JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 8, No. 6, December 2006, p. 2143 - 2147

Radiation power spectral distribution of the system of electrons moving in a spiral in vacuum

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Using the expressions for the average radiation power of three and four electrons moving one by one in a spiral in vacuum the synchrotron radiation spectrum for the first time is obtained and studied. The synchrotron radiation spectrum of a single electron is compared to these of two, three and four electrons moving in a spiral in vacuum. The influence of the coherence factor on the spectrum of synchrotron radiation is studied.

(Received October 31, 2006; accepted November 7, 2006)

Keywords: Synchrotron radiation spectrum, Coherence factor, Doppler effect

1. Introduction

Investigations of the radiation spectrum of a system of electrons moving in magnetic fields are important from the point of view of their applications in astrophysics, electronics, plasma physics, etc. [1-3].

The fine structure of synchrotron radiation spectrum for one and two electrons moving in magnetic field was studied in [4–12].

In this paper using the exact integral relationship for the spectral distribution of radiation power of three and four electrons moving one by one along a spiral in vacuum, the fine structure of synchrotron radiation spectrum is investigated for the first time by means of analytical and numerical methods in the case when the longitudinal component of velocity (the component parallel to magnetic induction vector) is much less than the velocity of light in vacuum. The synchrotron radiation spectrum of a single electron is compared with the radiation spectra of two, three, and four electrons moving in a spiral in vacuum. The influence of the Doppler effect and coherence factor on the spectrum of synchrotron radiation for one, two, three, and four electrons is studied.

2. Radiation power of the system of electrons moving along a spiral in vacuum

The time-averaged radiation power $\overline{P}^{rad}$ of charged particles moving in magnetic field is expressed in [4-6] as

$$\overline{P}^{rad} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left\{\int_{\tau}\left(\vec{j}(\vec{r},t)\frac{1}{c}\frac{\partial\vec{A}^{Dir}(\vec{r},t)}{\partial t}-\rho(\vec{r},t)\frac{\partial\varphi^{Dir}(\vec{r},t)}{\partial t}\right)d\vec{r}\right\}dt \cdot \tag{1}$$

Here $\vec{j}(\vec{r},t)$ is the current density and $\rho(\vec{r},t)$ is the charge density. The integration is over some volume τ. According to the hypothesis of Dirac [13], the scalar $\varphi^{Dir}(\vec{r},t)$ and vector $\vec{A}^{Dir}(\vec{r},t)$ potentials are defined as a half-difference of the retarded and advanced potentials:

$$\varphi^{Dir} = \frac{1}{2}\left(\varphi^{ret}-\varphi^{adv}\right),$$
$$\vec{A}^{Dir} = \frac{1}{2}\left(\vec{A}^{ret}-\vec{A}^{adv}\right). \tag{2}$$

Then according to [4], the source functions of N charged point particles are defined as

$$\vec{j}(\vec{r},t)=\sum_{l=1}^{N}\vec{V}_l(t)\rho_l(\vec{r},t), \qquad \rho(\vec{r},t)=\sum_{l=1}^{N}\rho_l(\vec{r},t),$$
$$\rho_l(\vec{r},\, t)=e\,\delta\left(\vec{r}-\vec{r}_l(t)\right), \tag{3}$$

where $\vec{r}_l(t)$ and $\vec{V}_l(t)$ are the motion law and the velocity of the l^{th} particle, respectively.

The law of motion and the velocity of the l^{th} electron moving in a spiral in vacuum are given by the expressions

$$\begin{gathered}\vec{r}_l(t)=r_0\cos\{\omega_0(t+\Delta t_l)\}\;\vec{i}\,+\\ +r_0\sin\{\omega_0(t+\Delta t_l)\}\;\vec{j}+V_{\parallel}(t+\Delta t_l)\;\vec{k}\end{gathered}, \quad \vec{V}_l(t)=\frac{d\vec{r}_l(t)}{dt}. \tag{4}$$

Here $r_0 = V_{\perp}\omega_0^{-1}$, $\omega_0 = ceB^{ext}\tilde{E}^{-1}$, $\tilde{E} = c\sqrt{p^2+m_0^2c^2}$, the magnetic induction vector $\vec{B}^{ext}\,\|0Z$, $V_{\perp}$ and $V_{\parallel}$ are the components of the velocity, $\vec{p}$ and $\tilde{E}$ are the momentum and energy of the electron, e and m_0 are its charge and rest mass.

