Reflection effects in stratified dielectric structures

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The electromagnetic response of a multi – layer half space by means of the ray analysis technique is presented. The influence of the thickness as well as of the electromagnetic constants of the dielectric layers on the reflection coefficient is studied, assuming both pure and lossy dielectrics. The zeros and the periodicities of the reflection coefficient are strongly influenced from the geometrical and the electrical characteristics of the multi – layer half space. Numerical results for several cases show the ability of the ray technique to analyze as well as to design multi – layer media in a given frequency range.

1. Introduction

The study of the mechanism of the EM response of a multilayered dielectric structure is one of the most interesting problems in Applied Science and Engineering. Several works given in the past ([1] – [4]) make use of ray analysis and give numerical solution of the EM response of a few – layer earth model. Our present study extends the same technique for up to N – layers model. A TM plane wave with oblique incidence is used and the reflection coefficient is found.
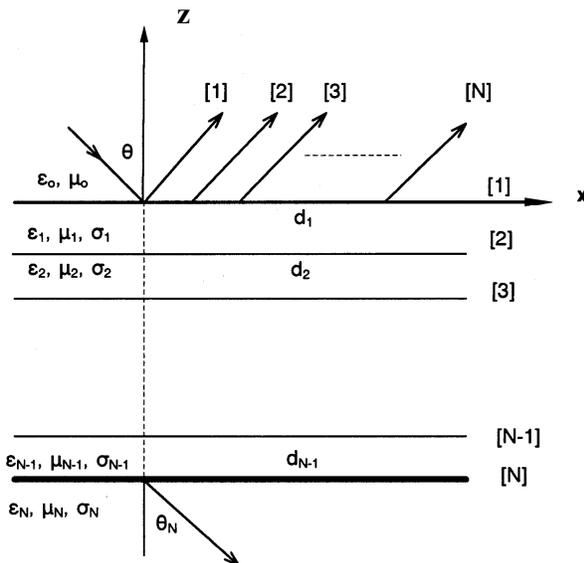

Fig. 1. An N – layer structure with an obliquely incident plane EM wave.

Fig. 1 shows a typical N – layer structure (N-1 slabs). We suppose, that the geometrical and the electromagnetic constants (thickness d_i, dielectric constant ε_i, magnetic permeability μ_i, conductivity σ_i, i = 1, 2, ... , N) of each layer are known. For a given angle of incidence, θ, it is easy to compute the angle of the outgoing ray applying a generalized form of Snell's law, $(\theta_N = \sin^{-1}(\sqrt{\varepsilon_o} \sin(\theta)/\sqrt{\varepsilon_N}))$, assuming $\sigma_N=0$). Our purpose is to study the behaviour of the reflection coefficient, in order to proceed to the investigation of the geometrical and electromagnetic characteristics of each layer and to design a multi – layer earth model

2. The reflection coefficient

It is well known, ([4], [6]), that for the structure of Fig. 1, the reflection coefficient is given by the expression

$$R = \frac{K_o - Z_1}{K_o + Z_1}, \qquad (1)$$

where

$$K_o = \sqrt{\frac{\mu_o}{\varepsilon_o}}C = \eta_o C,$$

$$Z_i = K_i \frac{Z_{i+1} + K_i \tanh(u_i d_i)}{K_i + Z_{i+1} \tanh(u_i d_i)}, \qquad (2)$$

$$K_i = \frac{u_i}{\sigma_i + j\varepsilon_i \omega}, \qquad u_i = \frac{\omega}{c}\sqrt{j\frac{\sigma_i}{\omega\varepsilon_o} - (\varepsilon_{ri} - S^2)},$$
$$(3)$$

$$Z_N = K_N \quad , \quad \varepsilon_{ri} = \frac{\varepsilon_i}{\varepsilon_o} \quad , \quad i = 1, 2, ... , N \quad ,$$

$$C = \cos(\theta) \quad , \quad S = \sin(\theta) \quad , \quad c = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \quad .$$

In the above expressions $\omega = 2\pi f$, where f is the frequency of the incident wave, d_i is the thickness of the i

– th layer, ε_o is the free space permittivity, ε_i and σ_i are the permittivity and conductivity of the i – th layer, μ_o is the magnetic permeability, which is assumed to be the same everywhere, and c is the velocity of the EM wave in free space. With K_o we denote the characteristic impedance of the free space (air), K_i is the characteristic impedance of the i – th layer and Z_i is the input impedance of the i – th layer.

