Reflection and transmission coefficients from chiral nihility slab

N. KAYA, K. DELIHACIOGLU[a,*]
Kilis 7 Aralık University Faculty of Science and Art, Physics Department, Kilis Turkey
[a]Kilis 7 Aralık University, Faculty of Engineering and Architecture, Electrical and Electronics Engineering Department, Kilis, Turkey

Chiral nihility media are known to produce two polarizations, yielding negative refraction at the free space-chiral nihility medium boundary. The electromagnetic wave reflection and transmission from a chiral nihility slab have been theoretically studied in this paper. This study has been done for both a linearly and a circularly polarized waves that is obliquely incident on a chiral nihility slab from the free space region. Since the relative permittivity and the permeability values are very small, the wave impedance of chiral nihility slab is automatically matched to that of free space. It is seen that total reflection and total transmission exist that cover a wide range of incident angles. Chiral nihility slab can be used to get a circularly polarized wave from linear polarization. The results are compatible with the previous study in literature.

(Received October 16, 2013; accepted July 10, 2014)

Keywords: Chiral media, Chiral nihility, Negative refraction

1. Introduction

In the last decade, there has been fast development of the study and design of materials with negative electromagnetic parameters, especially artificial chiral and metamaterial media supporting backward waves. Chiral media, known as the optically active media, were discovered in the beginning of the 19^{th} century. Chiral media are naturally realizable at visible light ranges. So far it has been widely studied in the scope of polarization, linear and nonlinear propagation in electromagnetic, physics and so forth. The chief property of optically active media is that the polarization plane of linearly polarized electromagnetic wave is rotated as wave passes through the medium. The polarization state of the transmitted wave depends significantly on the constitutive parameter values, as well as the thickness of the chiral medium and the incident angle [1]. In chiral media, the electric or magnetic incident field simultaneously produces both electric and magnetic polarization that exhibits magneto–electric coupling. In 1968, Veselego theoretically demonstrated the electromagnetic properties of substances with simultaneously negative permittivity and permeability [2]. These materials are referred to as metamaterial, left handed materials, backward wave materials, and so forth which is not normally exist in nature. As well as in chiral medium, metamaterial medium supports backward waves and shows negative refraction too. Zhang et al. [3] have studied electromagnetic fields propagating through metamaterial slabs. The double negative nature of the constitutive parameters results in the propagation of electromagnetic waves exhibiting antiparallel phase and group velocities. Ziolkowski et al. [4] have studied metamaterials both analytically and numerically. This artificial material was experimentally verified in [5]. It was realized using split ring resonators and thin wire structures in the microwave frequency regime. But, without requiring metallic structures, by using Gyro-chiral media, the refractive index can be made negative [6]. The negative refraction can also be realized in chiral metamaterials with a strong chirality, with neither ε nor μ negative required [7].

Chiral nihility is special kind of chiral medium. The term “nihility” was first introduced by Lakhtakia [8] and later extended by Tretyakov et al. [9] for the isotropic chiral medium. Unlike the conventional chiral medium, the relative permittivity and permeability of chiral nihility medium are simultaneously zero. In this case, the refractive index becomes zero at certain frequency, which is called as nihility frequency. At the frequency where the relative permittivity and permeability values are zero, the chirality parameter is nonzero. The wave impedance of the chiral nihility medium has been considered close to wave impedance of free space. Although the relative permittivity and permeability are zero in chiral nihility medium, the modified values, which are very small, are considered in this study. The realization of the chiral nihility medium is discussed in [9-11]. Tretyakov et al. [9], showed that this exotic material can be realized as a mixture of small helical inclusions. In [9], the regular array of small chiral or omega shaped ideally conducting particles are used to have a null valued permittivity and permeability. The chiral nihility medium exhibits very interesting properties about the wave propagation. When an electromagnetic wave is incident from free space to chiral nihility slab, double refraction takes place at the interface. An incident wave splits into two circularly polarized waves, such that one of them is refracted positively but the other one is refracted negatively (backward wave with negative phase velocity) like in left handed media. For backward waves, the phase velocity is antiparallel to corresponding Poynting vector. Recently, chiral nihility is very popular and has many interesting applications such as focusing [12], chiral fibers [13], scattering [14], cloaking [15], fractional dual solutions in grounded chiral nihility slab [16], wave tunneling and rejection properties in chiral nihility photonics [17], the plane wave in chiral nihility-chiral nihility interface [18]. The propagation of electromagnetic plane waves in an isotropic chiral medium is characterized, and a special interest is shown in chiral nihility and the effects of chirality on energy transmission [19]. Negative refraction in chiral nihility media is visualized in [20] by showing the propagation of Gaussian beam, both reflected and refracted from an air-chiral interface and through layered chiral nihility media that are matched to free space. In [21], the problem of reflection from and transmission through a dielectric-chiral interface and wave propagation in an infinitely chiral slab is discussed in detail.