The time-averaged radiation power of system of electrons we obtain after substituting expressions (2) – (4) into (1). Then

$$\overline{P}^{rad} = \int_0^{\infty}W(\omega)d\omega, \tag{5}$$

$$W(\omega)=\frac{2e^2}{\pi c^2}\int_0^{\infty}dx\,\omega\,S_N(\omega)\frac{\sin\left\{\frac{1}{c}\omega\eta(x)\right\}}{\eta(x)}\cos\omega x\left[V_{\perp}^2\cos(\omega_0 x)+V_{\parallel}^2-c^2\right], \tag{6}$$

where $\eta(x)=\sqrt{V_{\|}^2 x^2+4\frac{V_{\perp}^2}{\omega_0^2}\sin^2\left(\frac{\omega_0}{2}x\right)}$, ω is the cyclic frequency, and c is the velocity of light in vacuum.

In the case of electrons moving one by one along a spiral the coherence factor $S_N(\omega)$ takes the form [4]:

$$S_N(\omega)=\sum_{l,j=1}^{N}\cos\{\omega(\Delta t_l-\Delta t_j)\}. \quad (7)$$

This factor determines a redistribution of the charged particles radiation power between harmonics.

3. Spectral distribution of synchrotron radiation power of two, three, and four electrons

Starting from relationships (5) and (6) the contribution of separate harmonics to the averaged radiation power can be written as

$$\overline{P}^{rad}=\frac{e^2}{c^3}\sum_{m=1}^{\infty}\int_0^{\infty}d\omega S_N(\omega)\omega^2\int_0^{\pi}\sin\theta d\theta\delta\left\{\omega\left(1-\frac{1}{c}V_{\|}\cos\theta\right)-m\omega_0\right\}\times$$
$$\times\left\{V_{\perp}^2\left[\frac{m^2}{q^2}J_m^2(q)+J_m'^2(q)\right]+\left(V_{\|}^2-c^2\right)J_m^2(q)\right\}. \quad (8)$$

where $q=\frac{1}{c}\frac{\omega}{\omega_0}V_{\perp}\sin\theta$, $J_m(q)$ and $J_m'(q)$ are the Bessel function with integer index and its derivative, respectively.

From relationship (8) one can conclude that each harmonic is a set of the frequencies, which are determined from the solution of the equation

$$\omega\left(1-\frac{1}{c}V_{\|}\cos\theta\right)-m\omega_0=0. \quad (9)$$

The influence of the Doppler effect determines the band's boundaries of separate harmonics in the radiation spectrum of the considered systems of two, three, and four electrons moving one by one n along a spiral in vacuum.

The coherence factor $S_1(\omega)$ of a single electron is defined as

$$S_N(\omega)=S_1(\omega)=1. \quad (10)$$

In the case of two electrons the coherence factor $S_2(\omega)$ is defined as

$$S_2(\omega)=2+2\cos(\omega\Delta t_{12}). \quad (11)$$

Here $\Delta t_{12}=\Delta t_2-\Delta t_1$ is the time shift between the first and second electrons moving along a spiral. The analogous expression for the coherence factor was obtained by Bolotovskii [14].

The coherence factor $S_3(\omega)$ of three electrons takes the form

$$S_3(\omega)=3+2\cos(\omega\Delta t_{12})+2\cos(\omega\Delta t_{23})+2\cos\{\omega(\Delta t_{12}+\Delta t_{23})\} \quad (12)$$

Here Δt_{23} is the time shift between the second and third electrons.

The coherence factor $S_4(\omega)$ of four electrons is defined as

$$S_4(\omega)=4+2\cos(\omega\Delta t_{12})+2\cos(\omega\Delta t_{23})+2\cos(\omega\Delta t_{34})+$$
$$+2\cos\{\omega(\Delta t_{12}+\Delta t_{23})\}+2\cos\{\omega(\Delta t_{23}+\Delta t_{34})\}+$$
$$+2\cos\{\omega(\Delta t_{12}+\Delta t_{23}+\Delta t_{34})\}, \quad (13)$$

where Δt_{34} is the time shift between the third and fourth electrons.