Special cases

(i) Homogeneous half space

For the case of incidence on a homogeneous half space, the reflection coefficient (1) becomes

$$R = \frac{\left(\dfrac{\sigma_1}{\varepsilon_o\omega} + j\varepsilon_{r1}\right)C - \sqrt{-\left(\varepsilon_{r1} - S^2\right) + j\dfrac{\sigma_1}{\varepsilon_o\omega}}}{\left(\dfrac{\sigma_1}{\varepsilon_o\omega} + j\varepsilon_{r1}\right)C + \sqrt{-\left(\varepsilon_{r1} - S^2\right) + j\dfrac{\sigma_1}{\varepsilon_o\omega}}}, \quad (4)$$
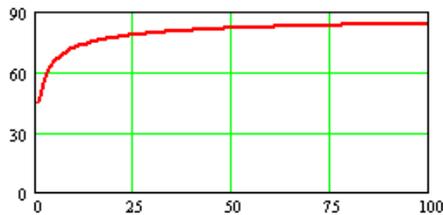
where
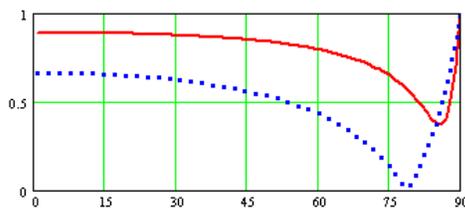
$$\varepsilon_{r1} = \frac{\varepsilon_1}{\varepsilon_o}$$

is the relative permittivity of the lower region. For $\sigma_1 = 0$ (pure dielectric), (4) is reduced to

$$R = \frac{\varepsilon_{r1}C - \sqrt{\varepsilon_{r1} - S^2}}{\varepsilon_{r1}C + \sqrt{\varepsilon_{r1} - S^2}}. \quad (5)$$

For this lossless case, since the angle θ takes values between 0° and 90°, for small θ we have $R > 0$ while for $\theta \rightarrow 90^o$ we have $R \rightarrow -1$. At the known as *Brewster angle*, θ_c, ([5] – [7]), we have $R = 0$. For $\theta > \theta_c$, the reflection coefficient becomes negative.

a

b

Fig. 2. (a) Brewster angle as a function of ε_{r1}. (b) |R| as a function of θ [(a) solid trace: $\sigma_1 \neq 0$, (b) dotted trace: $\sigma_1 = 0$].

Fig. 2(a) shows the variation of the Brewster angle θ_c as a function of ε_{r1} ($\theta_c = \tan^{-1}\sqrt{\varepsilon_{r1}}$, from (5) putting R = 0). It is obvious, that we cannot have $\theta_c < 45^o$ and $\theta_c \rightarrow 90^o$ as ε_{r1} increases. In Fig. 2(b) we can see the variation of |R| (equs. (4) and (5) above) with respect to θ, the angle of incidence. The solid trace of this graph corresponds to a configuration with $\varepsilon_{r1} = 25$, $\sigma_1 = 0.001$ (S/m) and f = 100 (kHz), while the dotted trace corresponds to a pure dielectric, i.e. $\sigma_1 = 0$ and the same ε_{r1}. In the first case ($\sigma_1 \neq 0$) there is no Brewster angle, but |R| has a minimum value near - and after - the corresponding θ_c of the pure dielectric.

(ii) ''Semi – two'' - layer half space

This case of a two – layer half space (one dielectric slab into air) is similar to that given in [4], but with $\varepsilon_2 = \varepsilon_o$. The argument of tanh( ) of (2), u_1d_1, is

$$u_1d_1 = \frac{2\pi d_1}{\lambda_o}\sqrt{-\left(\varepsilon_{r1} - S^2\right) + j\frac{\sigma_1}{\varepsilon_o\omega}}.$$

$$(6)$$

For a lossless case, $\sigma_1 = 0$, (6) takes the form

$$u_1d_1 = j2\pi\frac{d_1}{\lambda_o}\sqrt{\varepsilon_{r1} - S^2}.$$

Substituting Z_1 from (2) to (1), we take

$$R = \frac{\left(K_o^4 - K_1^4\right)\tan^2|u_1d_1|}{D} + j\frac{2K_oK_1\left(K_o^2 - K_1^2\right)\tan|u_1d_1|}{D},$$