In this paper, the reflection from and transmission through the chiral nihility slab were investigated for obliquely incident linear and circular polarizations from the free space region. Since the relative permittivity and the permeability values are very small, the wave impedance of chiral nihility slab is automatically matched to that of free space. After matching the boundary conditions at the interfaces a matrix equation is obtained. The reflection and transmission coefficients can be obtained from the matrix inversion. The reflection and transmission coefficients due to chiral nihility slab are plotted for obliquely incident linear and circular polarizations.

2. Chıral Nıhılıty slab

The reflection from and transmission through an infinite chiral nihility slab of thickness d are considered for an obliquely incident linear and circular polarizations as shown in Fig. 1. The Cartesian coordinate system (x,y,z) is employed, and x-z plane is considered the plane of incidence. Assuming $e^{j\omega t}$ time dependence, the constitutive relations of an isotropic, lossless, and reciprocal chiral slab can be written as [1],

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} - j\xi \boldsymbol{H} \tag{1}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} + j\xi \boldsymbol{E} \tag{2}$$

where, ε and μ are the permittivity and permeability values, $\xi = \xi_o\sqrt{\varepsilon_o\mu_o}$ and ξ_o is the chirality parameter of the chiral nihility slab. ε_o and μ_o are the free space permittivity and permeability, respectively.

For an isotropic chiral nihility medium the constitutive relations reduce to

$$\boldsymbol{D} = -j\xi \boldsymbol{H} \tag{3}$$

$$\boldsymbol{B} = j\xi \boldsymbol{E} \tag{4}$$

As shown in Fig. 1, the incident and reflected field equations corresponding to perpendicular polarization can be written as,

$$\boldsymbol{E}_i = \boldsymbol{a}_1 e^{-jk_x x - jk_z z} \tag{5}$$

$$\boldsymbol{H}_i = \frac{\boldsymbol{a}_2^i}{\eta} e^{-jk_x x - jk_z z} \tag{6}$$

$$\boldsymbol{E}_r = \left(R_{LCP}(\boldsymbol{a}_1 + j\boldsymbol{a}_2^r) + R_{RCP}(\boldsymbol{a}_1 - j\boldsymbol{a}_2^r)\right)e^{-jk_x x + jk_z z} \tag{7}$$

$$\boldsymbol{H}_r = \frac{1}{\eta}\left(R_{LCP}(-j\boldsymbol{a}_1 + \boldsymbol{a}_2^r) + R_{RCP}(j\boldsymbol{a}_1 + \boldsymbol{a}_2^r)\right)e^{-jk_x x + jk_z z} \tag{8}$$

where $k_z = k_o cos\theta_i$, $k_o = \omega\sqrt{\varepsilon_o\mu_o}$, and $\boldsymbol{a}_{1,2}$ are the unit vectors and $\boldsymbol{a}_1 = \boldsymbol{a}_y$ as shown in Fig. 1. R_{LCP} and R_{RCP} are the amplitude of the left circularly polarized (LCP) and right circularly polarized (RCP) reflected fields. η is the free space wave impedance and θ_i is the incident angle.

