \quad 0 \le x \le a \quad (3a)$$

$$\frac{d^2\psi_k(x)}{dx^2} + \frac{n_2^2 \cdot \cos^2\theta_2 \cdot \omega_k^2}{c^2}\psi_k(x) = 0; \quad -b \le x \le a \quad (3b)$$

where θ_1 and θ_2 are ray angle in the layer of refractive index n_1 and n_2 respectively.

The periodic nature of the problem allows the application of Bloch's theorem where the solution of equations (3) can be written as $\psi_K(x) = u_K(x)e^{iKx}$ where K is known as Bloch wave number and $u_K(x)$ is the eigen function. Thus, using this theorem and after some mathematical calculations, we get the following relation

$$\cos(K \cdot d) = \cos(\alpha a) \cdot \cos(\beta b) - \frac{1}{2}\left(\frac{n_1 \cdot \cos\theta_1}{n_2 \cdot \cos\theta_2} - \frac{n_2 \cdot \cos\theta_2}{n_1 \cdot \cos\theta_1}\right) \cdot \sin(\alpha a) \cdot \sin(\beta b)$$

$$(4)$$

where, $\quad \alpha = \left(\frac{n_1\omega}{c}\cos\theta_1\right)$, $\beta = \left(\frac{n_2\omega}{c}\cos\theta_2\right)$,

and $\quad \theta_1 = \cos^{-1}[1 - \frac{\sin^2\theta}{n_1}]^{\frac{1}{2}}$,

$$\theta_2 = \cos^{-1}[1 - \frac{\sin^2\theta}{n_2}]^{\frac{1}{2}}$$

Now, abbreviating the L.H.S. as $L(\lambda)$, Equation (4) may be written as

$$L(\lambda) = \cos(K \cdot d) \qquad (5)$$

The maximum and minimum values of right hand side of equation (5) will be +1 and -1 respectively, because of the cosine function.
Now again by using equation (4) we can obtain the expression for the wave vector as

$$K(\lambda) = \cos^{-1}\left[\cos(\alpha a) \cdot \cos(\beta b) - \frac{1}{2}\left(\frac{n_1 \cdot \cos\theta_1}{n_2 \cdot \cos\theta_2} - \frac{n_2 \cdot \cos\theta_2}{n_1 \cdot \cos\theta_1}\right) \cdot \sin(\alpha a) \cdot \sin(\beta b)\right]$$

$$(6)$$

Here $K(\lambda)$ is the function of λ as α and β are the functions of ω and ω is the function of λ. We can use either equation

(5) or equation (6) for calculating the allowed bands of the considered structures.

3. Results and discussion

3.1 Optical filter

For the proposed filter, we have chosen two dielectric materials namely, ZnS and Ge arranged in the form of alternate periodic layers with low and high refractive indices in ultraviolet region. The refractive index for ZnS is 2.2 (n_1), and that of Ge is 4.2 (n_2). The thicknesses of the two layers of the unit cell are a and b respectively. Taking the values of a and b of such a structure as $a = 0.85d$ and $b = 0.15d$ where $d=a+b$ is the lattice period.
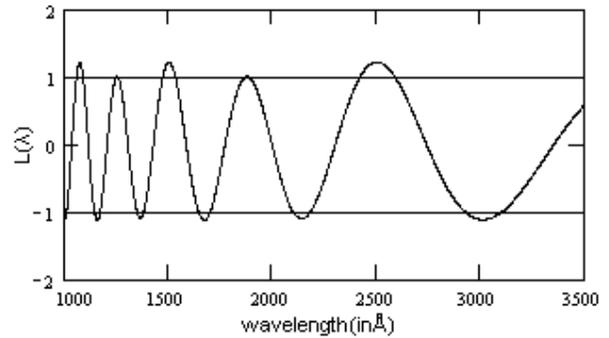
Fig. 2. Variation of L(λ) with wavelength (λ) for n$_1$ = 2.2, n$_2$ = 4.2, d = 300 nm, a = 0.85d and b = 0.15d and θ = 0˚.

Using these parameters, the function $L(\lambda)$ given by Equation (5) is plotted against the wavelength λ. The resulting curves are shown in the Figs. 2 to 4 for different angles of incidence.
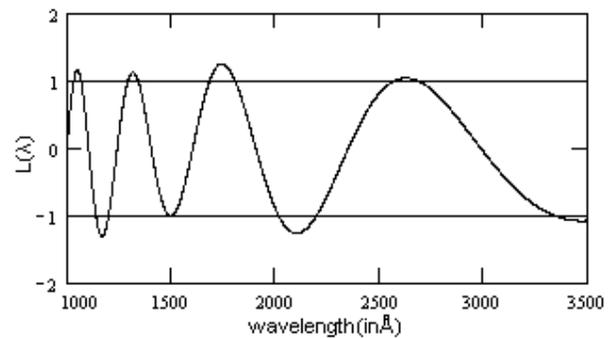
Fig. 3. Variation of L(λ) with wavelength (λ) for n$_1$ = 2.2, n$_2$ = 4.2, d = 300 nm, a = 0.85d and b = 0.15d and θ = 45˚.

