

Performance investigation of free space optical communication system using Gaussian beam wave

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In this paper the propagation of Gaussian beam in turbulent atmosphere for free space optical communication has been studied. The intensity on axis of Gaussian beam wave, beam radius and radius of curvature at the receiver has been evaluated and discussed. The effect of aperture averaging on Gaussian beam wave for different turbulence strength of atmosphere has been shown graphically. The aperture averaging factor decreases under high atmospheric strength and averaging ability of the receiving system increases by increasing receiving aperture diameter. Additionally an improved expression of scintillation loss has been evaluated using threshold power approach. This expression takes into account the loss due to scintillation when Gaussian wave propagates through atmospheric turbulence. Results show that probability of fading and losses due to scintillation are considerably lower when threshold power level is set low.

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1. Introduction

Previous studies on free space optical communication have been emphasized on the atmosphere that degrades an optical signal in free space optics. [1]

When optical wave propagate through the turbulent atmospheric condition, it experiences a random irradiance fluctuations, called scintillation. This is due to perturbations by refractive index fluctuations. Optical wireless scintillation is considered to be most important atmospheric effects and can attract much attention in practical applications, especially in laser tracking ranging systems and laser imaging systems [1-6].

In the early 1960s, Tatarskii and Cherenkov focussed on Rytov approximation method and evaluated the scintillation index expressions of unbounded plane wave and spherical wave but their scintillation indices results have limited to weak turbulent only. After that lots of theoretical and experimental work based on irradiance fluctuation under strong turbulence regimes have been in process. Several mathematical models of scintillation theory has later developed and modified by others [7-12].

Andrews et al. gave a comprehensive summary of these studies about plane wave, spherical wave, and Gaussian beam. Wu and Wei discussed the scintillation index of a Gaussian beam considering inner-scale and outer-scale on slant path. Previous work concerned with the spread and average intensity of partially coherent beam [1, 13-15].

Then Eyyuboglu and Baykal presented scintillation evaluation in weak atmospheric turbulent system for somewhat coherent general beams based on the extended Huygens- Fresnel principle [16, 17].

A simplified block diagram of FSO system is shown in Fig. 1.

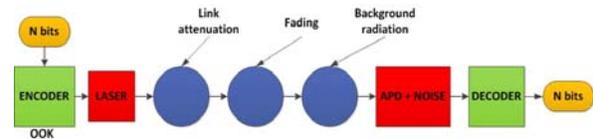


Fig. 1. Block diagram optical wireless link.

2. Gaussian beam wave at axis on z plane

The transmitting aperture of the gaussian beam has been placed in the plane $Z = 0$ and the distribution of amplitude has a gaussian function with effective beam radius W_0 . The phase distribution is parabolic with radius of curvature F_0 . The optical field of the wave on this plane having amplitude A_0 at the axis is given below [1]

$$U_0(\mathbf{r}, 0) = A_0 \exp(-r^2/W_0^2 - ik r^2/2F_0) \quad (1)$$

Here r is perpendicular distance from center line of beam.

Introducing the complex parameter α_0

$$\alpha_0 = \frac{2}{kW_0^2} + i \frac{1}{F_0} \quad , \quad [m^{-1}] \quad (2)$$

Here W_0 is waist size, $K=2\pi/\lambda$ denote wave number and i represent imaginary unit. From (2), the gaussian beam wave (1) can also be represented as [18]

$$U(r, L) = A_0 \exp\left(-\frac{1}{2} \alpha_0 k r^2\right) \quad (3)$$

Optical field of the gaussian-beam wave (3) at distance $z = L$ along the positive z axis represented by the Huygens-Fresnel integral [18, 19].

$$U_0(r, L) = -2ik \int_{-\infty}^{\infty} G(s, r; L) U_0(s, 0) d^2 s \quad (4)$$

Where $U_0(s, 0)$ represent the optical field at the source (transmitter) plane and $G(s, r; L)$ represent the green's function defined under the paraxial approximation by

$$G(s, r; L) = \frac{1}{4\pi L} \exp \left[ikL + \frac{ik}{2L} |s - r| \right] \quad (5)$$

By inserting the gaussian-beam formulation (2) into (3) and evaluate the resulting integrals, resulting a Gaussian-beam wave similar to (3) but with complex amplitude $A_0/(1 + i\alpha_0 L)$ [19]

$$U_0(r, L) = \frac{A_0}{1 + i\alpha_0 L} \exp \left[ikL - \frac{1}{2} \left(\frac{\alpha_0 k r^2}{1 + i\alpha_0 L} \right) \right] = \frac{A_0}{p(L)} \exp \left[ikL - \frac{1}{2} \left(\frac{\alpha_0 k r^2}{p(L)} \right) \right] \quad (6)$$

Where $p(L) = 1 + i\alpha_0 L$ called the propagation parameters [19].

