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Poynting singularities, angular momentum and "anticorrelation" in heterogeneously polarized vector field

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Poynting singularities and their networks in heterogeneously polarized vector field are considered. A new approach for experimental modeling of elementary field cells with heterogeneous polarization is proposed. It is shown that such cells may be obtained by superposition of orthogonally linearly polarized waves with relatively simple phase surfaces and close intensities. The relation between the behavior of intensity and parameters of the transversal component of the Poynting vector is analyzed. The experimental results and computer simulation data are presented.

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1. Introduction

The most well known optical singularities are optical (or phase) vortices (see, for example, ref. [1-7], which appear in the vicinity of the points, in which the amplitude of scalar field has exact zero. The vortices may be combined into vortex network, governed by the field phase at each field point [3-5].

The second type of traditional optical singularities is polarization ones [2,5-7,10-13].

As it is known C-point is the point, where field is circularly polarized. Poincare index of C-point defines the rotation direction of ellipses around this point. Correspondingly with the sign of this index C-points may be either positive or negative. The azimuth and vibration phase are indeterminate in the C-point. The rotation direction of field vector is indefinite along s-contour, because polarization is linear. C-points similarly to the vortices may be combined by the lines of constant azimuth into some singular network [2,5,7].

The last type of coherent singularities are concerned with the behavior of Poynting vector transversal component [7, 14-16]. They are the singularities of the azimuth of Poynting vector transversal component (briefly, Poynting singularities, or P-singularities) [17, 18] and they are closely connected with optical current [14] and magnitude of angular momentum of a field [15,16].

Poynting singularity is observed in the field point, where *x*- and y-components of the Poynting vector simultaneously achieve the exact zero and the orientation of transversal component is indeterminate.

The typical behavior of transversal component in the area of P-singularity is presented in Fig. 1.

Vortex singularity is presented in Figure 1 (a). It can be seen that the circulation of the Poynting component is very similar to the one around common phase vortex [14-16]. The physical consequence of this fact is the maximal magnitude of averaged field angular momentum in the field area. Vortex P-singularity is additionally characterized by chirality, which defines the direction of such circulation [7, 17, 18].

Passive singularities are in Figure 1 (c)-(d), like star (Figure 1 (b),(c)) and saddle ones (Fig. 1 (b)). The averaged angular momentum tends to zero in their areas.

As traditional singularities P-singularities may be combined in the network, which governs the behavior of Poynting vector in each field point [17].

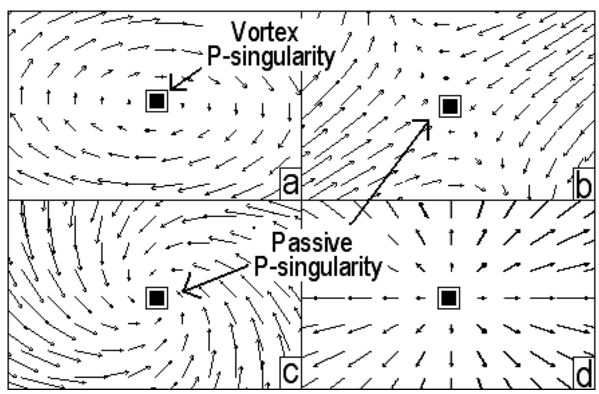

Fig. 1. Typical behavior of the component in the small area with Poynting singularity. a – vortex Poynting singularity, b-d – passive Poynting singularities

The idea of our consideration is based on one of the main (in our opinion) statement, which follows from the next conception of singular optics: any singular network of some field parameter forms the skeleton of this parameter, which governs the space distribution of them [17].

Let us explain this statement.

Practically for each space-distributed field characteristics the set of singular points may be derived, for which this parameter is indeterminate.

The topological charge or topological index may be defined for each such point. This charge or index determines the changes of parameter immediately around the singular point. All singular points may be connected by specific parameter lines, for example equiphase or equiazimuth lines etc. In other words singular network may be formed. The information about topological characteristics of its elements and location of parameter lines determines the behavior of this parameter in each field points.