$$(7)$$

where

$$D = 4K_o^2K_1^2 + \left(K_o^2 + K_1^2\right)^2\tan^2|u_1d_1|.$$

Assuming the lossless case ($\sigma_1 = 0$), from (7) we see, that |R| = 0 if

$$K_o = K_1 \quad \Rightarrow \quad S = \sin\theta = \sqrt{\frac{\varepsilon_{r1}}{\varepsilon_{r1} + 1}}$$

for any d_1/λ_o,
or

$$\tan|u_1d_1| = 0 \quad \Rightarrow \quad S = \sin\theta = \sqrt{\varepsilon_{r1} - \left[\frac{k}{2\left(\dfrac{d_1}{\lambda_o}\right)}\right]^2}$$

$$k = 1, 2, 3, \ldots.$$

provided that

$$0 < \varepsilon_{r1} - \left[\frac{k}{2\left(\frac{d_1}{\lambda_o}\right)} \right]^2 < 1 \cdot \tag{8}$$

The condition (8) denotes that we have $|R| = 0$ if

$$\frac{k}{2\sqrt{\varepsilon_{r1}}} < \frac{d_1}{\lambda_o} < \frac{k}{2\sqrt{\varepsilon_{r1} - 1}} \cdot \tag{9}$$

for any ε_{r1}, or

$$\left[\frac{k}{2\left(\frac{d_1}{\lambda_o}\right)} \right]^2 < \varepsilon_{r1} < 1 + \left[\frac{k}{2\left(\frac{d_1}{\lambda_o}\right)} \right]^2 \tag{10}$$

for any d_1/λ_o.

This means that for any d_1/λ_o satisfying (9) or for any ε_{r1} satisfying (10) we have two angles θ, for which $|R| = 0$.
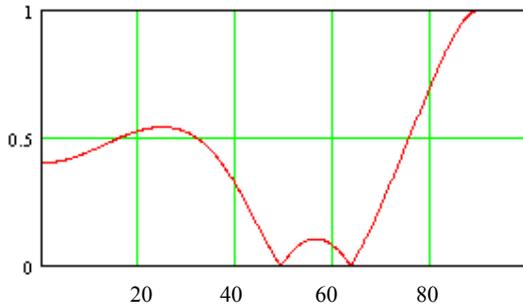

Fig. 3. Variation of $|R|$ as a function of θ for $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 1$, $k = 10$.

In Fig. 3 we see the case $\varepsilon_{r1} = 4$ and $d_1/\lambda_o = 0.27k$. We have $|R| = 0$ for $\theta_1 = 49.06^o$ and $\theta_2 = 63.4^o$. The first null of $|R|$ ($\theta_1 = 49.06^o$) corresponds to an effective $\varepsilon_{r1} = 1.33$.

Since

$$\tanh(jx) = j\tan(x) = j\tan(x + \pi),$$

it is evident that R has a periodicity with respect to the thickness, d_1, the frequency, f, and the permittivity, ε_{r1}. These periodicities are

$$\Delta d_1 = \frac{c}{2f\sqrt{\varepsilon_{r1} - S^2}} ,$$

$$\Delta f = \frac{c}{2d_1\sqrt{\varepsilon_{r1} - S^2}} ,$$

and

$$\Delta \varepsilon_{r1} = \frac{c}{d_1 f} \left[\sqrt{\varepsilon_{r1} - S^2} + \frac{c}{4d_1 f} \right]$$
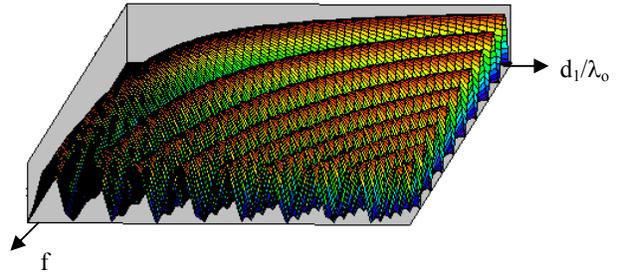
respectively.

Fig. 4. The - double - periodicity of $|R|$ with respect to d_1/λ_o and f for an "air – sandwiched" dielectric structure.

Fig. 4 shows the double variation of $|R|$ with respect to d_1/λ_o and f.

The values of d_1/λ_o or ε_{r1}, for which $|R|$ appears min or max, can be easily found. It is evident, that these values depend on the angle of incidence, θ.

(iii) Two - layer half space

The difference from the above (ii) case is that here we have $\varepsilon_2 \neq \varepsilon_o$, in general $\varepsilon_{r2} > \varepsilon_{r1}$.