The optical field $U_0(r, 0)$ passing through a aperture of radius r at transverse plane $Z=0$

$$U_0(r, 0) = A_0 \exp \left[-\frac{1}{2} (\alpha_0 k r^2) \right] e^{i\omega t} \quad (7)$$

Where $e^{i\omega t}$ has time dependence factor that may be suppressed in the further equations.

The optical intensity associated with Gaussian beam at radial distance r from the axis [19]

$$I(r, L) = I_0 \frac{W_0^2}{W^2(L)} \exp \left[-\frac{2r^2}{W^2(L)} \right] \quad (8)$$

The corresponding time-averaged intensity or irradiance for the beam located in plane $Z=0$

$$I(r, 0) = \frac{|E(r, 0)|^2}{2\eta} = I_0 \exp \left(\frac{2r^2}{W_0^2} \right) \quad (9)$$

Where $I_0 = I(0, 0)$ is the intensity at the centre of beam at its waist.

The incident power P on receiver aperture diameter D at a distance L given as [20,21]

$$P(D, L) = P_0 \left[1 - e^{-\left(\frac{D^2}{2W^2(L)} \right)} \right] \quad (10)$$

The power P passing through a aperture of diameter D at transverse plane $Z=0$

$$P(D, 0) = P_0 \left[1 - e^{-D^2/2W_0^2} \right] \quad (11)$$

Where total power transmitted by the beam $P_0 = \frac{1}{2} \pi I_0 W_0^2$. For a point receiver the diameter D approaches to zero, so the peak intensity on the axis of Gaussian beam has calculated using L'Hospital rule

$$I(0, 0) = \lim_{D \rightarrow 0} \frac{P_0 \left[1 - e^{-D^2/2W_0^2} \right]}{\frac{\pi D^2}{4}} \quad (12)$$

$$I(0, 0) = \frac{2P_0}{\pi W_0^2} \quad (13)$$

For physical interpretation it is needful to express beam wave equation in other form of beam radius and phase front radius of curvature at the receiver side. For this, introduce the following notation [19].

$$1 + i\alpha_0 L = \theta_0 + i_0, \quad (14)$$

Where θ_0 and i_0 are non dimensional real parameters at transmitter is defined by

$$\theta_0 = \text{Re}(1 + i\alpha_0 L) = 1 - \frac{L}{F_0}, \quad (15)$$

$$i_0 = \text{Im}(1 + i\alpha_0 L) = \frac{2L}{kW_0^2} \quad (16)$$

The notation of the beam radius and phase front radius of curvature at receiver is represented by W and F respectively. These beam characteristics at the receiver plane are related to the beam parameters at transmitter side as defined in (14) by [19]

$$F = F_0 (\theta_0^2 + i_0^2) (\theta_0 - 1) / (\theta_0^2 + i_0^2 - \theta_0) \quad (17)$$

Due to atmospheric effects, these beam parameters associated with gaussian wave in an indiscriminate medium, makes analysis more complicated. So it requires a characterization of related beam parameters of gaussian beam at the plane of receiver side.

From conformal transformation $1/(\theta_0 + i_0) = \theta - i$, where the (non dimensional) real parameters θ and i are defined by [22, 23]

$$\theta = \frac{\theta_0}{\theta_0^2 + i_0^2}, \quad i = \frac{i_0}{\theta_0^2 + i_0^2} \quad (18)$$

$$\theta = 1 + \frac{L}{F}, \quad i = \frac{2L}{kW^2} \quad (19)$$

By using above parameters, beam radius and phase front radius of curvature at the receiver side can be expressed in terms transmitter set of beam parameters as follows.

$$W = \left(\frac{2L(\theta_0^2 + i_0^2)}{K_0} \right)^{1/2} \quad (20)$$

$$F = \frac{L(\theta_0^2 + i_0^2)}{\theta_0^2 + i_0^2 + \theta_0} \quad (21)$$

By introducing two sets of non dimensional beam parameters at both sides, the beam spot radius and phase front radius of curvature as well as other beam parameters are determined from either set of beam parameters. We

can use these beam parameters to find out the location and size of the geometric focus and the beam waist.

When the Gaussian beam wave reach at the receiver side, the effect of scintillation can be minimized by aperture averaging technique that is discussed in next section.