The quick changes of parameter are observed just in the vicinity of a singular point. In other areas the characteristics changes smoothly. The saddles points are located in these regions. Consequently, if one knows the coordinates of a singular point, its topological charge or index and the location of, at least, one of specific parameter lines then one can estimate the magnitude of the parameter (with certain probability) in the point. The accuracy of such estimation as a rule may be characterized by some physical limitations.

It has been noted that all singularity systems are connected to each other. As a result, it can be stated that the behavior of all field parameters in each point is also interconnected, at least, in the statistical sense.

The objective of this paper is: to find out whether the above mentioned hypothesis is valid).

In the first section some important definitions will be given and some limitations (or criteria) will be stated.

In other sections different types of relations between different field parameters will be considered.

2. Assumptions, limitations, criteria

The main limitations of the performed analysis deal with the meaning of rapid changes of different field parameters. Obviously this value is closely connected with that of the parameter gradient. At the same time the direct use of this value is problematic because the gradient depends on the wave length, distance from the field forming object, dimension of this object, etc.

Thus, neglecting the similar characteristics influence we considered the problem under the following conditions:

1. The analysis has been performed for far zone. Under this assumption the field correlation length is a universal field parameter, which defines the scale of changes of practically all field characteristics, while the field characteristics may be defined only by x and y coordinates.

For example, the basic equations for the analysis of the Poynting vector behavior are the following [19]:

$$\begin{cases} \overline{P}_x \approx -\dfrac{c}{8\pi k}\{[A_x^2\Phi_x^x + A_y^2\Phi_y^x] - \dfrac{\partial}{\partial y}(A_x A_y \sin\Delta\Phi)\}, \\ \overline{P}_y \approx -\dfrac{c}{8\pi k}\{[A_x^2\Phi_x^y + A_y^2\Phi_y^y] + \dfrac{\partial}{\partial x}(A_x A_y \sin\Delta\Phi)\} \\ \overline{P}_z \approx \dfrac{c}{8\pi}\{A_x^2 + A_y^2\} \end{cases} \qquad (1)$$

where $A_i, \Phi_i\ (i = x, y)$ – amplitudes and phases of corresponding field components, $A_i^l, \Phi_i^l\ (i, l = x, y)$ – their partial derivatives, $\Delta\Phi = \Phi_x - \Phi_y$ – local phase difference.

2. All considered fields were expressed in such generalized coordinates:

$$\begin{cases} X = x / l_{Xcorr} \\ Y = y / l_{Ycorr} \end{cases}, \qquad (2)$$

where l_{Xcorr}, l_{Ycorr} – are the correlation lengths along x- and y-directions.

In that case the structure of the analyzed field did not depend on wave length, dimension of the scattering object, distance between the object and observation plane.

Obviously the singular network and its structure should depend on the level of field polarization homogeneity.

Let us introduce the value – level of integral depolarization for characterizing the field polarization homogeneity.

$$D = 1 - \overline{P}, \qquad (3)$$

where P is the degree of "integral" polarization.

Certainly, the field is polarized completely at any point, if the wave is monochromatic and the degree of polarization P associated with any field point is unit. At the same time, due to the space averaging this value, which is associated with some area, is less then unit for the field, which is inhomogeneous by polarization. Such a value, averaged by space coordinates, will be called the degree of "integral" polarization.

In the case, when mean intensities of orthogonal components are equal, the following relation takes place [20]:

$$\overline{P} = \sqrt{\overline{s}_2^2 + \overline{s}_3^2} = \gamma, \qquad (4)$$

where γ – correlation coefficient of orthogonal components, $\overline{s}_2, \overline{s}_3$ – normalized Stokes parameters, and correspondingly

$$D = 1 - \gamma. \qquad (5)$$

Let us introduce the criteria for smooth changes of different field parameters.

It has been noted that the choice of such criteria has relatively arbitrary nature. Practically a rather accurately defined criterion may be formulated on the base of Rayleigh criterion [21] (or on the base of the similar one) only for phase changes of scalar field. Due to that the criteria presented in our paper cannot be regarded as "universal" and should be specified for any particular task.

2.1. Smooth changes of polarization

Let us arrange that all limitations will be connected with the field area $q = l_{corr} \times l_{corr}$, which corresponds to the unit square for the field presented in generalized coordinates.

