Optical soliton perturbation with spatio-temporal dispersion using modified simple equation method

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This paper studies optical soliton perturbation by modified simple equation method. The spatio-temporal dispersion term is included, in addition to group velocity dispersion. There are four types of nonlear media that are addressed. These are Kerr, power, parabolic and dual-power law. This scheme only reveals topological and singular optical solitons.

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1. Introduction

Optical soliton perturbation is a core area of research in the field of nonlinear optics. There are various aspects in this topic that are touched upon by several scientists across the globe [1-12]. A few such issues are soliton perturbation theory, quasi-stationary solutions [3], quasi-patrticle theory, optical soliton cooling. This paper is going to take up the study of integrability of the model, namely the perturbed nonlinear Schrödinger's equation (NLSE). There are a variety of integration schemes that are applied to study the model in order to extract soliton and shock wave solutions. Some of these algorithms are traveling wave hypothesis, method of undetermined coefficients, semi-inverse variational principle, G'/G-expansion, Lie symmetry [6], extended Kudryashov's method [11] and countless others. This paper adopted the modified simple equation approach to secure soliton solutions to the perturbed NLSE. This will only reveal topological and singular soliton solutions to the model. Bright soliton solutions cannot be recovered with this integration tool. Such is the limitation to this scheme. There are four types of nonlinear media that are studied in this paper. They are Kerr law, power law, parabolic law and dual-power law.

2. The modified simple equation method

Suppose we have a nonlinear evolution equation in the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \tag{1}$$

where P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [11, 10, 2].

Step-1: We use the transformation

$$u(x,t) = u(\xi),\ \ \xi = x - ct, \tag{2}$$

where c is a constant to be determined, to reduce Eq. (1) to the following ODE :

$$Q(u, u', u'', \ldots) = 0, \tag{3}$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}$.

Step-2: We suppose that Eq. (3) has the formal solution.

$$u(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{4}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3).

Step-4: We substitute (4) into (3), then we calculate all the necessary derivatives $u', u'', \ldots$ of the unknown function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi),\ j = 0,1,2,\ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find $a_k$and $\psi(\xi)$. Consequently, we can get the exact solutions of Eq. (1) .

3. Application to NLSE

The dimensionless form of the improved perturbed NLSE is given by [1, 2]

$$iq_t + aq_{tx} + bq_{xx} + cF\left(|q|^2\right)q = i\left\{\alpha q_x + \lambda\left(|q|^{2m} q\right)_x + \upsilon\left(|q|^{2m}\right)_x q\right\}, \quad (5)$$

In (5), the two independent variables are x and t which represent the spatial and temporal variables respectively. The dependent variable $q(x,t)$ is the soliton pulse profile. The first term is the linear evolution term while the coefficients of a and b accounts for the dispersion term where the coefficient of a represents the improved term that introduces stability to the NLSE which is otherwise an ill-posed problem. The coefficient of b is the usual group velocity dispersion. From the perturbation terms on the right hand side, the coefficient of α is the inter-modal dispersion, while λ is the self-steepening perturbation term and finally υ is the nonlinear dispersion coefficient. The parameter m is the full nonlinearity factor that is studied on a generalized setting. These perturbation terms are all of Hamiltonian type and hence the perturbed NLSE given by (5) makes it integrable.

In Eq. (5), F is a real-valued algebraic function and it is necessary to have the smoothness of the complex function $F(|q|^2)q : C \mapsto C$. Considering the complex plane C as a two-dimensional linear space R_2, the function $F(|q|^2)q$ is k times continuously differentiable, so that

$$F(|q|^2)q \in \cup_{m,n=1}^{\infty} C^k\big((-n,n) \times (-m,m); R^2\big). \quad (6)$$

In order to solve Eq. (5), we use the following wave transformation

$$q(x,t) = U(\xi)e^{i\Phi(x,t)} \quad (7)$$

where $U(\xi)$ represents the shape of the pulse and

$$\xi = k(x - vt), \quad (8)$$

$$\Phi(x,t) = -\kappa x + \omega t + \theta. \quad (9)$$

In Eq. (7), the function $\Phi(x,t)$ is the phase component of the soliton. Then, in Eq. (9), κ is the soliton frequency, while ω is the wave number of the soliton and θ is the phase constant. Finally in Eq. (8), v is the velocity of the soliton. Substituting Eq. (7) into Eq. (5) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v = \frac{a\omega - 2b\kappa - \alpha}{1 - a\kappa}, \quad (10)$$

while the real part gives

$$k^2(b - av)U'' - \left(\omega + \alpha\kappa - a\omega\kappa + b\kappa^2\right)U - \lambda\kappa U^{2m+1} + cF\left(U^2\right)U = 0. \quad (11)$$

