Proportional effect in refractive index of optical mediums for single layer homogeneous modeling of surface roughness

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In the present work, the single layer model of surface roughness was studied in the context of proportional contribution of refractive index of neighboring layers. Using three effective medium approximation namely, Drude, Maxwell-Garnett and Lorentz-Lorenz, a relation between film thickness of homogeneous layer and surface roughness is established. It is shown that the reflectivity of nonabsorbing thin film varies even for a same value of surface roughness if the composition of roughness is changed.

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1. Introduction

The surface roughness is modeled by introducing a homogeneous layer between material surface and ambient. The refractive index and thickness of homogeneous film is always related to the refractive index of material and it's ambient refractive indexes [1-3]. In this way the effect of scattering due to surface roughness can be incorporated in spectral data analysis e.g. Ellipsometry [2].

Due to scattering of incident light from rough surface both reflectance and transmittance is found to be decreased. The random nature of roughness makes the spectral analysis difficult. Surface roughness can be defined as vertical deviations from the reference line of horizontal smooth boundary on the surface. The extent of peaks and valleys in horizontal direction also contributes to the roughness evaluation. Assuming the roughness portion of material surface as a nonabsorbing film, Carniglia et al [1] reported a relation between rms roughness and thickness of the film. This model gives a good account for the reflectance reduction (not for the transmission reduction) in nonabsorbing film. It uses Drude Effective Medium approximation [4, 6] and assume that refractive index of homogeneous film is a equal mixture of the refractive index of neighboring optical mediums.

Due to random nature of surface roughness it may not be always true that the refractive index of homogeneous layer is intermediate to the neighboring optical medium refractive index. The nature of roughness can be monodisperse or polydisperse. That means peaks and valleys may have the same size, shape and mass (monodisperse) or it may vary (polydisperse). It is important to study how roughness can be modeled to homogeneous layer when the proportions of neighboring layer refractive index are unequal. In this article, single layer homogeneous model for surface roughness is studied for unequal proportions of neighboring layer refractive indexes. This study is performed on nonabsorbing layer that can be used for absorbing layers in a similar manner.

2. Modeling and results

According to Drude Effective Medium approximation the refractive index of homogeneous layer can be related to neighboring layers refractive index by the following relation:

$$n^2 = (1-\varphi)n_a^2 + \varphi n_s^2 \qquad (1)$$

Here n represents the refractive index of homogeneous layer while n_a and n_s represents the refractive index of ambient and material surface respectively. The value of φ denotes the percentage contribution of substrate material. In most of the cases the rest percentage can be assumed to be proportion of ambient. Using eq.1 rms roughness of nonabsorbing layer can be related to the thickness of homogeneous layer by the eq.2 [1]

$$d = \sigma\sqrt{\frac{n_a}{n_s\varphi(1-\varphi)}} \qquad (2)$$

Fig. 1 shows the relationship between d and σ for various value of φ. d and σ have straight line relationship. The slope of straight line is a function of φ. It means, more rough surface will correspond to comparatively thicker film. The thickness of the film will be decided by the value of φ. Eq.2 and Fig. 1 makes it clear that for the same rms roughness value the thickness of the homogeneous film is not constant and depends upon φ.

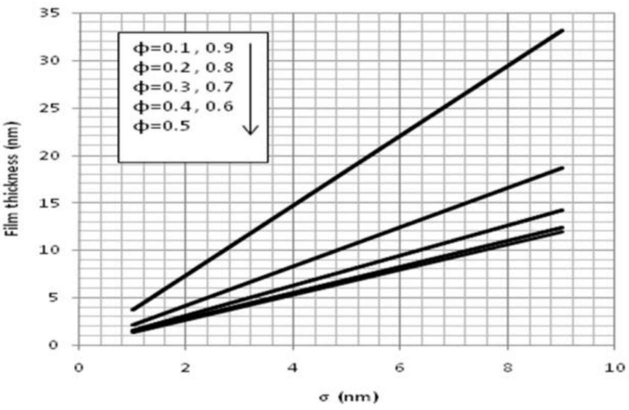

Fig. 1. Relationship between film thickness and surface roughness. Calculation based upon Drude effective medium approximation

Another way to look into the relationship between homogeneous layer thickness and φ, Fig. 2 is drawn using the constant value of rms roughness as 50A^0. This graph makes it clear that in either side of equal proportion (φ = 0.5) of na and ns (various combination of unequal proportion) the thickness of homogeneous film vary in a similar manner. That means the thickness of modeled layer will remain same if the proportion of neighboring refractive index is interchanged e.g. (0.1, 0.9) and (0.9, 0.1), (0.2, 0.8) and (0.8, 0.2), (0.3, 0.7) and (0.7, 0.3) etc. In other words it can be said that same thickness can be modeled for two different but interchangeable proportions of peaks or valleys representing the substrate material (ns) or void space between the peaks and valleys (na). This feature can be linked to the monodisperse nature of roughness.