We will assume that polarization changes smoothly in some field point if the phase difference and ratio of the components amplitudes satisfy the following relations:

$$\begin{cases} |\nabla \Delta\Phi| \le \pi/2 \\ \left|\nabla \arctan(\frac{A_y}{A_x})\right| \le \pi/4 \end{cases}. \quad (6)$$

The example of polarization behavior obeying to relations (6) is presented in Figure 2. Grey areas are the regions, where polarization changes rapidly. The example of such polarization behavior is marked by the rectangle 2. Smooth polarization changes are illustrated by rectangles 1. Additionally the magnitude of the correlation length is denoted in the figure.

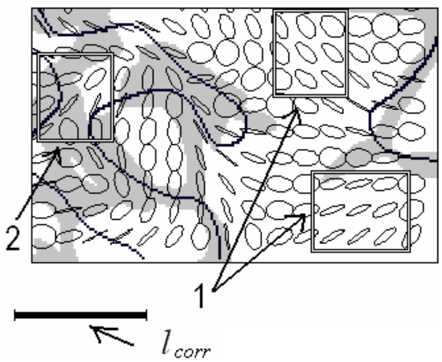

Fig. 2. Polarization changes in the vector field. Grey areas are the regions, where polarization changes rapidly.

2.2. Smooth changes of intensity and parameters of the Poynting vector

Let us assume that intensity changes smoothly in some field point if the value of its gradient is limited by the following relation:

$$|\nabla I|_n = \frac{|\nabla I|}{\bar{I}} \le 0.2\,. \quad (7)$$

where $\bar{I}$ is mean intensity in the analyzed field area.

It has been noted that the additional normalization of the intensity gradient (division on $\bar{I}$) is necessary because the value of the gradient depends on the level of field intensity. The field, similar in structure, but with different mean intensities, has different gradients.

The limitation for the changes of Poynting vector characteristics may be introduced as follows:

$$|\nabla\alpha| \le \pi/4, \quad (8)$$

for the azimuth of transversal component of the Poynting vector, and

$$|\nabla P_t^2|_n = \frac{|\nabla P_t^2|}{\bar{P}^2} \le 0.2, \quad (9)$$

for the modulus of this component. Like the intensity gradient, the gradient of square of transversal component modulus is additionally normalized by mean magnitude of this value.

3. "Anticorrelation" between polarization and intensity

It can be stated that different by type singularity systems are interconnected. Figure 3 illustrates such relationship. All types of optical singularities are presented in the figure. Percentage in the rectangles of each figure corresponds to the level of integral depolarization *D*. It can be seen that different field peculiarities bunch into some groups.

Obvious conclusion immediately follows from this fact. Namely, if different singular systems are connected, then one can not consider the behavior of all field parameters independently, even for random field.

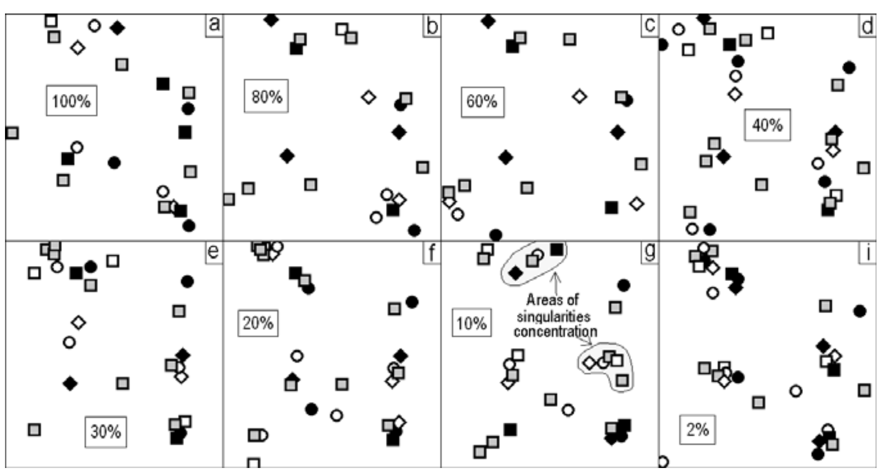

Fig. 3. Connection between different types of singularities of vector field.

