

# Realization of a general single qubit gate in a double quantum dot structure

YUXIA LIU, XINZHU SANG, CHONGXIU YU, DAXIONG XU, JUNYING ZENG

Key Laboratory of Optical Communication and Lightwave Technologies of Ministry of Education, School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, 100876 China

Based on the effect of a resonant laser pulse on an electron, the realization of a general single qubit gate  $U(\alpha, \phi)$  in a double-quantum-dot structure is investigated theoretically. By varying the voltage biases and the parameters of the pulse which acts on the double-dot structure, all single qubit gates can be implemented.

(Received April 4, 2007; accepted December 4, 2007)

Keywords: Quantum dot, Qubit gate

## 1. Introduction

Recently, there has been great interest in the implementation of quantum logic gates by manipulating two-level electron systems in semiconductor quantum dots [1]. Several interesting phenomena have been recognized when the quantum-dot structure contains one or two electrons. In 1999, Openov [2] first reported the resonant electron transferred between quantum dots, where the electron confined in the double-quantum-dot structures is interacted with external electromagnetic pulse. According to the above resonant phenomenon, a quantum NOT gate [3] and controlled qubit rotations were investigated [4]. In addition, quantum transmission and measurement were also analyzed [5-11]). We believe that progress in quantum information processing will boost the achievement of quantum computers.

Here, the potential of coherent manipulation of a double-quantum-dot structure is investigated. A method of realizing a general single qubit gate based on double-dot nanostructure is proposed. Moreover, the implementation of several single qubit gates, such as NOT, Hadamard and unit gates are discussed.

## 2. Model and analysis

Two semiconductor quantum dots (QDs) A and B, and an excess electron in the nanostructure are considered. Energy levels diagram of the nanostructure is shown in Fig. 1. The assumed conditions are given as the following: the QDs A and B are different, and the electron locates in the conduction band of A and B. Provided that the distance between the dots A and B is sufficiently large, the wave functions of the ground states  $\langle r|A\rangle$  and  $\langle r|B\rangle$  are localized in the corresponding QDs, and the overlap is negligible. Besides,  $U$  is the height of the energy barrier which separates the dots and one excited state  $|TAB\rangle$  whose energy lies just below  $U$  serves as a “transport state” for the electron.

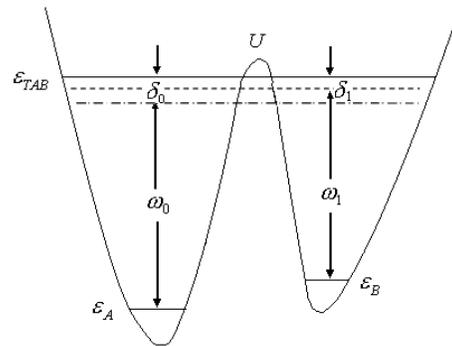


Fig.1. Energy levels diagram of two different quantum dots nanostructure.

Here, we use electron locations-ground states to encode qubits in semiconductor dots. If the electron locates in the dots A (B), the corresponding state is  $|A\rangle$  ( $|B\rangle$ ), which is viewed as the Boolean states 0 (1), respectively.

The external electromagnetic pulse which acts on the nanostructure is in the form of

$$\vec{E}(t) = [\vec{E}_0 \cos(\omega_0 t + \varphi_0) + \vec{E}_1 \cos(\omega_1 t + \varphi_1)] [\theta(t) - \theta(t - T)] \quad (1)$$

where  $T$  is the pulse duration, that is, the operation time.  $\vec{E}_0 \cos(\omega_0 t + \varphi_0)$  and  $\vec{E}_1 \cos(\omega_1 t + \varphi_1)$  act on dots A and B, respectively. We assume that  $\omega_{0,1}$  is close to the resonant frequency  $\omega_{A,B} = \mathcal{E}_{TAB} - \mathcal{E}_{A,B}$ , which equals to the energy difference between the ground state and the excited state of the dots, so that the detuning  $\delta_{0,1} = \omega_{A,B} - \omega_{0,1}$  satisfies  $|\delta_{0,1}| \ll \omega_{0,1}$ .