It is clear from Equation (5), that the values of the expression on the right hand side of equation (5) are between ± 1. Hence, the region for which $L(\lambda)$ lying between ± 1, is called allowed region of λ and for which

$L(\lambda)$ lying outside ± 1 is called forbidden region of λ. The allowed bands are given in Table1.

From the study of Figures and Table 1 it is clear that for the fixed values of a, b, n_1 and n_2 the photonic band width increases as wavelength increases. As we increase the angle of incidence, the total number of allowed bands decreases but the band width of all bands increases.
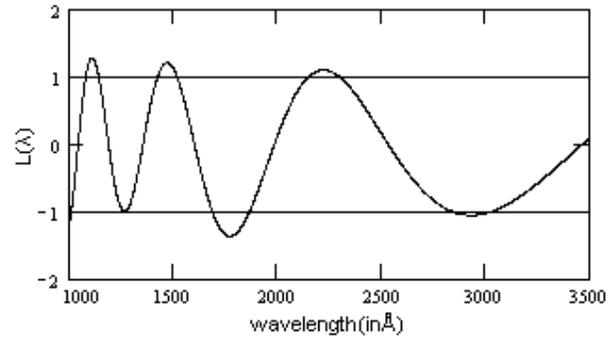

Fig. 4. Variation of $L(\lambda)$ with wavelength (λ) for $n_1 = 2.2$, $n_2 = 4.2$, $d = 300$ nm, $a = 0.85d$ and $b = 0.15d$ and $\theta = 75°$.

Table 1: Photonic bands for $n_1 = 2.2$, $n_2 = 4.2$, $d = 300$ nm, $a = 0.85d$ and $b = 0.15d$ for various incident angles.

Allowed Bands	$\theta = 0°$		$\theta = 45°$		$\theta = 75°$	
	Allowed Ranges (in Å)	Band Width (in Å)	Allowed Ranges (in Å)	Band Width (in Å)	Allowed Ranges (in Å)	Band Width (in Å)
1	1008-1056	48	1000-1027	27	1007-1077	70
2	1087-1141	54	1062-1133	71	1139-1257	118
3	1170-1248	78	1195-1287	92	1268-1422	154
4	1254-1345	91	1336-1481	145	1514-1683	169
5	1378-1470	92	1505-1680	175	1864-2149	285
6	1531-1642	111	1802-2010	208	2292-2832	540
7	1699-1872	173	2196-2574	378	-	-
8	1879-2097	218	2676-3331	655	-	-
9	2181-2418	237	-	-	-	-
10	2586-2923	337	-	-	-	-
11	3102-3500	398	-	-	-	-

The wavelengths corresponding to the allowed ranges are transmitted through the filter structure but the wavelengths corresponding to the forbidden ranges are reflected from the filter structure if the absorption part is negligible. The band widths depend upon the controlling parameters *i.e.* d (lattice period), n_1 and n_2. If we increased lattice period d (without changing the percentage of a and b), then the number of allowed bands increases but band width decreases and if d is decreased, then number of allowed bands decreases but band width increases.
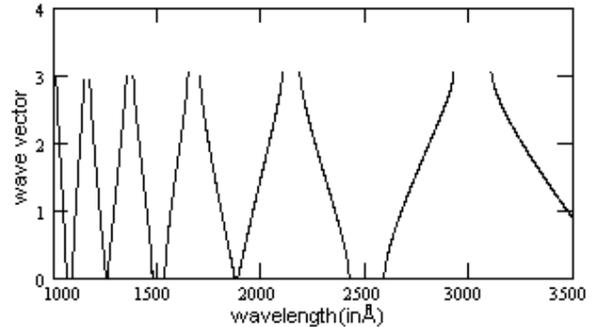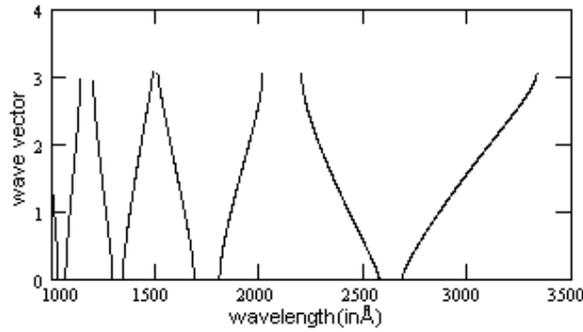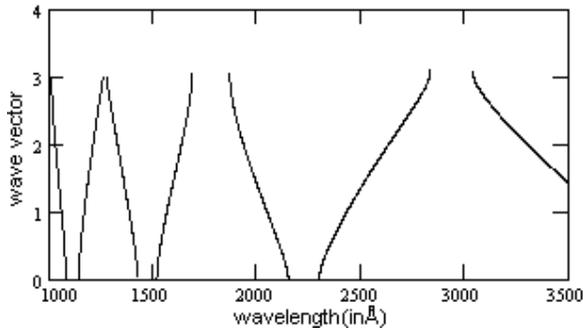