The total radiation power emitted by a single electron is determined, according to [15], as

$$P_{vac}^{tot}=\frac{2}{3}\frac{e^2\omega_0^2V_{\perp}^2}{c^3}\left(1-\frac{V^2}{c^2}\right)^{-2},\ \omega_0=\frac{eB^{ext}}{m_0c}\sqrt{1-\frac{V^2}{c^2}} \quad (14)$$

It is interesting to compare the radiation power spectral distribution for two, three and four electrons (curve 2 in Fig. 2, curve 3 in Fig. 3, and curve 4 in Fig. 4, respectively) was to that of a single electron (curve 1 in Fig. 1). Our numerical calculation of the radiation spectra were carried out on the basis of equations (5) and (6) taking into account the corresponding coherence factors. Spectral distribution of synchrotron radiation power is obtained for $B^{ext}=1$ Gs, $V_{\perp vac}=0.24\cdot10^{11}$ cm/s, $V_{\| vac}=0.15\cdot10^{10}$ cm/s, $c=0.2997925\cdot10^{11}$ cm/s, $r_{0j}=2285$ cm, $\omega_{0j}=0.105\cdot10^{8}$ rad/s (j=1,2,…,7).

The total radiation power of a single electron in vacuum $P_{vac1}^{tot}=0.2852\cdot10^{-14}$ erg/s calculated according to relationship (14) is in good agreement to the power $P_{vac1}^{\text{int}}=0.2839\cdot10^{-14}$ erg/s determined after integration of relationships (5) and (6) at $S_N(\omega)=S_1(\omega)=1$.

For the time shifts $\Delta t_{12}^{(2)}=0.001\cdot\pi/\omega_{02}$ the coherence factor $S_2(\omega)\cong4$ and two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$ (curve 2 in Fig. 2), i.e. by a factor of four higher than a single electron ($P_{vac2}^{\text{int}}\cong4\cdot P_{vac1}^{\text{int}}=0.1136\cdot10^{-13}$). Here the upper index (2) and other similar ones further mark the corresponding curves in the plots.

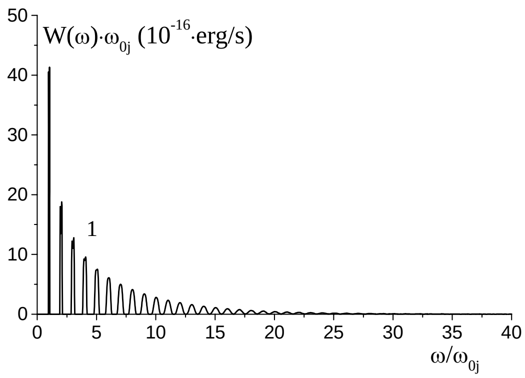

Fig. 1. Synchrotron radiation spectrum of a single electron moving in a spiral with radiation power $P_{vac1}^{\text{int}} = 0.2839 \cdot 10^{-14}$ *erg /s.*

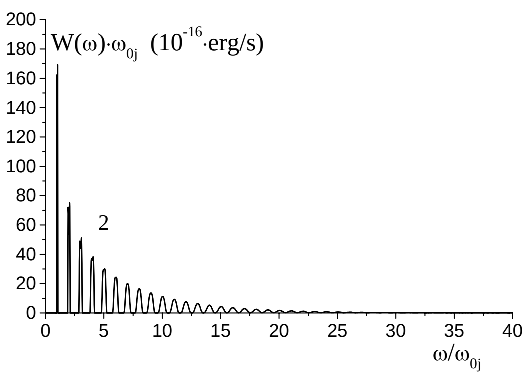

Fig. 2. Synchrotron radiation spectrum of two electrons moving in a spiral for the time shift $\Delta t_{12}^{(2)} = 0.001 \cdot \pi / \omega_{02}$ *with radiation power* $P_{vac2}^{\text{int}} = 0.1135 \cdot 10^{-13}$ *erg /s.*

For the time shifts $\Delta t_{12}^{(3)} = \Delta t_{23}^{(3)} = 0.001 \cdot \pi / \omega_{03}$ the coherence factor $S_3(\omega) \cong 9$ and three electrons radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$ (curve 3 in Fig. 3), i.e. by a factor of nine higher than a single electron ($P_{med3}^{\text{int}} \cong 9 \cdot P_{med1}^{\text{int}} = 0.2555 \cdot 10^{-13}$ erg/s).