The relation Eq. (10) gives the velocity of the soliton in terms of the wave number while Eq. (11) can be integrated to compute the soliton profile provided the functional is known.

3.1. Kerr law nonlinearity

The Kerr law nonlinearity is the case when $F(s) = s$. For Kerr law nonlinear medium, $m = 1$ in order for Eq. (5) to be integrable. Thus, (5) reduces to

$$iq_t + aq_{tx} + bq_{xx} + c|q|^2 q = i\left\{\alpha q_x + \lambda\left(|q|^2 q\right)_x + \upsilon\left(|q|^2\right)_x q\right\}, \quad (12)$$

and Eq. (11) simplifies to

$$k^2(b - av)U'' - \left(\omega + \alpha\kappa - a\omega\kappa + b\kappa^2\right)U + (c - \lambda\kappa)U^3 = 0. \quad (13)$$

Balancing U'' with U^3 in Eq. (13) gives $N = 1$. Consequently we reach

$$U(\xi) = a_0 + a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right), a_1 \neq 0. \quad (14)$$

Substituting Eq. (14) in Eq. (13) and then setting the coefficients of $\psi^{-j}(\xi), j = 0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving a_0, a_1, k, κ, v and ω as follows:

ψ^{-3} coeff.:

$$a_1(\psi')^3\left(2k^2(b-av)+a_1^2(c-\kappa\lambda)\right)=0, \tag{15}$$

ψ^{-2} coeff.:

$$3a_1\psi'(k^2\psi''(av-b)+a_0a_1\psi'(c-\kappa\lambda))=0, \tag{16}$$

ψ^{-1} coeff.:

$$-a_1\begin{pmatrix}\psi'\begin{pmatrix}-3a_0^2(c-\kappa\lambda)-a\kappa\omega+\alpha\kappa+\\ b\kappa^2+\omega\end{pmatrix}\\ +k^2\psi^{(3)}(av-b)\end{pmatrix}=0, \tag{17}$$

ψ^{0} coeff.:

$$-a_0\begin{pmatrix}a_0^2(\kappa\lambda-c)-a\kappa\omega+\\ \alpha\kappa+b\kappa^2+\omega\end{pmatrix}=0. \tag{18}$$

Solving this system, we obtain

$$a_0=\pm\sqrt{\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{c-\kappa\lambda}},\quad a_1=\mp\sqrt{-\frac{2k^2(b-av)}{c-\kappa\lambda}}. \tag{19}$$

and

$$\psi''=\sqrt{-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}}\psi', \tag{20}$$

$$\psi'''=-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}\psi'. \tag{21}$$

From Eqs. (20) and (21), we can deduce that

$$\psi'=\sqrt{-\frac{k^2(b-av)}{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}}\,k_1e^{\sqrt{-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}}\xi}, \tag{22}$$

and

$$\psi=-\frac{k^2(b-av)}{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}\,k_1e^{\sqrt{-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}}\xi}+k_2, \tag{23}$$

where k_1 and k_2 are constants of integration. Substituting Eq. (22) and Eq. (23) into Eq. (14), we obtain following the following exact solution to Eq. (12)

$$q(x,t)=\pm\left\{\sqrt{\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{c-\kappa\lambda}}+\frac{\frac{k^2(b-av)}{\sqrt{(c-\kappa\lambda)(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}}k_1e^{\sqrt{-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}}\xi}}{-\frac{k^2(b-av)}{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}k_1e^{\sqrt{-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}}\xi}+k_2}\right\}\times e^{i(-\kappa x+\omega t+\theta)}. \tag{24}$$

If we set

$$k_1=-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}e^{\sqrt{-\frac{2(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)}{k^2(b-av)}}\xi_0},\quad k_2=\pm1,$$

we obtain:

(i) When

$$(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)(av-b)>0,$$

we have

$$q(x,t)=\pm\sqrt{\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{c-\kappa\lambda}}\tanh\left[\sqrt{\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{2k^2(av-b)}}\left(k(x-vt)+\xi_0\right)\right]\times e^{i(-\kappa x+\omega t+\theta)} \tag{25}$$

$$q(x,t)=\pm\sqrt{\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{c-\kappa\lambda}}\coth\left[\sqrt{\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{2k^2(av-b)}}\left(k(x-vt)+\xi_0\right)\right]\times e^{i(-\kappa x+\omega t+\theta)}, \tag{26}$$

where (25) and (26) represent dark soliton and singular soliton solutions.

(ii) When

$$(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega)(av-b)<0,$$

we have the following periodic singular solutions:

$$q(x,t)=\pm\sqrt{-\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{c-\kappa\lambda}}\tan\left[\sqrt{-\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{2k^2(av-b)}}\left(k(x-vt)+\xi_0\right)\right]\times e^{i(-\kappa x+\omega t+\theta)}, \tag{27}$$

$$q(x,t)=\mp\sqrt{-\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{c-\kappa\lambda}}\cot\left[\sqrt{-\frac{-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega}{2k^2(av-b)}}\left(k(x-vt)+\xi_0\right)\right]\times e^{i(-\kappa x+\omega t+\theta)}, \tag{28}$$

where v is given by Eq. (10) and ω is an arbitrary constant.

3.2 Power law nonlinearity

The power law nonlinearity is the case when $F(s)=s^n$. For Power law nonlinearity, $m=n$ in order for Eq. (5) to be integrable. Thus, (5) reduces to

$$iq_t+aq_{tx}+bq_{xx}+c|q|^{2n}q=i\left\{\alpha q_x+\lambda\left(|q|^{2n}q\right)_x+\upsilon\left(|q|^{2n}\right)_x q\right\}, \tag{29}$$

and Eq. (11) simplifies to

$$k^2(b-av)U''-\left(\omega+\alpha\kappa-a\omega\kappa+b\kappa^2\right)U+(c-\lambda\kappa)U^{2n+1}=0. \tag{30}$$

Set

$$U=V^{\frac{1}{n}} \tag{31}$$

so that (30) transforms to

$$k^2(b-av)\left(nVV''+(1-n)(V')^2\right)-n^2\left(\omega+\alpha\kappa-a\omega\kappa+b\kappa^2\right)V^2+n^2V^4(c-\kappa\lambda)=0. \tag{32}$$

Balancing VV'' with V^4 in Eq. (32) gives $N=1$. Consequently we reach

$$V(\xi)=a_0+a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right),\quad a_1\neq 0. \tag{33}$$

Substituting Eq. (33) in Eq. (32) and then setting the coefficients of $\psi^{-j}(\xi)$, $j=0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving a_0, a_1, k, κ, v and ω as follows:

ψ^{-4} coeff.:

$$a_1^2(\psi')^4\left(k^2(n+1)(b-av)+a_1^2n^2(c-\kappa\lambda)\right)=0, \tag{34}$$

ψ^{-3} coeff.:

$$a_1(\psi')^2\begin{pmatrix}2a_0n\psi'\left(k^2(b-av)+2a_1^2n(c-\kappa\lambda)\right)-\\ a_1k^2(n+2)\psi''(b-av)\end{pmatrix}=0, \tag{35}$$

ψ^{-2} coeff.:

$$\begin{gathered}-3a_1a_0k^2n\psi'\psi''(b-av)-a_1^2n^2(\psi')^2\\ \left(-6a_0^2(c-\kappa\lambda)-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)-\\ a_1^2\begin{pmatrix}k^2(n-1)(\psi'')^2(b-av)-\\ a_1^2k^2n\psi^{(3)}\psi'(av-b)\end{pmatrix}=0,\end{gathered} \tag{36}$$

ψ^{-1} coeff.:

$$a_0a_1(-n)\begin{pmatrix}2n\psi'\left(-2a_0^2(c-\kappa\lambda)-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)\\ +k^2\psi^{(3)}(av-b)\end{pmatrix}=0, \tag{37}$$

