

Physical features of left-handed mirrors in millimeter wave band

C. SABAH*, S. UCKUN

University of Gaziantep, Electrical and Electronics Engineering Department, 27310, Gaziantep, Turkey

In this work, we describe a left-handed mirror as a structure that forms an array of left-handed layers sandwiched between free spaces. Under this assumption, we construct the left-handed mirrors and investigate their physical features for the millimeter wave applications when the incident electric field with any arbitrary polarization. Transfer matrix method is used in the analysis to find the relations among the incident, reflected and transmitted electric fields. Finally, the reflectance and transmittance are calculated in numerical results for the s-polarization case to present their physical characteristics in the millimeter wave band with the emphasis on the layer numbers and thicknesses.

(Received May 4, 2007; accepted June 27, 2007)

Keywords: Left-handed, Mirror, Transfer matrix method, Millimeter wave, Multilayer

1. Introduction

A material in which both the permittivity ϵ and permeability μ are simultaneously negative is called the left-handed and it shows specific physical properties not commonly found in nature. Negative permittivity could be realized by periodic rods and negative permeability by split-rings, over a certain frequency band. The left-handed medium was first investigated and theoretically characterized to illustrate the electromagnetic wave propagation in hypothetical lossless left-handed medium by Veselago in 1968 [1]. Pendry and his group presented their studies in 1996 [2] on the artificial metallic construction which shows the negative permittivity and in 1999 [3] on the split rings which show the negative permeability. The first experiment using the combination of the split rings and wires to construct the left-handed material at microwave frequencies goes back to 2000 [4]. In 2001, Shelby and his group performed the experimental investigation of the negative refraction on the left-handed materials at microwave frequencies [5]. The results of this experiment done for the negative refraction are criticized by Valanju *et al.* in 2002 [6]. They stated that negative refraction is impossible for any real physical signal within the finite bandwidth. This statement was found incorrect by Pendry and Smith in 2003 [7]. The reflection and transmission analysis for the isotropic left-handed materials, field solution of guided waves, and linear and dipole antennas in stratified left-handed materials are investigated in detail by Kong [8]. The analysis for guided modes in parallel-plate waveguides filled with pairs of layers made of any two of the lossless epsilon-negative (ENG), mu-negative (MNG), double-positive (DPS), and double-negative (DNG) materials is analyzed extensively by Engheta [9]. Chew examined some reflection on the left-handed materials and the realistic Sommerfeld problem of a point source over a left-handed material half space and a left-handed slab in 2005 [10]. The theoretical and numerical investigation of the reflected and

transmitted powers due to the interaction of electromagnetic waves with a hypothetical lossless left-handed slab were presented by Sabah *et al.* in 2006 [11]. Due to the progress of the fabrication technologies, the left-handed media are widely used in the components and apparatuses, such as absorbers, lens, microwave components and antennas, etc. The topic continues to be of great interest and practical importance due to a variety of potential applications (many references can be found in the studies included in [12]).

In this work, physical properties of left-handed mirrors for millimeter wave applications are presented in detail. The left-handed mirrors are described as the structure consisting an array of alternating left-handed layers similar to dielectric or chiral mirrors. Under this assumption, left-handed mirrors are constructed with varying parameters and their behaviors are investigated in millimeter wave frequencies when the incident electric field has any arbitrary polarization. Although the multilayer left-handed media have been analyzed in the literature, there exists no work which directly relates to the left-handed mirrors, and this absence has given rise to the present study.

2. Theoretical analysis

A left-handed mirror is the special case of the multilayer left-handed media. It is formed from left-handed layers similar to dielectric or chiral mirrors. The concept of chiral mirrors is investigated extensively and their scattering characteristics are presented by Sabah and Uckun in 2006 [13]. Here, we intend to construct the left-handed mirrors to observe their physical characteristics in the millimeter wave band.

The left-handed mirrors, shown in Figure 1, are comprised of an array of alternating left-handed layers with two different refraction indices (n_A and n_B) and thicknesses (d_A and d_B) sandwiched between free spaces.

In the analysis, $\exp(j\omega t)$ time dependence is assumed and suppressed throughout this work.