Results of computer simulation. Effective phase difference between orthogonal components is equal to zero. Percentage in the rectangles of each figure corresponds to the level of integral depolarization *D*.

▣ – P-singularity; □,◇,■,◆ – positive and negative vortices of *x*- and *y*-components correspondingly; ○,● – positive and negative C-points.

Isaac Freund and colleagues [22, 23] was the first to show such relationship between intensity and phase of scalar field.

The similar “anticorrelation” between intensity and polarization is observed in the vector field. Such anticorrelation must depend on the level of polarization homogeneity because similarity of orthogonal components increases with the decrease of integral depolarization and in case of limitation of uniformly polarized field such anticorrelation transforms into the relationship between the intensity and phase.

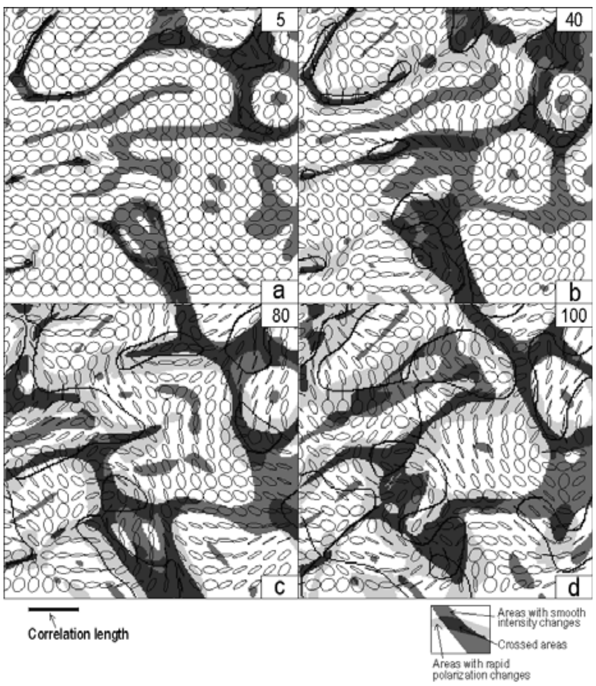

Fig. 4. Anticorrelation between intensity and polarization a – corresponds to the integral depolarization D=5%; b – D=40%; c – D=80%; d – D=100%. Light grey areas are the regions, where polarization changes quickly. The darker ones are the areas, where intensity has relatively small gradient. Black regions represent the crossed areas.

The connection between polarization and intensity depending on the polarization homogeneity is illustrated by the results of computer simulation (see Fig. 4). In the right upper corner of each figure the integral depolarization is marked. Fig. 4 (d) corresponds to absolutely random vector field with integral depolarization 100%. Light grey areas are the regions where polarization changes quickly. The darker ones are the area with relatively small intensity gradient. Black color corresponds to the crossed areas.

It can be seen that:

1. Areas with rapid changes of polarization tend to the regions with relatively small intensity gradient independently on the level of field polarization homogeneity.

2. The square of grey areas with mean depth is practically the same for all levels of depolarization.

3. Squares of light grey and black areas decrease, if the polarization homogeneity increases. At the same time dark grey square decreases slower than the light one. So, it can be stated that interconnection between the behaviors of intensity and polarization becomes strict, when field becomes homogeneous by polarization.

4. “Anticorrelation” between the intensity distribution and behavior of Poynting vector transversal component azimuth

Let us consider the relationship between the intensity distribution and Poynting vector parameters.

The connection between the intensity distribution and that of the modulus of transversal component is obvious, because, as it is well known that the modulus of Poynting z-component is proportional to the intensity (see, for example, [15, 20]), and the behavior of the transversal component of modulus must be similar.

The connection between the intensity distribution and behavior of component azimuth has different nature.