ψ^0 coeff.:

$$a_0^2\left(-n^2\right)\left(a_0^2(\kappa\lambda-c)-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)=0. \tag{38}$$

Solving this system, we obtain

$$a_0=0,\ a_1=\pm k\sqrt{-\frac{(n+1)(b-av)}{n^2(c-\kappa\lambda)}}. \tag{39}$$

and

$$\psi''=0, \tag{40}$$

$$\psi''' = \frac{n\left(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)}{k^2(b-av)}\psi'. \tag{41}$$

Eqs. (40) and (41) give a trivial solution. This leads to recapitulation of a well known fact. Power law nonlinearity does not support topological solitons ot singular solitons of this type unless the nonlinearity collapses to Kerr type [4].

3.3 Parabolic law nonlinearity

The parabolic law nonlinearity is the case when $F(s)=\eta s+\zeta s^2$. For Parabolic law nonlinearity, $m=1$ in order for Eq. (5) to be integrable. Thus, (5) reduces to

$$\begin{aligned} &iq_t + aq_{tx} + bq_{xx} + \left(c_1|q|^2 + c_2|q|^4\right)q = \\ &i\left\{\alpha q_x + \lambda\left(|q|^2 q\right)_x + \upsilon\left(|q|^2\right)_x q\right\}, \end{aligned} \tag{42}$$

and Eq. (11) simplifies to

$$\begin{aligned} &k^2(b-av)U'' - \left(\omega+\alpha\kappa-a\omega\kappa+b\kappa^2\right)U \\ &+(c_1-\lambda\kappa)U^3 + c_2U^5 = 0, \end{aligned} \tag{43}$$

where $c_1 = c\eta, c_2 = c\zeta.$ Set

$$U = V^{\frac{1}{2}} \tag{44}$$

so that (43) transforms to

$$\begin{aligned} &k^2(b-av)\left(2VV''-(V')^2\right) \\ &-4\left(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)V^2 \\ &+4\left(c_1-\kappa\lambda\right)V^3 + 4c_2V^4 = 0. \end{aligned} \tag{45}$$

Balancing VV'' with V^4 in Eq. (45) gives $N=1.$ Consequently we reach

$$V(\xi) = a_0 + a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right), \quad a_1 \neq 0. \tag{46}$$

Substituting Eq. (46) in Eq. (45) and then setting the coefficients of $\psi^{-j}(\xi),\ j=0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving a_0, a_1, k, κ, v and ω as follows:

ψ^{-4} coeff.:

$$a_1^2\left(\psi'\right)^4\left(3k^2(b-av)+4a_1^2c_2\right)=0, \tag{47}$$

ψ^{-3} coeff.:

$$4a_1\left(\psi'\right)^2\begin{pmatrix} a_1\left(k^2\psi''(av-b)+a_1\psi'\left(c_1-\kappa\lambda\right)\right) \\ +a_0\psi'\left(k^2(b-av)+4a_1^2c_2\right) \end{pmatrix}=0, \tag{48}$$

ψ^{-2} coeff.:

$$\begin{aligned} &-4a_1^2\left(\psi'\right)^2\begin{pmatrix} 3a_0\left(\kappa\lambda-c_1\right)-6a_0^2c_2- \\ a\kappa\omega+\alpha\kappa+b\kappa^2+\omega \end{pmatrix} \\ &-6a_0a_1k^2\psi'\psi''(b-av)a_1^2 + k^2\left(\psi''\right)^2(av-b) \\ &+2a_1^2k^2\psi^{(3)}\psi'(b-av)=0, \end{aligned} \tag{49}$$

ψ^{-1} coeff.:

$$-2a_0a_1\left(\psi'\begin{pmatrix} 4\left(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)+ \\ 6a_0\left(\kappa\lambda-c_1\right)-8a_0^2c_2+k^2\psi^{(3)}(av-b) \end{pmatrix}\right)=0, \tag{50}$$

ψ^0 coeff.:

$$\begin{aligned} &4a_0^2(a_0\left(c_1-\kappa\lambda\right)+a_0^2c_2+ \\ &a\kappa\omega-\alpha\kappa-b\kappa^2-\omega \end{aligned} \tag{51}$$