3. Effect of aperture averaging for different atmospheric turbulence conditions

According to Rytov hypothesis, the intensity distribution shows lognormal behaviour only in weak to intermediate turbulence. Numerical analysis and experimental verification shows that lognormal action of the received intensity also implement good estimate in all turbulence regime (weak, intermediate, strong, saturation) except when extreme amounts of aperture averaging take place [24-26].

The aperture averaging factor has usually been used to measure the fading that decreases by aperture averaging process. The aperture averaging parameter is expressed as below [1][27].

$$A = \frac{\sigma_I^2(D)}{\sigma_I^2(0)} \quad (22)$$

Where, $\sigma_I^2(D)$ and $\sigma_I^2(0)$ denotes the scintillation index of a receiver aperture of diameter D and a point receiver whose diameter is approximately equals to zero respectively.

The minimum probable value of A is needed in order to overcome signal fading due to atmospheric instability [28].

The scintillation index obtained as follows [29,30].

$$\sigma_I^2(D) = \exp \left[\frac{0.49\sigma_R^2}{(1+0.653d^2+1.11\sigma_R^{12/5})^{7/6}} + \frac{0.51\sigma_R^2(1+0.69\sigma_R^{12/5})^{-5/6}}{1+0.9d^2+0.621d^2\sigma_R^{12/5}} \right] - 1 \quad (23)$$

$$\sigma_I^2(0) = \exp \left\{ \frac{0.49\sigma_B^2}{[1+0.56(1+\theta)\sigma_B^{12/5}]^{7/6}} + \frac{0.51\sigma_B^2}{(1+0.69\sigma_B^{12/5})^{5/6}} \right\} - 1 \quad (24)$$

The aperture averaging factor can be approximated as [1]

$$A \approx \left[1 + 1.062 \left(\frac{D^2 k}{4L} \right) \right]^{-7/6} \quad (25)$$

Also note that, above equation assumes $l_0 \ll \sqrt{L/k}$

According to kolmogorov spectrum, the spectral density for refractive-index fluctuations over the internal sub range is defined by [1]

$$\phi_n(K) = 0.033C_n^2 K^{-11/3} \quad 1/L_0 \ll K \ll 1/l_0 \quad (26)$$

The atmospheric turbulence strength factor can be obtained from equation (27) as follows

$$30.3\phi_n(K)K^{11/3} = C_n^2 \quad (27)$$

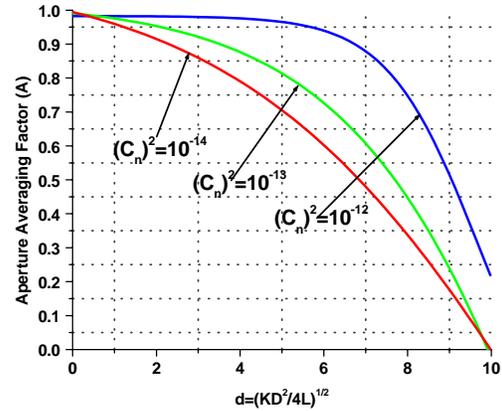


Fig. 2. Aperture averaging parameter for different turbulence strength of atmosphere as a function of circular aperture radius d which depend on receiving aperture radius $D/2$.

By considering the three regime of turbulence for Gaussian beam wave, results shows that the effect of aperture averaging bound by different propagation scenarios. The aperture averaging factor move downward very slowly under higher atmospheric turbulence strength. Moreover, results shown that the aperture averaging ability of the receiving system increases by increasing receiving aperture radius.

Now next section discusses the scintillation loss calculation using threshold power approach.

4. Scintillation loss evaluation by threshold approach

The probability of fading received in Gaussian wave signal is determine using threshold approach technique. This approach is based on the theory that due to fading and within a certain time interval, the received optical signal power or its intensity drops below the receiver sensitivity (threshold level).

The threshold approach reduces the complication in analysis of fading as it does not require a complete and in depth investigation of a particular receiver performance. The probability of fading can be evaluated by cumulative distribution function (CDF) [19]

$$P_r[I \leq I_t(0, L)] = \int_0^{I_r(0, L)} P(I) dI \quad (28)$$

Where $I_r(0, L)$ is intensity at receiver.

For Gaussian optical wave, the intensity of the Gaussian wave at the radial distance r from the axis can be given by [19]

$$I^0(r, L) = I_0 \left[\frac{W_0}{W(L)} \right]^2 \exp \left[-\frac{2r^2}{W^2(L)} \right] \quad (29)$$

Where I_0 is the transmitter output intensity at the centre axis line.