For the case of four electrons at the time shifts $\Delta t_{12}^{(4)} = \Delta t_{23}^{(4)} = \Delta t_{34}^{(4)} = 0.001 \cdot \pi / \omega_{04}$, the coherence factor $S_4(\omega) \cong 16$ and four electrons radiate as a charged particle with the charge $4e$ and the rest mass $4m_0$ (curve 4 in Fig. 4), i.e. by a factor of sixteen higher than a single electron ($P_{vac4}^{\text{int}} \cong 16 \cdot P_{vac1}^{\text{int}} = 0.4543 \cdot 10^{-13}$ erg/s).

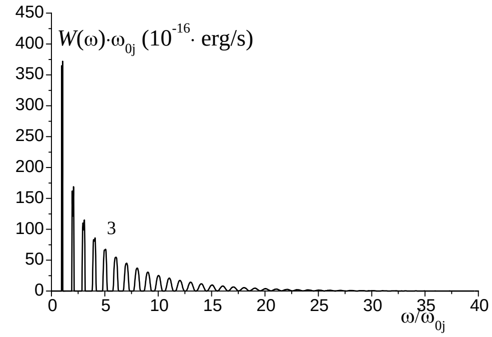

Fig. 3. Synchrotron radiation spectrum of three electrons moving in a spiral for the time shifts $\Delta t_{12}^{(3)} = \Delta t_{23}^{(3)} = 0.001 \cdot \pi / \omega_{03}$ *with radiation power* $P_{vac3}^{\text{int}} = 0.2554 \cdot 10^{-13}$ *erg/s.*

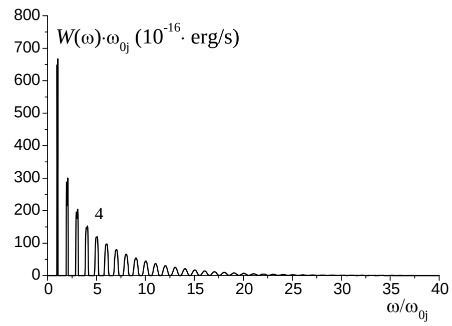

Fig. 4. Synchrotron radiation spectrum of four electrons moving in a spiral for the time shifts $\Delta t_{12}^{(4)} = \Delta t_{23}^{(4)} = \Delta t_{34}^{(4)} = 0.001 \cdot \pi / \omega_{04}$ *with radiation power* $P_{vac4}^{\text{int}} = 0.4540 \cdot 10^{-13}$ *erg /s.*

In the case of uniform location of two electrons (one the opposite gides of a convolution) at the time shifts $\Delta t_{12}^{5} = \pi / \omega_{05}$ any radiation at the frequencies $(2i-1)\,\omega_{05}$ (i=1,2,…,20) is absent (curve 5 in Fig. 5).

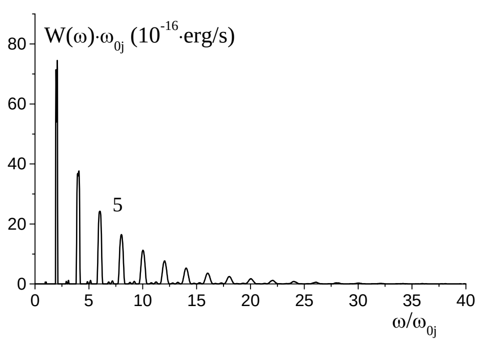

Fig. 5. Spectral distribution of synchrotron radiation power of two electrons moving one by one in a spiral for $\Delta t_{12}^{(5)} = \pi / \omega_{05}$ *and* $P_{vac5}^{\text{int}} = 0.4441 \cdot 10^{-14}$ *erg/s.*

In the case of uniform location of three electrons along spiral at the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = 2\pi/(3\cdot\omega_{03})$ we have found that any radiation also is absent at the frequencies $(3i-2)\omega_{06}$ and $(3i-1)\omega_{06}$ (i=1,2,…,13) (curve 6 in Fig. 6).