Following the same procedure as in the previous case (ii), we find that

$$R = \frac{K_1^2(K_o^2 - K_2^2) + (K_o^2 K_2^2 - K_1^4)\tan^2|u_1 d_1|}{A} + j\frac{2K_1 K_o(K_2^2 - K_1^2)\tan|u_1 d_1|}{A}, \tag{11}$$

where

$$A = K_1^2(K_o + K_2)^2 + (K_o K_2 + K_1^2)\tan^2|u_1 d_1|.$$

K_1, K_2, K_o are defined with equs. (2) and (3), which are simplified for $\sigma_i = 0$.

The periodicities, maxima and minima or R are analog as in the previous case (ii).

We can find in this case, too, the angle θ, for which $|R| = 0$. From (11) we have $|R| = 0$ if

$$K_o = K_2 \qquad \text{and} \qquad \tan|u_1 d_1| = 0, \tag{12}$$

From (12) we find that

$$S = \sin\theta = \sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r2} + 1}} \tag{13}$$

and

$$\frac{d_1}{\lambda_o} = \frac{k}{2\sqrt{\varepsilon_{r1} - \dfrac{\varepsilon_{r2}}{\varepsilon_{r2} + 1}}} , \qquad\qquad k = 1, 2, 3, ... \qquad (14)$$

This result means that there is an angle

$$\theta = \sin^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r2} + 1}}\right) = \tan^{-1}\left(\sqrt{\varepsilon_{r2}}\right),$$

for which $|R| = 0$, provided that d_1/λ_o satisfies (14).

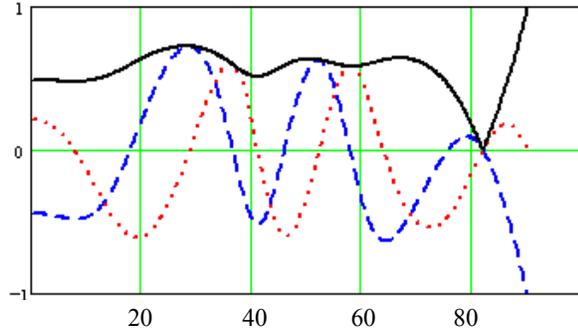*Fig. 5. |R| (solid), Re(R) (dashed) and Im(R) (dotted)
for a two – layer half – space.*

Fig. 5 shows the variation of $|R|$, Re(R) and Im(R) with respect to the angle θ for the case of $\varepsilon_{r1} = 2.5$, $\varepsilon_{r2} = 50$, $\sigma_i = 0$ and $d_1/\lambda_o = 3.65$, which is calculated from (14) with $k = 9$.

The difference from case (ii) is that here there are specific values of d_1/λ_o for having $|R| = 0$, while in case (ii) we have one $|R| = 0$ for any value of d_1/λ_o and ε_{r1} and a second one $|R| = 0$ for specific values of d_1/λ_o or ε_{r1}.

All the above results hold for $\sigma_i = 0$. For the case of lossy dielectrics, the conductivity of the media plays a degenerative role concerning the periodicities and, of course, there are no angles for which $|R| = 0$. The degenerative affect of the conductivity on the mentioned periodicities is shown in Fig. 6.
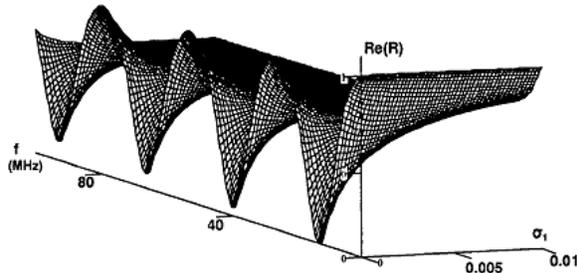

*Fig. 6. Re(R) as a function of f and σ_1 for a lossy
two – layer half space.*

We see, that for small values of σ_1, Re(R) has a periodicity with the frequency. For large values of σ_1 this periodicity is not clear and finally it is disappeared.