With the effect of the external electromagnetic pulse, the electron in the dots will do a resonant movement. Its

wave function  $|\Psi(t)\rangle$  satisfies the Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle \quad (2)$$

and the Hamiltonian is given as

$$H(t) = \sum_{K=A,B,TAB} \varepsilon_K a_K^\dagger a_K + \frac{1}{2}(\lambda_A a_{TAB}^\dagger a_A e^{-i\omega_A t} + \lambda_B a_{TAB}^\dagger a_B e^{-i\omega_B t} + h.c.) \quad (3)$$

where  $a_K^\dagger$  ( $a_K$ ) is the operator of creation (annihilation) of the electron in the state  $|K\rangle$ , and

$$\lambda_{A,B} = \vec{E}_{0,1} \vec{d}_{A,B} e^{-i\varphi_{0,1}} \quad (4)$$

in which

$$\vec{d}_{A,B} = -e\langle TAB|\vec{r}|A,B\rangle \quad (5)$$

is the dipole matrix element for the transitions  $|A,B\rangle \leftrightarrow |TAB\rangle$ .

From the resonant approximation, the state vector  $|\Psi(t)\rangle$  can be written as

$$|\Psi(t)\rangle = \sum_{K=A,B,TAB} C_K(t) e^{-i\varepsilon_K t} |K\rangle \quad (6)$$

Here, the detuning condition  $\delta_{0,1} = 0$  is considered for facilitating analytical solution. When the initial condition is

$$|\Psi(0)\rangle = |A\rangle \quad (7)$$

and the Hamiltonian is as described in Eq.(3), the solution of Eq.(2) is

$$\begin{aligned} |\Psi(t)\rangle = & e^{-i\varepsilon_A t} \left[ 1 - \frac{\lambda_A^* \lambda_A}{8\Omega^2} \sin^2(\Omega t) \right] |A\rangle + e^{-i\varepsilon_B t} \left[ -\frac{\lambda_B^* \lambda_A}{8\Omega^2} \sin^2(\Omega t) \right] |B\rangle \\ & - i e^{-i\varepsilon_{TAB} t} \frac{\lambda_A}{4\Omega} \sin(2\Omega t) |TAB\rangle \end{aligned} \quad (8)$$

where  $\Omega = \frac{\sqrt{|\lambda_A|^2 + |\lambda_B|^2}}{4}$  is the Rabi frequency for the system.

From Eq. (8), it can be seen that at the operation time  $T = \pi n / 2\Omega$  ( $n = 1, 2, \dots$ ), the electron state vector is completely localized in the logic qubit subspace  $\{|A\rangle, |B\rangle\}$ . Then the discussion can be divided into two parts according to the parity of  $n$ .

When  $T = \pi K / \Omega$  ( $K = 1, 2, \dots$ ), one has

$$|\Psi(t)\rangle = e^{-i\varepsilon_A T} |A\rangle \quad (9)$$

and when  $T = \pi(2K - 1) / 2\Omega$  ( $K = 1, 2, \dots$ ),

$$|\Psi(t)\rangle = e^{-i\varepsilon_A T} \frac{|\lambda_B|^2 - |\lambda_A|^2}{|\lambda_B|^2 + |\lambda_A|^2} |A\rangle - e^{-i\varepsilon_B T} \frac{2\lambda_A \lambda_B^*}{|\lambda_B|^2 + |\lambda_A|^2} |B\rangle \quad (10)$$

Alternatively, if the initial condition is

$$|\Psi(0)\rangle = |B\rangle \quad (11)$$

the solution of Eq. (2) is

$$\begin{aligned} |\Psi(t)\rangle = & e^{-i\varepsilon_A t} \left[ -\frac{\lambda_A^* \lambda_B}{8\Omega^2} \sin^2(\Omega t) \right] |A\rangle + e^{-i\varepsilon_B t} \left[ 1 - \frac{\lambda_B^* \lambda_B}{8\Omega^2} \sin^2(\Omega t) \right] |B\rangle \\ & - i e^{-i\varepsilon_{TAB} t} \frac{\lambda_B}{4\Omega} \sin(2\Omega t) |TAB\rangle \end{aligned} \quad (12)$$