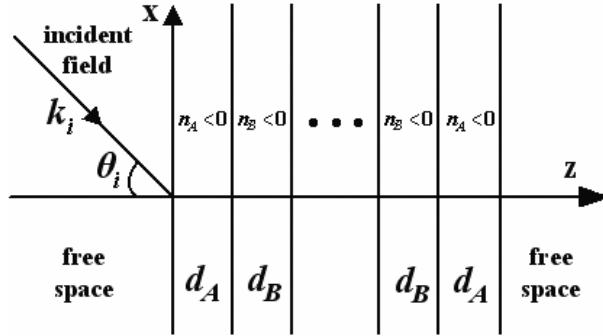


Fig. 1. The geometry of left-handed mirrors.

Referring to Fig. 1, the incident electric field of any arbitrary polarization with the wave number k_i and the incidence angle θ_i can be written as follows:

$$\mathbf{E}_i = [E_{i//}(\cos \theta_i \mathbf{a}_x + \sin \theta_i \mathbf{a}_z) + E_{i\perp} \mathbf{a}_y] \cdot [\exp -j(-k_{ix} x + k_{iz} z)] \quad (1)$$

where $k_{ix} (= k_i \sin \theta_i)$ and $k_{iz} (= k_i \cos \theta_i)$ are the x- and z-components of the wave number $k_i (= \omega \sqrt{\mu_i \epsilon_i})$. Note that, the subscripts // and \perp represent the p- and s-polarized components of the electric field vector, respectively. According to the incident electric field given in Equation 1, the reflected (\mathbf{E}_r) and the transmitted (\mathbf{E}_t) electric fields can be expressed as:

$$\mathbf{E}_r = [E_{r//}(\cos \theta_i \mathbf{a}_x - \sin \theta_i \mathbf{a}_z) + E_{r\perp} \mathbf{a}_y] \cdot [\exp -j(-k_{ix} x - k_{iz} z)] \quad (2)$$

$$\mathbf{E}_t = [E_{t//}(\cos \theta_t \mathbf{a}_x + \sin \theta_t \mathbf{a}_z) + E_{t\perp} \mathbf{a}_y] \cdot [\exp -j(-k_{tx} x + k_{tz} z)] \quad (3)$$

where θ_t is the transmission angle, $k_t (= \omega \sqrt{\mu_t \epsilon_t})$ is the wave number of the transmitted medium, $k_{tx} = k_t \sin \theta_t$ and $k_{tz} = k_t \cos \theta_t$ are the x- and z-components of the wave number k_t . The other fields inside and outside the left-handed mirrors can easily be determined by using the Maxwell's equations. Note that, the wave number and the refractive index of the left-handed medium A (B) must be negative and they can be given as:

$$k_{A(B)} = -\omega \sqrt{\mu_{A(B)} \epsilon_{A(B)}} \quad (4)$$

$$n_{A(B)} = -\sqrt{|\mu_{A(B)} \epsilon_{A(B)}| / \epsilon_o \mu_o} \quad (5)$$

where $\epsilon_{A(B)}$ and $\mu_{A(B)}$ are the permittivity and permeability of the left-handed medium A (B), respectively. Here, ϵ_o and μ_o are the free-space permittivity and permeability.

To solve the general problem for the reflectance and transmittance through the left-handed mirrors shown in Fig. 1, it is necessary to analyze the free space-left-handed interfaces and two left-handed interfaces in detail. The tangential components of the fields must be continuous at the interfaces both in magnitude and in phase. Thus, imposing the boundary conditions at the interfaces, the relationships among the incident, reflected and transmitted electric fields can be obtained by the transfer matrix [U] which is expressed as:

$$\begin{bmatrix} E_{i\perp} \\ E_{r\perp} \\ E_{i//} \\ E_{r//} \end{bmatrix} = [\mathbf{U}] \begin{bmatrix} E_{t\perp} \\ E_{t//} \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \\ u_{41} & u_{42} \end{bmatrix} \begin{bmatrix} E_{t\perp} \\ E_{t//} \end{bmatrix} \quad (6)$$