Solving this system, we obtain

$$\begin{aligned} &a_0 = -\frac{3\left(c_1-\kappa\lambda\right)}{4c_2},\ a_1 = \pm\sqrt{\frac{3k^2(av-b)}{4c_2}}, \\ &\omega = \frac{16c_2\left(\alpha\kappa+b\kappa^2\right)+3\left(c_1-\kappa\lambda\right)^2}{16c_2(a\kappa-1)}. \end{aligned} \tag{52}$$

and

$$\psi'' = \pm\sqrt{\frac{3\left(c_1-\kappa\lambda\right)^2}{4c_2k^2(av-b)}}\,\psi', \tag{53}$$

$$\psi''' = \frac{3\left(c_1-\kappa\lambda\right)^2}{4c_2k^2(av-b)}\,\psi'. \tag{54}$$

From Eqs. (53) and (54), it is possible to deduce

$$\psi' = \pm\sqrt{\frac{4c_2k^2(av-b)}{3\left(c_1-\kappa\lambda\right)^2}}\,k_1 e^{\pm\sqrt{\frac{3\left(c_1-\kappa\lambda\right)^2}{4c_2k^2(av-b)}}\xi}, \tag{55}$$

and

$$\psi = \frac{4c_2k^2(av-b)}{3(c_1-\kappa\lambda)^2} k_1 e^{\pm\sqrt{\frac{3(c_1-\kappa\lambda)^2}{4c_2k^2(av-b)}}\xi} + k_2, \qquad (56)$$

where k_1 and k_2 are constants of integration. Substituting Eq. (55) and Eq. (56) into Eq. (46), we obtain the following exact solution to Eq. (42)

$$q(x,t) = \left\{ -\frac{3(c_1-\kappa\lambda)}{4c_2} + \frac{\sqrt{\frac{k^4(av-b)^2}{(c_1-\kappa\lambda)^2}}\, k_1 e^{\pm\sqrt{\frac{3(c_1-\kappa\lambda)^2}{4c_2k^2(av-b)}}\xi}}{\frac{4c_2k^2(av-b)}{3(c_1-\kappa\lambda)^2} k_1 e^{\pm\sqrt{\frac{3(c_1-\kappa\lambda)^2}{4c_2k^2(av-b)}}\xi} + k_2} \right\}^{\frac{1}{2}}$$
$$\times e^{i\left(-\kappa x + \left(\frac{16c_2(\alpha\kappa+b\kappa^2)+3(c_1-\kappa\lambda)^2}{16c_2(a\kappa-1)}\right)t+\theta\right)} \qquad (57)$$

If we set

$$k_1 = \frac{3(c_1-\kappa\lambda)^2}{4c_2k^2(av-b)} e^{\pm\sqrt{\frac{3(c_1-\kappa\lambda)^2}{4c_2k^2(av-b)}}\xi_0}, \quad c_2 = \pm 1,$$

we obtain:

$$q(x,t) = \left\{ \frac{3(c_1-\kappa\lambda)}{8c_2} \left(-1 \pm \tanh \left[\sqrt{\frac{3(c_1-\kappa\lambda)^2}{16c_2k^2(av-b)}} (k(x-vt)+\xi_0) \right] \right) \right\}^{\frac{1}{2}}$$
$$\times e^{i\left(-\kappa x + \left(\frac{16c_2(\alpha\kappa+b\kappa^2)+3(c_1-\kappa\lambda)^2}{16c_2(a\kappa-1)}\right)t+\theta\right)} \qquad (58)$$

$$q(x,t) = \left\{ \frac{3(c_1-\kappa\lambda)}{8c_2} \left(-1 \pm \coth \left[\sqrt{\frac{3(c_1-\kappa\lambda)^2}{16c_2k^2(av-b)}} (k(x-vt)+\xi_0) \right] \right) \right\}^{\frac{1}{2}}$$
$$\times e^{i\left(-\kappa x + \left(\frac{16c_2(\alpha\kappa+b\kappa^2)+3(c_1-\kappa\lambda)^2}{16c_2(a\kappa-1)}\right)t+\theta\right)} \qquad (59)$$

where v is given by Eq. (10) and

$$c_2(av-b) > 0.$$

Equations (58) and (59) represent dark and singular soliton solutions respectively.