When  $T = \pi K / \Omega$  ( $K = 1, 2, \dots$ ),

$$|\Psi(T)\rangle = e^{-i\varepsilon_B T} |B\rangle \quad (13)$$

and when  $T = \pi(2K - 1) / 2\Omega$  ( $K = 1, 2, \dots$ ),

$$|\Psi(T)\rangle = -e^{-i\varepsilon_A T} \frac{2\lambda_A^* \lambda_B}{|\lambda_B|^2 + |\lambda_A|^2} |A\rangle - e^{-i\varepsilon_B T} \frac{|\lambda_B|^2 - |\lambda_A|^2}{|\lambda_B|^2 + |\lambda_A|^2} |B\rangle \quad (14)$$

At  $T = \pi K / \Omega$  ( $K = 1, 2, \dots$ ), it can be seen from Eqs.(9) and (13), if  $\varepsilon_A \neq \varepsilon_B$ , qubit rotations are performed. Now, let us see how to realize a general single qubit gate at  $T = \pi(2K - 1) / 2\Omega$  ( $K = 1, 2, \dots$ ). Assume that

$$|\lambda_B| = C |\lambda_A| \quad (15)$$

in which  $C$  is a constant. Then, we can obtain

$$\begin{aligned} \frac{|\lambda_B|^2 - |\lambda_A|^2}{|\lambda_B|^2 + |\lambda_A|^2} &= \frac{C^2 - 1}{C^2 + 1}, \quad \frac{2\lambda_A \lambda_B^*}{|\lambda_B|^2 + |\lambda_A|^2} = \frac{2C}{C^2 + 1} e^{i(\varphi_1 - \varphi_0)}, \\ \frac{2\lambda_A^* \lambda_B}{|\lambda_B|^2 + |\lambda_A|^2} &= \frac{2C}{C^2 + 1} e^{-i(\varphi_1 - \varphi_0)}. \end{aligned}$$

$$\text{If } (\varepsilon_B - \varepsilon_A)T = 2m\pi + \pi(m \in \mathbb{Z}) \quad (16)$$

and

$$\varphi_1 - \varphi_0 = 2n\pi - \frac{\pi}{2} + \phi(n \in \mathbb{Z}, \phi \in [-\pi, \pi)) \quad (17)$$

Eqs.(10) and (14) can be rewritten as

$$|\Psi(T)\rangle = e^{-i\varepsilon_A T} [\cos \alpha |A\rangle - i e^{i\phi} \sin \alpha |B\rangle] \quad (18)$$

$$|\Psi(T)\rangle = e^{-i\varepsilon_A T} [\cos \alpha |B\rangle - ie^{-i\phi} \sin \alpha |A\rangle] \quad (19)$$

where  $\cos \alpha = \frac{C^2 - 1}{C^2 + 1}$  and  $\sin \alpha = \frac{2C}{C^2 + 1}$ .

Now, from Eqs.(18) and (19) we can see that, when  $T = \pi(2K - 1)/2\Omega(K = 1, 2, \dots)$ , the  $U(\alpha, \phi)$  gate is implemented if Eqs.(15), (16) and (17) hold, where

$$\alpha = \arccos \frac{C^2 - 1}{C^2 + 1} \quad (\alpha \in [0, \pi]) \quad (20)$$

$$\phi = 2n\pi + \frac{\pi}{2} + (\varphi_1 - \varphi_0) \quad (n \in \mathbb{Z}, \phi \in [-\pi, \pi]) \quad (21)$$

Actually, from Eqs.(4), (5), (15) and (20), it can be seen that  $\alpha$  is a function of the distance between the QDs ( $r$ ) and the electric field amplitudes ( $E_{0,1}$ ), and the value of  $\alpha$  will be changed with the variation of  $E_{0,1}$ .

$\varepsilon_A$  and  $\varepsilon_B$  are dependent on the electron effective mass, the dots size and the energy barrier. And the energy barrier is dependent on the voltage biases which apply to the surface dots. So through adjusting the voltage biases, Eq.(16) is easy to be satisfied. Besides, it can be seen from Eq.(21),  $\phi$  can reach all its intended values by changing the initial phases of the electric field. Therefore, a single qubit gate  $U(\alpha, \phi)$  can be implemented by varying the voltage biases and pulse parameters such as intensity, phases, duration and frequencies. In the following we give some examples.