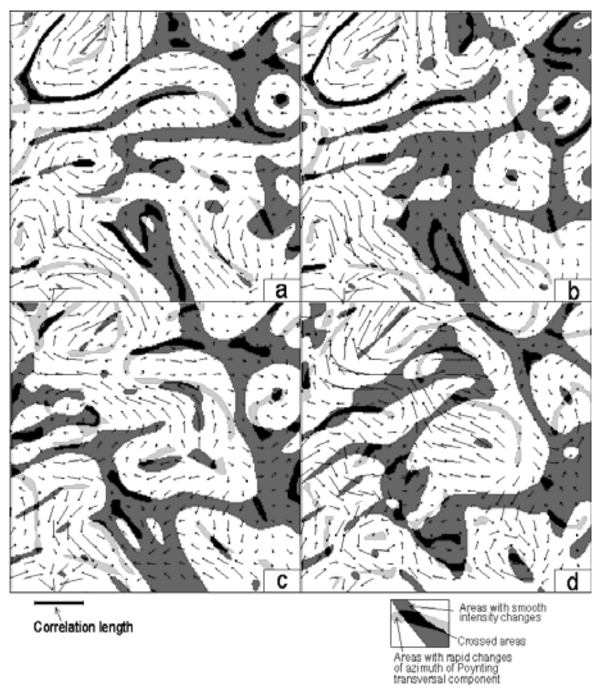

Fig. 5. Anticorrelation between intensity and azimuth of transversal component of the Poynting vector. a – corresponds to the integral depolarization D=5%; b – D=40%; c – D=80%; d – D=100%. Light grey areas are the regions, where azimuth of Poynting transversal component changes quickly. The darker ones are the areas, where intensity has relatively small gradient. Black regions are the crossed areas. The arrows illustrate the behavior of Poynting component. Their length corresponds to modulus of the component. Direction of arrows denotes the azimuth of the component.

It is known [17] that C-points and P-singularutues may be combined into some pairs. The vortex of Poynting singularity is observed in the area of negative C-points.

The passive Poynting singularities are positioned in the area of positive C-points.

Note that P- and C-singularities are located in the area with rapid changes of corresponding field parameters. As a result, the consequence of the relationship between P- and C-singularities leads to the following. In areas with relatively small intensity gradient, azimuth of Poynting component must change quickly.

This assumption was confirmed by computer simulation (see Fig. 5).

Light grey areas are those, where the Poynting component azimuth changes quickly. Darker areas are the regions with relatively small intensity gradient. Black color corresponds to the crossed areas.

It can be seen that the areas with rapid changes of azimuth tend to the regions with relatively small intensity gradient, but in contrast to the polarization-intensity connection, the square of crossed areas does not depend on polarization homogeneity of a field.

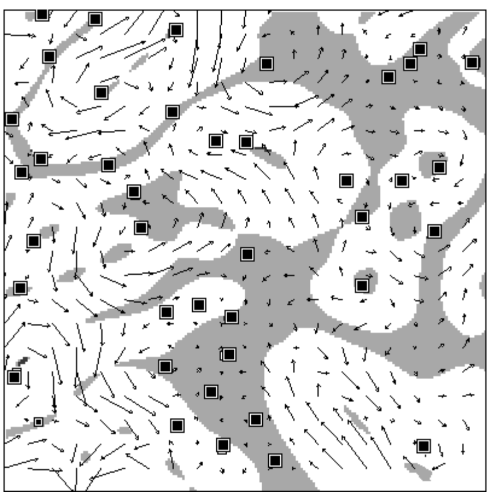

Fig. 6. Positions of P-singularities and areas with relatively small intensity gradient. Integral depolarization D=30%. Grey areas are the areas, where intensity has relatively small gradient. ▣ – P- singularities.

In Fig. 6 the positions of P-singularities are presented. It can be seen that most of them are located in the areas with smooth intensity changes. Correspondingly it can be stated that angular momentum of a field achieves its minimal and maximal magnitudes right in these regions.

Integral depolarization *D*=30%. Light grey areas are the regions, where azimuth of Poynting transversal component changes quickly. The darker ones are the areas, where polarization also changes rapidly. Black regions are the crossed areas.

Note that the Poynting vector orientation coincides with z-axis, the preferred direction of wave propagation. Thus in 3-D P-singularities form the corresponding lines, which define this direction. It can be said that vortex P-singularities form helical energy tubes while passive P-singularities generate still tubes with zero angular momentum in each their crossing.