where $[\mathbf{U}] = [\mathbf{K}][\mathbf{L}_1][\mathbf{L}_2][\mathbf{L}_3] \cdots [\mathbf{L}_m] \cdots [\mathbf{L}_{N-1}][\mathbf{S}]$. Note that, [K] and $[\mathbf{L}_m]$ are both square matrices of order 4, [S] is a 4×2 matrix and [U] is in the form of 4×2 matrix. The elements of [K], $[\mathbf{L}_m]$, and [S] are expressed as a function of the incidence angle, the structure parameters, the thickness of each left-handed layer, and the frequency. Then, according to Equation (6), we can write the reflected and the transmitted electric fields in terms of the incident electric field as:

$$E_{r\perp} = \frac{V_2 E_{i\perp} + V_3 E_{i//}}{V_1} \quad (7)$$

$$E_{r//} = \frac{V_4 E_{i\perp} + V_5 E_{i//}}{V_1} \quad (8)$$

$$E_{t\perp} = \frac{(u_{32})E_{i\perp} - (u_{12})E_{i//}}{V_1} \quad (9)$$

$$E_{t//} = \frac{-(u_{31})E_{i\perp} + (u_{11})E_{i//}}{V_1} \quad (10)$$

where u_{ab} ($a = 1, 2, 3, 4; b = 1, 2$) are the elements of the 4×2 transfer matrix [U] given in Equation (6) and V_c ($c = 1, 2, 3, 4, 5$) are given by

$$V_1 = (u_{11} u_{32} - u_{12} u_{21})$$

$$V_2 = (u_{21} u_{32} - u_{22} u_{31})$$

$$V_3 = (u_{11} u_{22} - u_{12} u_{21}) \quad (11)$$

$$V_4 = (u_{32} u_{41} - u_{31} u_{42})$$

$$V_5 = (u_{11} u_{42} - u_{12} u_{41})$$

Finally, the reflectance and transmittance can easily be obtained using the equations between (7) and (11).

3. Numerical results

The computations for the reflectance (R) and transmittance (T) have been presented to observe their characteristics in the millimeter wave band using the results obtained in Section 2, when the incident electric field is normalized to unity. Two mirrors are considered in the numerical results as: (ABA) and $(AB)^7A$. The mirror I has three left-handed layers and the mirror II has fifteen left-handed layers. In all figures, the solid lines correspond to mirror I and the dotted lines to mirror II. Furthermore, to verify the computations, the conservation of power, as a first method, is satisfied for all examples. As a second method, a transmission line equivalent is obtained for the structure given in Fig. 1 [14]. Both methods give the same numerical values for all computations. Thus, the results are verified by means of two methods.

In the first example, the reflectance and transmittance are calculated as a function of the frequency and the incidence angle when the incident electric field is a plane electromagnetic wave with the s-polarization ($E_{//} = 0$). The operation frequency is selected to be $f_o = 60$ GHz. The thicknesses d_A and d_B are arranged from $|n_A d_A| = \lambda_o/2$ and $|n_B d_B| = \lambda_o/4$ where λ_o is the wavelength in free-space at the operation frequency. The permittivity and permeability of the left-handed medium A and left-handed medium B are given as in [9]:

$$\varepsilon_A = -2\varepsilon_o, \mu_A = -3\mu_o, \varepsilon_B = -5\varepsilon_o, \text{ and } \mu_B = -2\mu_o.$$

Fig. 2 points out the reflectance and transmittance as a function of the frequency at normal incidence. As it is seen, the frequency response of the reflectance and transmittance is periodic and symmetric according to the operation frequency. From Fig. 2(a), the mirror I transmits the most of the incident wave, because $|T|$ is greater than $|R|$ when the frequency changes. For the mirror II, $|R|$ and $|T|$ reach to unity at different frequency bands. Also, the mirror II acts as a band-pass filter and an anti-reflection filter at these frequency bands. At this point, it can be said that, the mirror shows both filter characteristics at some frequency bands for increasing the layer numbers.

Fig. 3 presents the reflectance and transmittance versus the incidence angle at the operation frequency. The transmittance is dominant between 0° and $\sim 50^\circ$ for the mirror I and between 0° and $\sim 60^\circ$ for the mirror II. The reflectance and transmittance for the mirror I show the monotonically increasing and decreasing behaviors with the incidence angle, respectively. Brewster angle occurs at the incidence angle of 47° for the mirror II which means there is no reflection at this angle. In addition, full reflection occurs at the incidence angle of 90° .