If  $\alpha = \frac{\pi}{2}$  and  $\phi = 0$ , according to Eqs.(7), (11), (18) and (19),

$$|\Psi(0)\rangle = |A\rangle \rightarrow |\Psi(T)\rangle = -ie^{-i\varepsilon_A T} |B\rangle \quad (22)$$

$$|\Psi(0)\rangle = |B\rangle \rightarrow |\Psi(T)\rangle = -ie^{-i\varepsilon_A T} |A\rangle \quad (23)$$

The right hand sides of Eqs.(22) and (23) have the same global phase factor  $-ie^{-i\varepsilon_A T}$ , which has no observed effects in a real implementation and can be ignored. Therefore, Eqs. (22) and (23) mean that a NOT gate is realized by using the double-dot structure.

Similarly, if  $\alpha = \frac{\pi}{4}$  and  $\phi = -\frac{\pi}{2}$ , we have

$$|\Psi(0)\rangle = |A\rangle \rightarrow |\Psi(t)\rangle = \frac{\sqrt{2}}{2} e^{-i\varepsilon_A T} [|A\rangle - |B\rangle] \quad (24)$$

$$|\Psi(0)\rangle = |B\rangle \rightarrow |\Psi(t)\rangle = \frac{\sqrt{2}}{2} e^{-i\varepsilon_A T} [|B\rangle + |A\rangle] \quad (25)$$

which corresponds to a Hadamard gate.

If  $\alpha = \frac{\pi}{2}$  and  $\phi = -\frac{\pi}{2}$ , then

$$|\Psi(0)\rangle = |A\rangle \rightarrow |\Psi(t)\rangle = ie^{-i\varepsilon_A T} (i|B\rangle) \quad (26)$$

$$|\Psi(0)\rangle = |B\rangle \rightarrow |\Psi(t)\rangle = ie^{-i\varepsilon_A T} (-i|A\rangle) \quad (27)$$

which implies that the realized operation in the double-dot structure is the unit operator  $\hat{Y}$ .

From the above discussion, we know that several single qubit gates can be realized by varying the voltage biases and the parameters of the pulse which act on the double-quantum-dot structure. The general gate can be conveniently used in experiments.

### 3. Conclusion

Charge qubit rotations in a double-dot structure were analyzed [3], including a NOT gate and a Hadamard gate, but the method how to choose proper parameters to realize other single qubit gates hadn't been discussed. Here we have shown that a general single qubit gate  $U(\alpha, \phi)$  can be realized by using the microwave pulses with different parameters. In addition, the particular implementations of some single qubit gates such as NOT gate, Hadamard gate and unit  $\hat{I}$  operator are presented.

### Acknowledgement

This paper is supported by the National Basic Research Program of China (2006CB921105), Research Grant of BUPT, Open Grant of Beijing Key Lab of Electromechanical System Control and Measurement and the Initiating Grant For PH.D. of School of Electronic Engineering, BUPT.

### References

- [1] A. Barenco, D. Deutsch, A. Ekert, R. Jozsa, Phys. Rev. Lett. **74**, 4083 (1995).
- [2] L. A. Openov, Phys. Rev. B **60**, 8798 (1999).
- [3] L. A. Openov, A. V. Tsukanov, Pis'ma Zh. Eksp. Teor. Fiz. **80**, 572 (2004).
- [4] E. Paspalakis, Z. Kis, E. Voutsinas, A. F. Terzis, Phys. Rev. B **69**, 155316 (2004).
- [5] A. Borzi, G. Stadler, U. Hohenester, Phys. Rev. A **66**, 053811 (2002).
- [6] C. Karrasch, T. Enss, V. Meden, Phys. Rev. B **73**, 235337 (2006).
- [7] Richard Berkovits, Boris Altshuler, arXiv.org: cond-mat/0610466 (2006).
- [8] V. Meden, F. Marquardt, Phys. Rev. Lett. **96**, 146801 (2006).
- [9] T. Gilad, S. A. Gurvitz, Phys. Rev. Lett. **97**, 116806 (2006).
- [10] HuJun Jiao, Xin-Qi Li, Phys. Rev. B **75**, 155333 (2007).
- [11] Hui Zhang, Guo-Ping Guo, Tao Tu, Guang-Can Guo, arXiv.org: cond-mat /0702230v1 (2007).

\*Corresponding author: liuyunxiaee@gmail.com