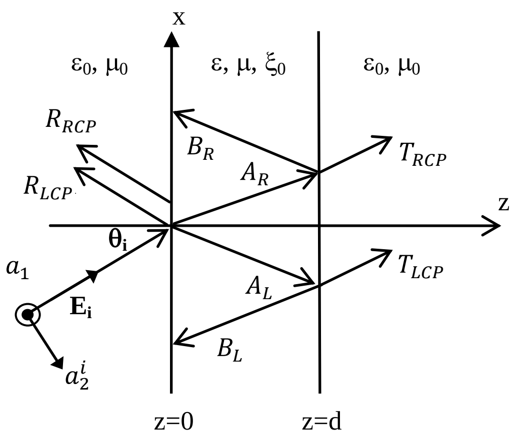


Fig. 1. The configuration of an infinitely long chiral nihility slab of thickness d located between z=0 and z=d.

The waves inside the chiral nihility slab are RCP and LCP waves with different phase velocities of ω/k_R and ω/k_L, respectively. The wavenumbers of RCP and LCP waves are,

$$k_{R,L} = \omega\sqrt{\varepsilon\mu} \pm \xi \tag{9}$$

where ω is the radial frequency. The right and left circularly polarized waves have the same impedance in the chiral nihility slab and equal to the wave impedance of free space. The total electric and magnetic field equations can be written as

$$\boldsymbol{E} = A_L(\boldsymbol{a}_1 + j\boldsymbol{a}_2^{a-})\, e^{jk_x x - jk_z^- z} + A_R(\boldsymbol{a}_1 - j\boldsymbol{a}_2^{a+})\, e^{-jk_x x - jk_z^+ z}$$
$$+B_L\left(\boldsymbol{a}_1 + j\boldsymbol{a}_2^{b-}\right) e^{jk_x x + jk_z^- z} + B_R\left(\boldsymbol{a}_1 - j\boldsymbol{a}_2^{b+}\right) e^{-jk_x x + jk_z^+ z} \tag{10}$$

$$\boldsymbol{H} = \frac{1}{\eta}\{A_L(-j\boldsymbol{a}_1 + \boldsymbol{a}_2^{a-})e^{jk_x x - jk_z^- z} + A_R(j\boldsymbol{a}_1 + \boldsymbol{a}_2^{a+})e^{-jk_x x - jk_z^+ z}$$

$$+B_L(-j\boldsymbol{a}_1 + \boldsymbol{a}_2^{b-})e^{jk_x x + jk_z^- z} + B_R(j\boldsymbol{a}_1 + \boldsymbol{a}_2^{b+})e^{-jk_x x + jk_z^+ z}\} \quad (11)$$

where $A_{R,L}$ and $B_{R,L}$ are the amplitude of the RCP and LCP waves. $k_z^{R,L} = k_{R,L}cos\theta_{R,L}$, $\theta_{R,L} = sin^{-1}\left(\frac{k_o sin\theta_i}{k_{R,L}}\right)$ and $\theta_{R,L}$ are the refracted angles for both circularly polarized waves. $\boldsymbol{a}_2^{a_{R,L}}$ *and* $\boldsymbol{a}_2^{b_{R,L}}$ are unit vectors such that:

$$\boldsymbol{a}_2^{a_{R,L}} = \boldsymbol{a}_z sin\theta_{R,L} - \boldsymbol{a}_x cos\theta_{R,L}$$

$$\boldsymbol{a}_2^{b_{R,L}} = \boldsymbol{a}_z sin\theta_{R,L} + \boldsymbol{a}_x cos\theta_{R,L}$$