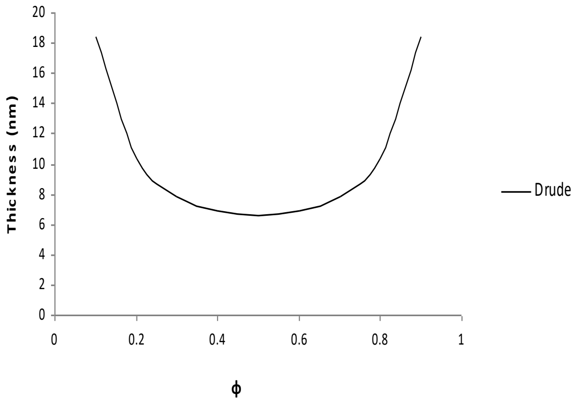

Fig. 2. Shows the relation between nonabsorbing film thickness and φ

The effect of unequal distribution of refractive index to the surface roughness is studied using Maxwell-Garnett and Lorentz-Lorenz approximation and results are compared with those of Drude effective medium approximation. Eq. 3 and 4 gives the Maxwel-Garnett and Lorentz-Lorenz formula relating film refractive index with neighboring layer refractive index.

$$n^2 = n_a^2\left[1 - \frac{3\varphi(n_a^2 - n_s^2)}{2n_a^2 + n_s^2 + \varphi(n_a^2 - n_s^2)}\right] \quad (3)$$

$$n^2 = \left[\frac{(n_a^2+2)(n_s^2+2)}{(1-\varphi)(n_s^2+2)+\varphi(n_a^2+2)} - 2\right] \quad (4)$$

Based on the relation (3) and (4) the expression for d in terms of σ can be given as follows:

$$d = \sigma \frac{2n_a^2 + n_s^2 + \varphi(n_a^2 - n_s^2)}{\sqrt{3 n_a n_s \varphi(1-\varphi)(2n_a^2 + n_s^2)}} \quad (5)$$

$$d = \sigma \sqrt{\frac{n_a}{n_s}} \frac{(1-\varphi)(n_s^2+2)+\varphi(n_a^2+2)}{\sqrt{\varphi(1-\varphi)(n_a^2+2)(n_s^2+2)}} \quad (6)$$

A graph between *d* and *σ* is drawn using three approximations, Drude (Fig. 1), Maxwell-Garnett (Fig. 3), Lorentz-Lorenz (Fig. 4). A linear relationship has been observed for all three approximation.

Although for the same roughness level, the thickness of the homogeneous film is not same and varies in case of three different approximations. This thickness variation is in the range of 10^{-9} to 10^{-8}.

It is also noticeable in Figs. 1, 3 and 4 that for comparatively less rough surface, film thickness variation is negligible but as the roughness level increases, the value of film thickness changes significantly when the value of φ is changed.

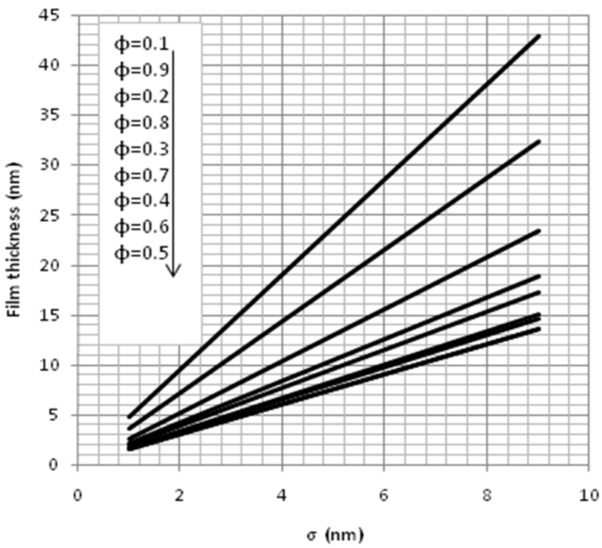

Fig. 3. Relationship between film thickness and surface roughness. Calculation based upon Maxwell-Garnett formula