Fig. 5. Variation of wave vector with wavelength (λ) for $n_1 = 2.2$, $n_2 = 4.2$, $d = 300$ nm, $a = 0.85d$ and $b = 0.15d$ and $\theta = 0°$.

Fig. 6. Variation of wave vector with wavelength (λ) for n₁ = 2.2, n₂ = 4.2, d = 300 nm, a = 0.85d and b = 0.15d and θ = 45˚.

Also, we can compute allowed and forbidden bands from dispersion curves (curves between wave vector and wavelength). Figures 5 to 7 represents the dispersion curves for angle of incidence 0˚, 45˚ and 75˚ respectively. From the observation of these figures it is found that the allowed and forbidden bands are identical to the Table 1.

Fig. 7. Variation of wave vector with wavelength (λ) for n₁ = 2.2, n₂ = 4.2, d = 300 nm, a = 0.85d and b = 0.15d and θ = 75˚.

Choosing different values of controlling parameters we can obtain desired ranges of reflection and transmission. On the other hand, the transmission through the filter generally decreases as the value of $\left(\dfrac{n_1 - n_2}{n_2}\right)$ increases for the fixed values of *a* and *b*.

3.2 Monochromator

To find the forbidden and allowed values of the wavelengths, equation (2.11) has been evaluated numerically. For the sake of illustration, here we have considered only two structures, which are cascaded to each other. In these cases, we have chosen alternate layers of ZnS and dielectric Ge. The refractive indices are chosen to be 2.2 and 4.2 for ZnS and the Ge respectively. For the structure-1, we take lattice period, d=1800Å in which the thickness of air, a, is 0.95d and the thickness of dielectric, b, is 0.05d and for structure-2, we take lattice period,

d=2700Å in which the thickness of two alternate layers are in the same ratio as structure-1. The resulting dispersion curves for both structures are shown in Fig. 8.
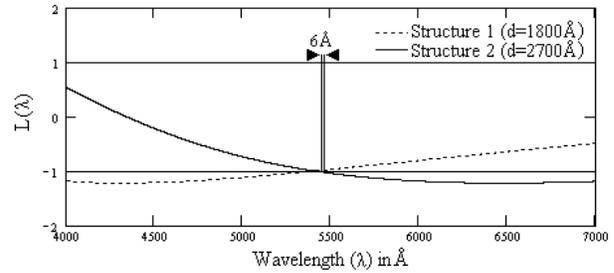

Fig. 8. Plot of the left side of the equation with respect to wavelength.

It is clear from Equation (6), that the value of right hand side lying between ± 1. Hence the region for which $L(\lambda)$ lying between ± 1, is called allowed region of λ and for which $L(\lambda)$ lying outside ± 1 is called forbidden region of λ. From the computation of the result, cascading of the two structures blocks the electromagnetic radiations having wavelengths below 5400Å and those electromagnetic radiations having wavelengths above 5405Å. The structure-1 is responsible for blocking wavelengths below 5400Å whereas the structure-2 is responsible for blocking wavelengths above 5405Å. In this way if we cascade the two structures parallel to the direction of propagation, we have a narrow bandwidth. The bandwidth, from which the radiation can transmit, is 6Å (5400-5405Å). We can achieve the desired ranges of bandwidth by changing the lattice period of the structures and the percentages of the two materials.

In the same manner, we can also design a monochromator, which can be used in lasers and other optical devices.

3. Conclusion

This type of filter and monochromator may be used in many optical devices and in other optical systems. By choosing appropriate values of controlling parameters, we can design a frequency selector or rejecter. Also, by cascading two, three or more filters we can design a perfect mirror or monochromator.

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