The transmitted electric and magnetic fields from the chiral nihility slab can be written as,

$$\boldsymbol{E}_t = \left(T_{LCP}(\boldsymbol{a}_1 + j\boldsymbol{a}_2^i) + T_{RCP}(\boldsymbol{a}_1 - j\boldsymbol{a}_2^i)\right)e^{-jk_x x - jk_z z} \quad (12)$$

$$\boldsymbol{H}_t = \frac{1}{\eta}\left(T_{LCP}(-j\boldsymbol{a}_1 + \boldsymbol{a}_2^i) + T_{RCP}(j\boldsymbol{a}_1 + \boldsymbol{a}_2^i)\right)e^{-jk_x x - jk_z z} \quad (13)$$

where T_{LCP}, T_{RCP} are the amplitude of the LCP and RCP transmitted waves.

To find the amplitudes of reflection and transmission coefficients, the boundary conditions on the tangential electric and magnetic fields are applied at two interfaces situated at z=0 and z=d. The wave impedance of chiral nihility slab is also matched to that of free space at the interfaces. When this is done, a system of eight nonhomogeneous equations with eight unknowns is obtained. This system of equations can be written in the following matrix form:

$$[V] = [Z][I] \quad (14)$$

where Z is the following matrix:

$$Z = \begin{bmatrix} -1 & -1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & C_1 & -C_2 & -C_1 & C_2 & 0 & 0 \\ -1 & -1 & -C_1 & -C_2 & C_1 & C_2 & 0 & 0 \\ -1 & 1 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & D_1 & D_2 & D_3 & D_4 & -F & -F \\ 0 & 0 & C_1D_1 & -C_2D_2 & -C_1D_3 & C_2D_4 & -F & F \\ 0 & 0 & D_1 & -D_2 & D_3 & -D_4 & -F & F \\ 0 & 0 & -C_1D_1 & -C_2D_2 & C_1D_3 & C_2D_4 & F & F \end{bmatrix}$$

where $C_{1,2} = \frac{cos\theta_{L,R}}{cos\theta_i}$, $D_{1,3} = e^{\mp jk_z^- d}$,

$$D_{2,4} = e^{\mp jk_z^+ d}, \; F = e^{-jk_z d}$$

and the excitation (V) corresponding to incident fields (perpendicular, parallel and circular polarizations) and unknown coefficient (I) matrices are,

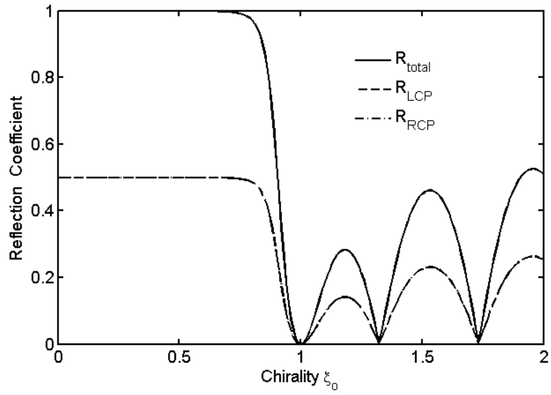
$$V_\perp = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V_\parallel = \begin{bmatrix} 0 \\ -j \\ 0 \\ -j \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad V_{L,R} = \begin{bmatrix} 1 \\ \pm 1 \\ -1 \\ \pm 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad I = \begin{bmatrix} R_{LCP} \\ R_{RCP} \\ A_L \\ A_R \\ B_L \\ B_R \\ T_{LCP} \\ T_{RCP} \end{bmatrix}$$

By using a numerical technique to invert the matrix equation (14), the reflection, (R_{LCP}, R_{RCP}) and transmission (T_{LCP}, T_{RCP}) coefficients can be determined.