3.4 Dual-power law nonlinearity

The Dual-Power law nonlinearity is the case when $F(s) = \eta s^n + \zeta s^{2n}$. For Dual-Power law nonlinearity, $m = n$ in order for Eq. (5) to be integrable. Thus, (5) reduces to

$$\begin{aligned} &iq_t + aq_{tx} + bq_{xx} + \left(c_1|q|^{2n} + c_2|q|^{4n}\right)q \\ &= i\left\{\alpha q_x + \lambda\left(|q|^{2n} q\right)_x + \upsilon\left(|q|^{2n}\right)_x q\right\}, \end{aligned} \qquad (60)$$

and Eq. (11) simplifies to

$$\begin{aligned} &k^2(b-av)U'' - \left(\omega + \alpha\kappa - a\omega\kappa + b\kappa^2\right)U \\ &+(c_1-\lambda\kappa)U^{2n+1} + c_2U^{4n+1} = 0, \end{aligned} \qquad (61)$$

where $c_1 = c\eta, c_2 = c\zeta$. Set

$$U = V^{\frac{1}{2n}} \qquad (62)$$

so that (61) transforms to

$$\begin{aligned} &k^2(b-av)\left(2nVV'' + (1-2n)(V')^2\right) \\ &-4n^2\left(-a\kappa\omega + \alpha\kappa + b\kappa^2 + \omega\right)V^2 \\ &+4n^2(c_1-\kappa\lambda)V^3 + 4c_2n^2V^4 = 0. \end{aligned} \qquad (63)$$

Balancing VV'' with V^4 in Eq. (13) gives $N = 1$. Consequently we reach

$$V(\xi) = a_0 + a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right), \quad a_1 \neq 0. \qquad (64)$$

Substituting Eq. (64) in Eq. (63) and then setting the coefficients of $\psi^{-j}(\xi),\ j = 0,1,2,3,$ to zero, then we obtain a set of algebraic equations involving $a_0,\ a_1,\ k,\ \kappa,\ v$ and ω as follows:

ψ^{-4} coeff.:

$$a_1^2(\psi')^4\left(\begin{aligned}&k^2(2n+1)(b-av)\\&+4a_1^2c_2n^2\end{aligned}\right) = 0, \qquad (65)$$

ψ^{-3} coeff.:

$$2a_1(\psi')^2\begin{pmatrix} a_1\begin{pmatrix} 2a_1n^2\psi'(c_1-\kappa\lambda) \\ -k^2(n+1)\psi''(b-av)\end{pmatrix} \\ +2a_0n\psi'\left(k^2(b-av)+4a_1^2c_2n\right)\end{pmatrix}=0, \tag{66}$$

ψ^{-2} coeff.:

$$\begin{aligned}&-6a_1a_0k^2n\psi'\psi''(b-av)-4a_1^2n^2(\psi')^2\\&\left(3a_0(\kappa\lambda-c_1)-6a_0^2c_2-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right)-\\&a_1^2k^2(2n-1)(\psi'')^2(b-av)\\&+2a_1^2k^2n\psi^{(3)}\psi'(b-av)=0,\end{aligned} \tag{67}$$

ψ^{-1} coeff.:

$$-2a_0a_1n\begin{pmatrix} 2n\psi'\begin{pmatrix} 2\left(-a\kappa\omega+\alpha\kappa+b\kappa^2+\omega\right) \\ +3a_0(\kappa\lambda-c_1)-4a_0^2c_2\end{pmatrix} \\ +k^2\psi^{(3)}(av-b)\end{pmatrix}=0, \tag{68}$$

ψ^0 coeff.:

$$\begin{aligned}&4a_0^2n^2(a_0(c_1-\kappa\lambda)+a_0^2c_2\\&+a\kappa\omega-\alpha\kappa-b\kappa^2-\omega\end{aligned} \tag{69}$$

Solving this system, we obtain

$$a_0=-\frac{(2n+1)(c_1-\kappa\lambda)}{2(n+1)c_2},$$

$$a_1=\pm\sqrt{\frac{(2n+1)k^2(av-b)}{4n^2c_2}},$$

$$\omega=\frac{4(n+1)^2c_2\left(\alpha\kappa+b\kappa^2\right)+(2n+1)(c_1-\kappa\lambda)^2}{4(n+1)^2c_2(a\kappa-1)}. \tag{70}$$

and

$$\psi''=\pm\sqrt{\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}}\,\psi', \tag{71}$$

$$\psi'''=\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}\,\psi'. \tag{72}$$