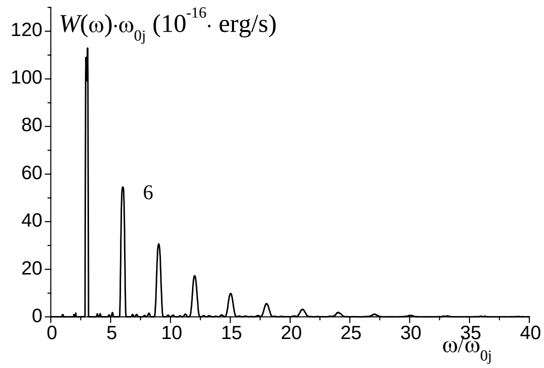

Fig. 6. Spectral distribution of synchrotron radiation power of three electrons moving one by one in a spiral for the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = 2\pi/(3\cdot\omega_{06})$ *and* $P_{vac6}^{\text{int}} = 0.7602\cdot 10^{-14}$ *erg/s.*

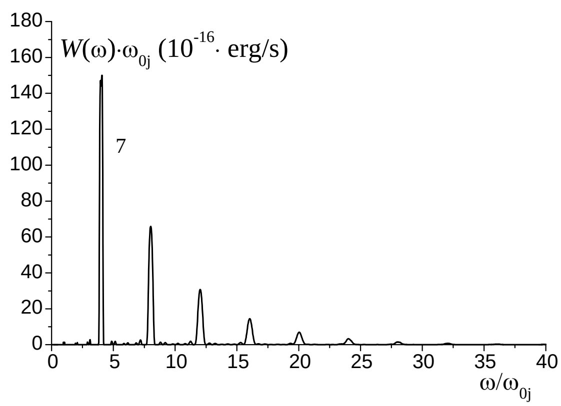

Fig. 7. Spectral distribution of synchrotron radiation power of four electrons moving one by one in a spiral for $\Delta t_{12}^{(7)} = \Delta t_{23}^{(7)} = \Delta t_{34}^{(7)} = \pi/(2\cdot\omega_{07})$ *and* $P_{vac7}^{\text{int}} = 0.1135\cdot 10^{-13}$ *erg/s.*

Similar in the case of uniform location of four electrons along spiral at the time shifts $\Delta t_{12}^{(7)} = \Delta t_{23}^{(7)} = \Delta t_{34}^{(7)} = \pi/(2\cdot\omega_{07})$ any radiation is absent at the frequencies $(4i-3)\omega_{07}$ $(4i-2)\omega_{07}$ and $(4i-1)\omega_{07}$ (i=1,2,…,10) (curve 7 in Fig. 7).

4. Conclusions

The influence of the Doppler effect determines the band's boundaries of separate harmonics in the radiation spectra of a single, two, three, and four electrons moving one by one along a spiral in vacuum.

For small time shifts two electrons radiate as a charged particle with the charge $2e$ and the rest mass $2m_0$, i.e. by a factor of four higher than a single electron.

For uniform location of two electrons along a spiral at the time shifts $\Delta t_{12}^{(2)} = \pi/\omega_{05}$ between them the radiation at the frequencies $(2i-1)\omega_{05}$ (i=1,2,…,20) is absent.

For small time shifts three electrons radiate as a charged particle with the charge $3e$ and the rest mass $3m_0$, i.e. by a factor of nine higher than a single electron.

For uniform location of three electrons along a spiral at the time shifts $\Delta t_{12}^{(6)} = \Delta t_{23}^{(6)} = 2\pi/(3\cdot\omega_{06})$ between them the radiation at the frequencies $(3i-2)\omega_{06}$ and $(3i-1)\omega_{06}$ (i=1,2,…,13) is absent.

At small time shifts four electrons radiate as a charged particle with the charge $4e$ and the rest mass $4m_0$, i.e. by a factor of sixteen higher than a single electron.

For uniform location of four electrons along a spiral with the time shifts $\Delta t_{12}^{(7)} = \Delta t_{23}^{(7)} = \Delta t_{34}^{(7)} = \pi/(2\cdot\omega_{07})$ between them the radiation at the frequencies $(4i-3)\omega_{07}$, $(4i-2)\omega_{07}$ and $(4i-1)\omega_{07}$ (i=1,2,…,10) is absent.

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