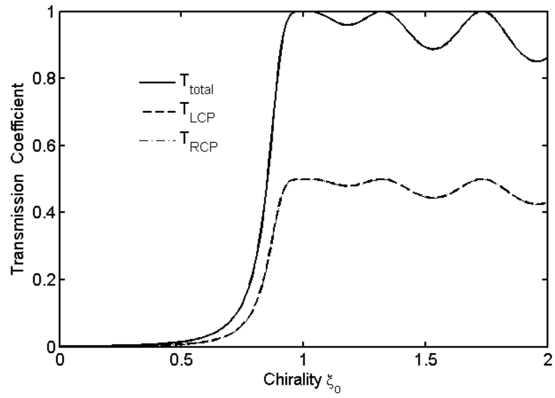
3. Numerical results

In this section, the numerical results of reflection and transmission from a chiral nihility slab were presented. The electric field of the incident wave was assumed to have a magnitude of 1 V/m. The chiral slab with both relative permittivity and relative permeability having values of $1x10^{-5}$ (very close to 0) are taken. The slab thickness was assumed to be d=λ, where λ is the wavelength of the incident wave in free space. The operating frequency of the incident wave is taken as 10 GHz. The chirality parameter is taking values between 0 and 2 in the plots. For a chiral medium to be negatively refractive, the chirality parameter should be chosen sufficiently large compared with the product of the relative permittivity and permeability. Unlike the chiral slab (the chirality admittance is restricted between zero and the square root of ε/μ [22]), the chiral nihility slab produces two equal amplitude reflection and transmission coefficients.

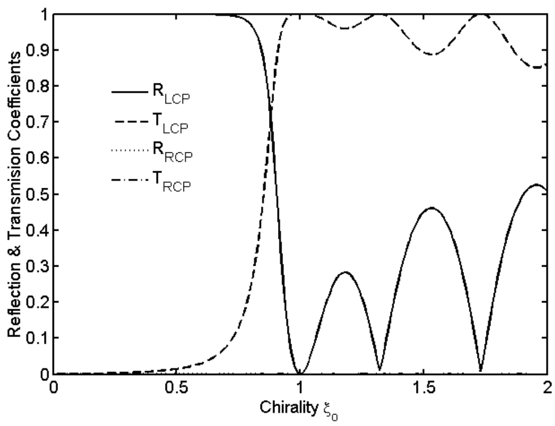
When a linearly polarized wave (either perpendicular or parallel) is obliquely incident on a chiral nihility slab, we have two circularly polarized reflected and transmitted waves with equal amplitudes which have half of the amplitude of incident wave. Due to this amplitude equivalence, the reflection and transmission coefficients of LCP and RCP waves are seen as overlapping. The total reflection (R_{total}) and transmission (T_{total}) coefficients are the sum of the reflection $(R_{LCP} + R_{RCP})$ and transmission $(T_{LCP} + T_{RCP})$ coefficients, as shown in Figs. 2a, and 2b. From this point of view the chiral nihility slab is used to obtain LCP and RCP waves (wave splitter) from linear polarizations and can also be used like a power divider due to amplitude reduction. In case of LCP (RCP) incident wave, the reflected and transmitted waves are LCP (RCP). Namely, LCP incident wave produces LCP reflected and transmitted waves and no RCP waves, or vice versa, as shown in Fig. 2c. As can be seen from Fig. 2, the amplitude of total reflected and transmitted waves are the same independently of polarization type. The reflected and transmitted wave polarizations corresponding to incident one is summarized in Table 1.



a



b



c

Fig. 2 (a) Reflection (b) Transmission coefficients for perpendicularly polarized incident wave (c) Reflection and transmission coefficients for LCP incident wave $(\theta_i = 60^\circ)$.

Table 1. Reflected and transmitted waves corresponding to polarization type of incident wave.

Incident wave	Reflected wave	Transmitted wave
Linear	LCP, RCP	LCP, RCP
LCP	LCP	LCP
RCP	RCP	RCP

In Fig. 3, the magnitudes of the reflection and transmission coefficients are plotted with respect to the chirality parameter for three different values of the angle of incidences. The reflection and transmission coefficients strongly depend on the values of chirality parameter. For three incident angles, full reflection and full transmission regions are observed. As the angle of incidence increases the full reflection region also increases but the full transmission region decreases. For the chirality parameter variations at different incident angles the reflection curve looks like a low pass filter whereas the transmission curve looks like a high pass filter.