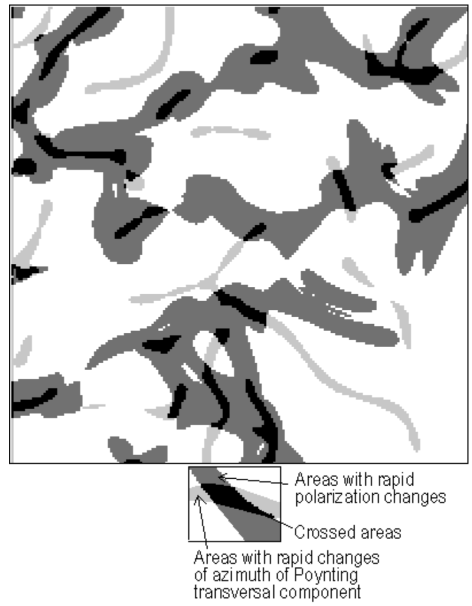

Fig. 7. Correlation between polarization and azimuth of transversal component of the Poynting vector.

Additional consequences, which follow from such behavior of Poynting vector, are the following:

1. "Correlation" between polarization and azimuth changes of transversal component must be observed.

Such statement was conformed by the data of computer simulation. Results of them are presented in Figure 7.

2. The following function may be considered a characteristic one:

$$U = P_t \exp(j\alpha), \qquad (10)$$

where P_t is <u>modulus</u> of Poynting vector transversal component, $\alpha = \arctan(\frac{P_y}{P_x})$ is its azimuth.

It can be shown that U satisfies the wave equation. Thus this function may be associated with some scalar filed with amplitude P_t and phase α. As a result the "Freund" anticorrelation must be observed, namely in areas, where square <u>modulus</u> of Poynting transversal component changes smoothly the azimuth changes relatively quickly. This fact was conformed by the computer simulation. The results of them are presented in Fig. 8.

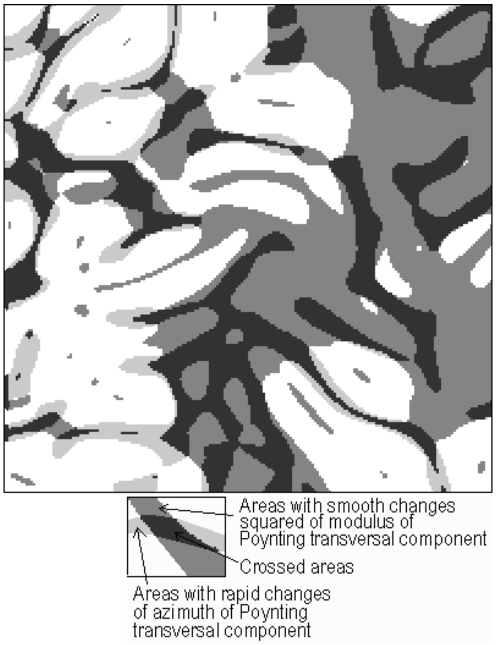

Fig. 8. Anticorrelation between azimuth and square of Poynting transversal component modulus.

Integral depolarization *D*=30%. Light grey areas with rapid changes of component azimuth. The darker areas are those, where intensity has relatively small intensity gradient. Black ones are the crossed regions.

Thus, it can be stated that not only all types of optical singularities are connected with each other. Different optical parameters cannot change independently.

5. Conclusions

1. Areas with rapid changes of polarization tend to the regions with relatively small intensity gradient. This “anticorrelation” becomes strict if polarization homogeneity of a field increases.

2. In areas with relatively small intensity gradient, the azimuth of Poynting vector transversal component changes quickly. Angular momentum of a field achieves its minimal and maximal magnitudes right in these regions.

3. In areas where modulus of Poynting transversal component changes smoothly while the azimuth of this component changes relatively quickly.

4. All field parameters, their singular systems are connected. As a result, one can predict (at least statistically) the behavior of any field parameter if one has information about the characteristics of any singular system or characteristics of intensity distribution.

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