From Eqs. (71) and (72), it is possible to deduce

$$\psi'=\pm\sqrt{\frac{(n+1)^2c_2k^2(av-b)}{(2n+1)n^2(c_1-\kappa\lambda)^2}}\;k_1e^{\pm\sqrt{\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}}\xi}, \tag{73}$$

and

$$\psi=\frac{(n+1)^2c_2k^2(av-b)}{(2n+1)n^2(c_1-\kappa\lambda)^2}\;k_1e^{\pm\sqrt{\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}}\xi}+k_2, \tag{74}$$

where k_1 and k_2 are constants of integration. Substituting Eq. (73) and Eq. (74) into Eq. (64), we obtain the following exact solution to Eq. (60)

$$q(x,t)=\left\{\frac{(2n+1)(\kappa\lambda-c_1)}{2(n+1)c_2}+\frac{\sqrt{\frac{(n+1)^2k^4(av-b)^2}{4n^4(c_1-\kappa\lambda)^2}}\,k_1e^{\pm\sqrt{\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}}\xi}}{\frac{(n+1)^2c_2k^2(av-b)}{(2n+1)n^2(c_1-\kappa\lambda)^2}\,k_1e^{\pm\sqrt{\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}}\xi}+k_2}\right\}^{\frac{1}{2n}}\times e^{i\left(-\kappa x+\left(\frac{4(n+1)^2c_2\left(\alpha\kappa+b\kappa^2\right)+(2n+1)(c_1-\kappa\lambda)^2}{4(n+1)^2c_2(a\kappa-1)}\right)t+\theta\right)} \tag{75}$$

If we set

$$k_1=\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}e^{\pm\sqrt{\frac{(2n+1)n^2(c_1-\kappa\lambda)^2}{(n+1)^2c_2k^2(av-b)}}\xi_0},$$
$$c_2=\pm1,$$

we obtain:

$$q(x,t)=\left\{\frac{(2n+1)(c_1-\kappa\lambda)}{4(n+1)c_2}\left(-1\pm\tanh\left[\sqrt{\frac{(2n+1)(c_1-\kappa\lambda)^2}{4(n+1)^2c_2k^2(av-b)}}\left(k(x-vt)+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}\times e^{i\left(-\kappa x+\left(\frac{4(n+1)^2c_2\left(\alpha\kappa+b\kappa^2\right)+(2n+1)(c_1-\kappa\lambda)^2}{4(n+1)^2c_2(a\kappa-1)}\right)t+\theta\right)} \tag{76}$$

$$q(x,t)=\left\{\frac{(2n+1)(c_1-\kappa\lambda)}{4(n+1)c_2}\left(-1\pm\coth\left[\sqrt{\frac{(2n+1)\left(c_1-\kappa\lambda\right)^2}{4(n+1)^2c_2k^2(av-b)}}(k(x-vt)+\xi_0)\right]\right)\right\}^{\frac{1}{2n}} \times e^{i\left(-\kappa x+\left(\frac{4(n+1)^2c_2\left(\alpha\kappa+b\kappa^2\right)+(2n+1)\left(c_1-\kappa\lambda\right)^2}{4(n+1)^2c_2(a\kappa-1)}\right)t+\theta\right)} \tag{77}$$

where v is given by Eq. (10) and

$$c_2(av-b)>0.$$

Equations (76) and (77) represent dark and singular soliton solutions respectively.

4. Conclusions

This paper obtains dark and singular soliton solutions to the perturbed NLSE that was considered with four forms of nonlinear media. The integration alorithm applied is the modified simple equation method. These soliton solutions appear with the necessary integrability criteria that are often referred to as constraint conditions. Apparently, the shrtcoming of this scheme is that bright soliton solution cannot be recovered using this interability criteria. Later, this scheme will be applied to other situations such as brefringent fibers, DWDM systems, optical couplers and others. The results of those research will be soon visible. Additionally, models with time-dependent coefficients will also be considered.

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