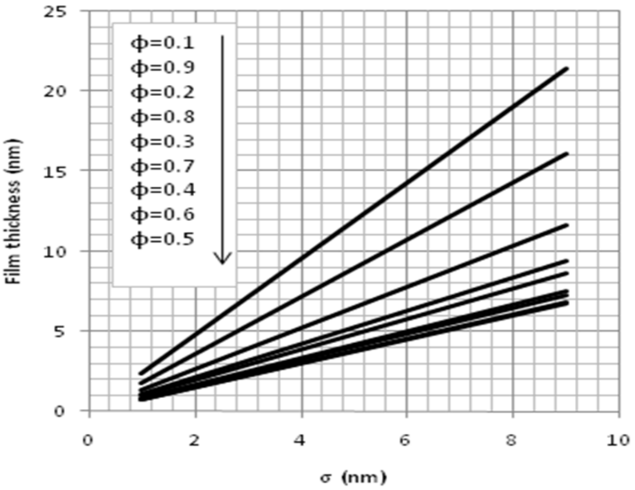

Fig. 4. Relationship between film thickness and surface roughness. Calculation based upon Lorentz-Lorentz approximation

Fig. 5 shows the relationship between *d* and φ for three approximations. A *U* like shape is obvious in all three cases. It indicates that in all three approximations if φ is in the range of 0.2–0.8 there is no remarkable change in the film thickness. But in the range of 0.1-0.2 and 0.8-0.9, film thickness increases significantly even for the small change in the value of φ. The interchangeability of Drude like approximation as discussed in relation to Fig. 2 does not exist in case of other two approximations.

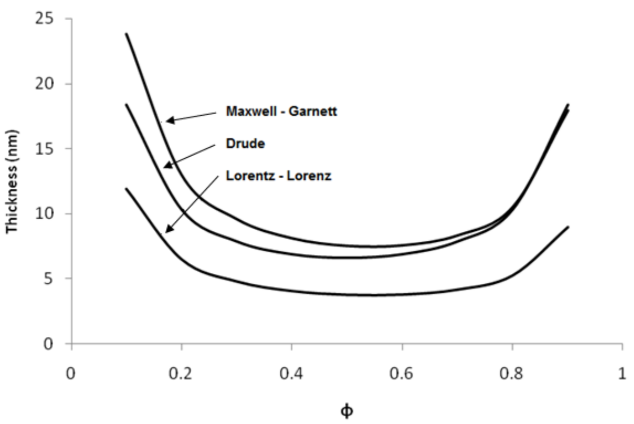

Fig. 5. shows the relationship between film thickness and φ or three approximations. This relation is based upon the fixed value of σ as 50 A^0

Therefore, from the above discussion it becomes clear that in homogeneous layer modeling of surface roughness its not only about the refractive index aproximation of thin film but also the proportinality of refractive index must be considered. The thickness of the homogeneous film is not only dependent upon the roughness level but the composition of roughness (proportional contribution of neighboring optical medium to roughness) also affects the thickness of the film. Therefore, the reflectivity of homogeneous film corresponding the same level of roughness can vary if the composition of roughness is changed. The present result can be used to design various thin film optics by manipulating the refractive index as well as composition of surface roughness. Because the reflectance of thin film is dependent upon its refractive index and optical thickness [7].

3. Conclusion

In the present work, the effect of proportional contribution of refractive index of neighboring optical medium is applied to single layer model of surface roughness. The results suggest that the thickness of the film is not only dependent upon the surface roughness but on its composition as well. Taking the three different approximations, linear relationship between the film thickness and surface roughness is shown. Using Drude Effective Model the interchangeability of refractive index of neighboring optical mediums for the same level of roughness was observed that may be usefull in the processing of many optical devices.

Reference

[1] C. K. Carniglia, D. G. Jensen, Applied Optics **41**(16), 3167 (2002).
[2] D. E. Aspnes, J. B. Theetan, F. Hottier, Phys. Rev. B **20**, 3292 (1979).
[3] C. K. Carniglia, Opt. Eng. **18**, 104 (1979).
[4] N. J. Hutchinson, T. Coquil, A. Navid, L. Pilon, Thin Solid Films **518**, 2141 (2010).
[5] A. Navid, L. Pilon, Thin Solid Films **516**, 4159 (2008).
[6] N. J. Hutchinson, T. Coquil, E. K. Richman, S. H. Tolbert, A. Navid, L. Pilon, Thin Solid Films **518**, 2134 (2010).
[7] A. Vasicek, Optics of Thin Films, Interscience Publishers Inc., 1960.

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