The relation between Gaussian wave intensity and the total beam power at the center line of the beam can be given by [20]

$$I^0(0, L) = I_0 \left[-\frac{W_0^2}{W^2(L)} \right] = \frac{2P_0}{\pi W^2(L)} \quad (30)$$

Here P_0 is the power of beam at transmitter side.

When beam reached at the receiver side, the incident power P at a distance L on the receiver lens of aperture diameter D is

$$P(D, L) = P_0 \left[1 - \exp \left(-\frac{D^2}{2W^2(L)} \right) \right] \quad (31)$$

Solving the equation (31) for P_0

$$P_0 = \frac{-\pi I_0 W_0^2}{2} \quad (32)$$

On substituting P_0 from equation (32) in equation (31) and assumed that the received power is approximately equals the receiver sensitivity that is $P(D, L) = P_r$ then intensity becomes threshold intensity that is $I_0 = I_{thr}$ then the threshold intensity represented as

$$I_{thr} = -\frac{2P_r}{\pi W_0^2} \left[1 - \exp \left(-\frac{D}{2w^2(D)} \right) \right]^{-1} \quad (33)$$

With this threshold approach, when received power P_r is below certain minimum power P_{min} during certain time, no data reception is possible.

The time during which $P_r < P_{min}$, then power margin between the received power P_r and minimum power P_{min} or threshold power P_{thr} should be calculated as an extra loss in the link budget computation. This additional loss is defined as the scintillation loss α_{sci} of the communication system, which is calculated in decibels as follows.

$$\alpha_{sci} = 10 \log_{10} \left(\frac{P_{min} \text{ or } P_{thr}}{P_r} \right), \quad \alpha_{sci} < 0 \quad (34)$$

The value of threshold power P_{thr} for point receiver can be evaluated from the equation below [31]

$$P_{thr} = \frac{1}{2} \left(1 + erf \left(\frac{\ln \left[\alpha_{sci} (\sigma_p^2 + 1)^{1/2} \right]}{\left[2 \ln (\sigma_p^2 + 1) \right]^{1/2}} \right) \right) \quad (35)$$

Where σ_p^2 is power scintillation index.

Solve the equation (35) using Taylor series expansion, the new modified expression of scintillation loss α_{sci}

$$\alpha_{sci} = \frac{e^{\frac{\sqrt{\pi}(2P_{thr}-1)\ln(\sigma_p^2+1)}{\sqrt{2}}}}{\sigma_p^2+1} \quad (36)$$

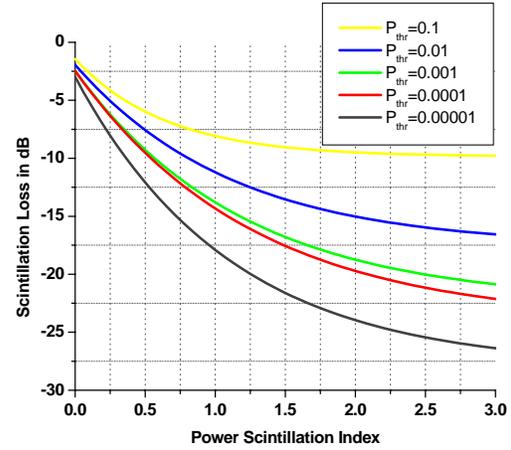


Fig. 3. Power scintillation index versus Scintillation loss with threshold power as parameter.

Results shows that losses due to scintillation and probability of fading are considerably low when threshold power level has low. The scintillation loss without difficulty exceeds 18 dB for minimum threshold power when power scintillation index is less than one, as compared to other threshold levels. When some portion of total data loss is considered then by threshold method, the losses due to scintillation can be evaluated according to newly evaluated equation (36). This approach is not restricted to Gaussian waves, spherical or plane waves but can be implemented to other beams whenever power scintillation index is known.

5. Conclusion

Representation of various parameters like field, intensity and power associated with gaussian beam wave at the axis by new expressions. By developing the gaussian beam parameters for transmitter and receiver side in terms of their respective opposite side, spot radius of beam, radius of curvature and other parameters can be determined, so that we can identify the location, size of focus and beam waist of gaussian beam wave. At the receiver side, effect of scintillation for gaussian wave can be minimized by aperture averaging technique. The aperture averaging effect depends on receiving aperture diameter and it increases with it in different propagation conditions defined by structure parameter C_n^2 . Results shows that aperture averaging effect becomes lower for higher atmospheric turbulence strength.

The scintillation loss in received Gaussian wave signal is determined using threshold approach technique. Scintillation loss can also be evaluated from new derived mathematical expression. This new approach is not limited to gaussian beam wave profile only but it can be implemented to plane or spherical beam wave profile whenever power scintillation index is known.

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