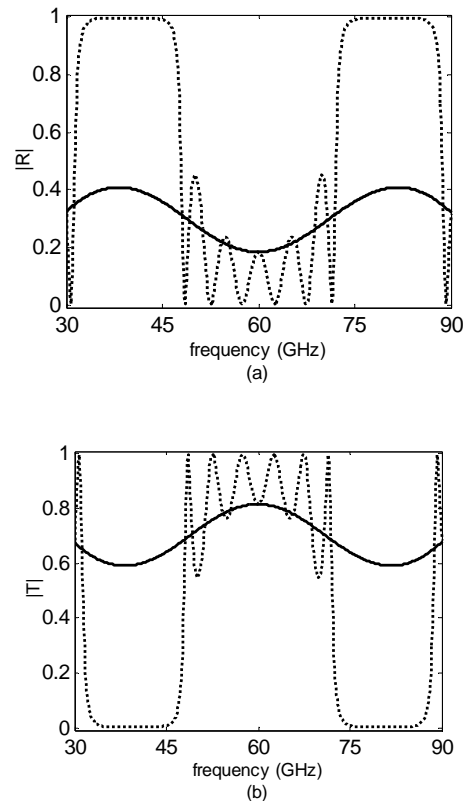


Fig. 2. Reflectance and transmittance for the mirror I and mirror II as a function of the frequency at normal incidence.

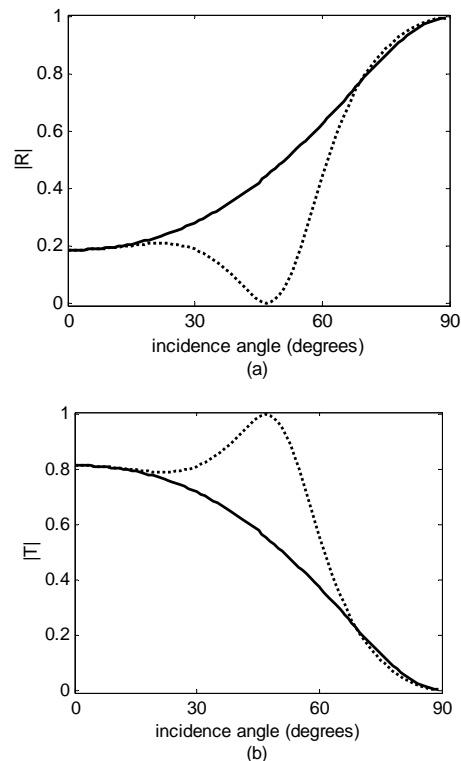


Fig. 3. Reflectance and transmittance for the mirror I and mirror II versus the incidence angle at the operation frequency.

In the second example, mirror I and mirror II are again considered to calculate the reflectance and transmittance as a function of the frequency and the incidence angle for the s-polarization. The structure parameters and the operation frequency are the same with the first example except for the thicknesses of the left-handed layers. Here, the thicknesses d_A and d_B are assumed as $d_A = \lambda_0/2$ and $d_B = \lambda_0/4$, respectively.

Fig. 4 displays the reflectance and transmittance against the frequency at normal incidence. As it is seen, the frequency response of $|R|$ and $|T|$ is not periodic and not symmetric according to the operation frequency. The mirror I again transmits the most of the incident wave as in the previous example. For the mirror II, $|R|$ and $|T|$ reach to unity at some frequency bands. Comparing Fig. 2 and Fig. 4, the frequency bands where $|R|$ becomes unity are not wide as in the first example, but they are more than two here. Also, the frequency bands where $|T|$ reaches to unity are more than one in this configuration. Thus, it can be said that, the mirror has more frequency bands where it shows a band-pass filter and an anti-reflection filter behavior at the narrow ranges.