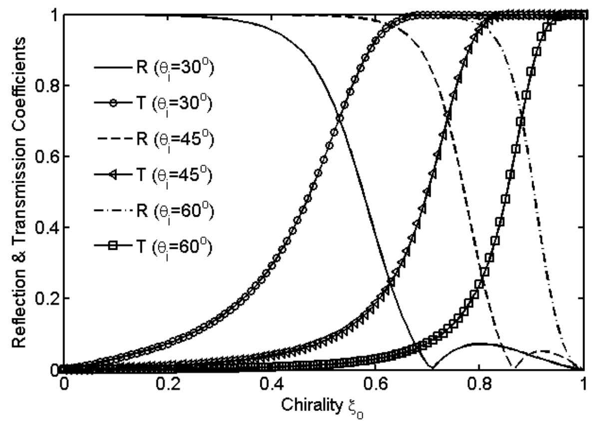
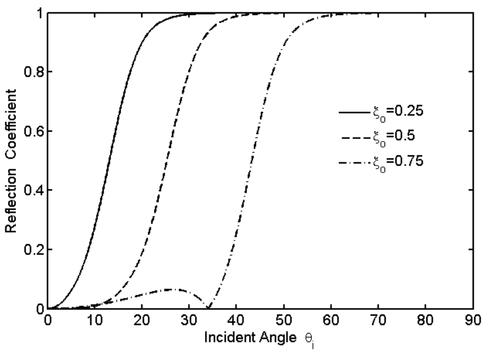
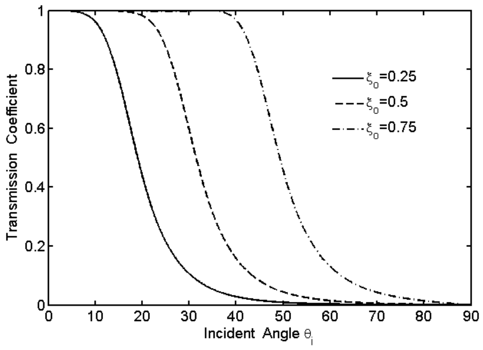


Fig. 3. Reflection and Transmission coefficients of chiral nihility slab versus the chirality parameter for different values of the incident angles.

Fig. 4 shows the variation of the reflection and transmission coefficients versus the angle of incidence for different values of chirality parameters. For three values of chirality parameter (0.25, 0.5, 0.75) there exist critical (total reflection) and Brewster angles (total transmission) which cover an extremely wide range of incident angles. In Fig. 4*a*, as the chirality parameter increases the critical angle moves to higher degrees. At normal incidences the incident wave passes from chiral nihility slab without reflection. It can be said that the chiral nihility slab matched to free space at normal incidence. In Fig. 4b, the full transmission region enlarges as the chirality parameter increases. At normal incidence the wave is a fully transmitted.



a



b

Fig. 4. (a) Reflection and (b) Transmission coefficients of chiral nihility slab versus the angle of incidence for different values of the chirality parameter.

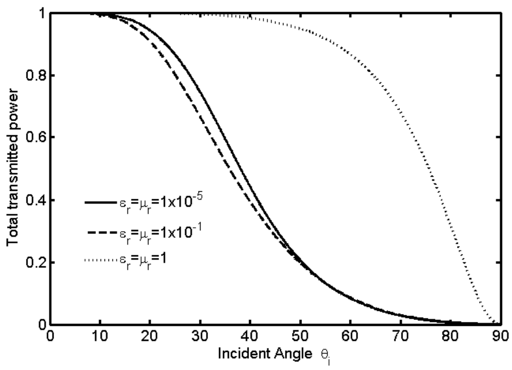


Fig. 5. Total transmitted power versus incident angle for three different values of ε_r and μ_r. ($\xi_o = 0.25$, f= 10 GHz, d=5 mm).

For a linearly polarized wave the total transmitted power is plotted in Fig. 5 so as to compare with previous study found in literature. As it is shown in the figure the results are in good agreement with the Fig. 9b of reference [19].

4. Conclusıon

In this paper, new parameters of low values of relative permittivity and permeability have been proposed. The reflection and transmission coefficients of an obliquely incident wave upon an impedance matched chiral nihility slab are investigated. It is seen that the reflection and transmission coefficients are strongly dependent on incident angle, polarization, and chirality parameter. Since the chiral nihility slab is matched to free space, it exhibits different wave propagating characteristics depending on the value of chirality parameters. Now that we observed two equal amplitude LCP and RCP waves, chiral nihility slab can be used to obtain circularly polarized waves from linear polarization. The chiral nihility slab can be used as wave splitter for the case of linearly polarized incident wave. It is also used as a power divider due to amplitude reduction of the incident wave. Total reflection and total transmission that cover a wide range of incident angles are observed.

References

[1] I. V. Lindel, A. H. Sihvola, S. A. Tretyakov, A. J. Viitanen, Norwood, MA, Artech House, (1994).
[2] V. G. Veselego, Sov. Phys. Usp. **10**, (1968).
[3] Y. Zhang, T. M. Gregorczyk, J. A. Kong, PIER **35**, (2001).
[4] R. W. Ziolkowski, E. Heyman, Phys Rev E, **64**, (2001).
[5] R. A. Shelby, D. R. Smith, S. Schultz, Science **292**, (2001).
[6] C. W. Qiu, H. Y. Yao, S. Zouhdi, L. W. Li, M. S. Leong, Microwave Opt. Technol. Lett. **48**, 12, (2006).
[7] B. Wang, J. Zhou, T. Koschny, M. Kafesaki, C. M Soukoulis, J. Opt. A: Pure Appl. Opt. **11** (2009).
[8] A. Lakhtakia, Int. J. Inf. and Mil. Wav., **22**, 12 (2001).
[9] S. Tretyakov, I. Nefedov, A. Sihvola, S. Maslovski, in Proceedings of the Progress in Electromagnetics Research Symposium, Honolulu, Hawaii, USA, (2003).
[10] S. Tretyakov, I. Nefedov, A. H. Sihvola, S. Maslovki, C. Simovski, Journal of Electromagnetic Waves and Applications, **17**, 5 (2003).
[11] Q. Cheng, T. J. Cui, C. Zhang, Optics Communications, **276**, 2 (2007).
[12] A. Illahi, Q. A. Naqvi, Journal of Electromagnetic Waves and Applications, **23**, 7 (2009).
[13] J. Dong, PIER, **99**, (2009).
[14] S. Ahmed, Q. A. Naqvi, PIER Letters, **18**, (2010).
[15] X. Cheng, H. Chen, X.-M. Zhang, B. Zhang, B.-I. Wu, PIER, **100**, (2010).
[16] Q. A. Naqvi, Journal of Electromagnetic Waves and Applications, **23**, 56 (2009).
[17] V. R. Tuz, C.-W. Qiu, PIER, **103**, (2010).
[18] M. Taj, A. Naqvi, A. A. Syed, Q. A. Naqvi, PIER, Letters, **22**, (2011).
[19] C. W. Qiu, N. Burokur, S. Zouhd, L. W. Li, J. Opt. Soc. Am. A., **25**, 1 (2008).
[20] X. Cheng, H. Chen, B. I. Wu, J. A. Kong, IEEE Antennas and Propagation Magazine, **51**, 4 (2009).
[21] S. Bassiri, C. H. Papas, N. Engheta, J. Opt. Soc. Am. A, **5**, (1988).
[22] N. Engheta, D. L. Jaggard, IEEE Antennas and Propagation Society Newsletter, **30**, (1988).

*Corresponding author: kemal@kilis.edu.tr