Fig. 5 illustrates $|R|$ and $|T|$ against the incidence angle at the operation frequency. $|T|$ for the mirror I is dominant over a wide range of the incidence angle. But in the mirror II, $|R|$ is more dominant. In addition, $|R|$ is the unity and $|T|$ is zero up to 60° . After this angle, $|T|$ has a sharp peak and it reaches to unity at 77° . In turn, $|R|$ has a reverse sharp peak and it becomes zero at 77° . This means that Brewster angle occurs at the incidence angle of 77° .

It is confirmed that, similar numerical results given in Fig. 2 – Fig. 5 can easily be obtained for the incident wave with the p-polarization.

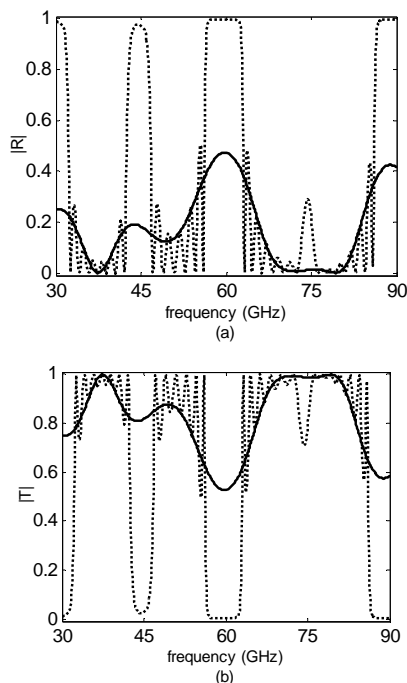


Fig. 4. Reflectance and transmittance for the mirror I and mirror II against the frequency at normal incidence.

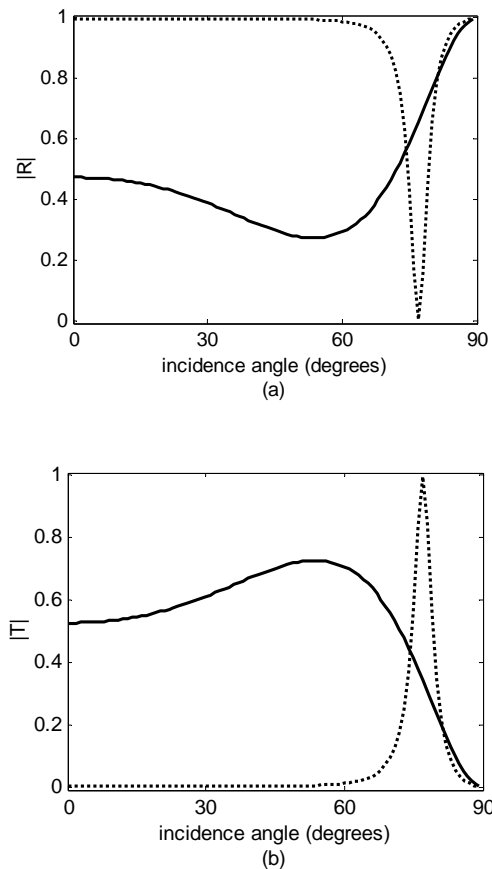


Fig. 5. Reflectance and transmittance for the mirror I and mirror II against the incidence angle at the operation frequency.

4. Conclusions

A general analysis of left-handed mirrors in the millimeter wave band has been presented extensively. The left-handed mirrors are described in the theory and constructed as a structure that forms an array of left-handed layers with two different refractive indices and different thicknesses embedded between free spaces. The problem of the wave propagation for the left-handed mirrors is solved when the incident electric field has any arbitrary polarization. Then, the reflectance and transmittance are computed in numerical results to illustrate the physical features of the left-handed mirrors in the millimeter wave frequencies. The effects of layer numbers and thicknesses are shown when the frequency and the incidence angle change. From the numerical results, we can say that high magnitude reflectance and transmittance in wide ranges can be obtained by arranging the layer numbers and thicknesses of the left-handed mirrors. In addition, the left-handed mirrors show the band-pass filter and the anti-reflection filter characteristics at some frequency regions within the millimeter wave band. Furthermore, the results obtained here can be

applied to design of the both filters at the millimeter wave, optical, and microwave regimes. Moreover, this study will make a foundation for future works and provide some insight into the filter application of LH materials.

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*Corresponding author: sabah@gantep.edu.tr