(iv) N - layer half space

The general case of an N – layer half space is a complicated problem. In order to calculate the reflection coefficient for such a structure, given the N, we need the quantities (2) and (3). For the case of a three – layer half space, in order to calculate R from (1)

$$R = \frac{K_o - Z_1}{K_o + Z_1} , \qquad (1)$$

we need the quantities

$$Z_1 = K_1 \frac{Z_2 + K_1 \tanh(u_1 d_1)}{K_1 + Z_2 \tanh(u_1 d_1)} ,$$

$$Z_2 = K_2 \frac{K_3 + K_2 \tanh(u_2 d_2)}{K_2 + K_3 \tanh(u_2 d_2)} ,$$

$$\begin{aligned}
K_1 &= \frac{u_1}{\sigma_1 + j\varepsilon_{r1}\omega} , \\
K_2 &= \frac{u_2}{\sigma_2 + j\varepsilon_{r2}\omega} , \qquad (15) \\
K_3 &= \frac{u_3}{\sigma_3 + j\varepsilon_{r3}\omega} ,
\end{aligned}$$

where

$$u_1 = \frac{\omega}{c}\sqrt{-\left(\varepsilon_{r1} - S^2\right) + j\frac{\sigma_1}{\varepsilon_o\omega}} , \qquad (16)$$

$$u_2 = \frac{\omega}{c}\sqrt{-\left(\varepsilon_{r2} - S^2\right) + j\frac{\sigma_2}{\varepsilon_o\omega}} , \qquad (17)$$

$$u_3 = \frac{\omega}{c}\sqrt{-\left(\varepsilon_{r3} - S^2\right) + j\frac{\sigma_3}{\varepsilon_o\omega}} \qquad (18)$$

and

$$u_1 d_1 = 2\pi\frac{d_1}{\lambda_o}\sqrt{-\left(\varepsilon_{r1} - S^2\right) + j\frac{\sigma_1}{\varepsilon_o\omega}} \qquad (19)$$

and

$$u_2 d_2 = 2\pi\frac{d_2}{\lambda_o}\sqrt{-\left(\varepsilon_{r2} - S^2\right) + j\frac{\sigma_2}{\varepsilon_o\omega}} . \qquad (20)$$

It is evident, that for pure dielectrics ($\sigma_i = 0$), the above equs. (15) – (20) take a simple form.

The zeros and the periodicities of R, which were mentioned previously, hold also in the general case. It is difficult - if not impossible - to calculate explicitly the angles of incidence for having $|R| = 0$. We can approach such zeros applying a kind of "trial – and – error" method.

By this procedure we found that for a three – layer half space, with $\varepsilon_{r1} = 4$, $\varepsilon_{r2} = 8$, $\varepsilon_{r3} = 100$, $d_1/\lambda_o = 0.5$, $d_2/\lambda_o = 0.24785$ and $\sigma_i = 0$, we have $|R| = 0$ for $\theta = 82^o$. This angle gives an effective $\varepsilon_r = 50.63$.

As we can see from (1), the zeros and the general behaviour of R depends on the input impedance of the first layer, Z_1. This Z_1 is a function of the physical characteristics of all the layers as well as it depends on the angle of incidence of the EM wave on the dielectric structure.
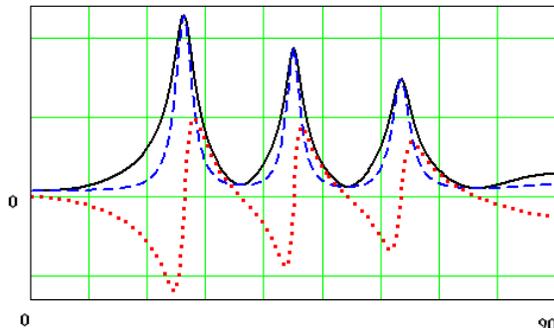

Fig. 7. The variation of Z_1 with the angle of incidence, θ, for a six layer half space (five slabs of pure dielectrics).

In Fig. 7 we see the variation of $|Z_1|$ (solid trace), $Re(Z_1)$ (dashed trace) and $Im(Z_1)$ (dotted trace) of a six layer half space (five slabs). We consider that all the layers have equal optical pathlength ($n_i d_i = 2$) and $\varepsilon_{r6} = 100$. For this case, we see that Z_1 has a tripple oscillation. The multiplicity of the oscillation of Z_1 depends on the optical pathlength of the dielectric structure and it increases with it. For the case of Fig. 7 we have $|R|_{min}$ (= 0.273, not seen here) for $\theta = 77^o$, where it takes place the third minimum of $Re(Z_1)$ and $|Z_1|$ and the 6-th zero of $Im(Z_1)$.

Although the non – zero conductivities of the dielectrics cause degenerative phenomena on the periodicities of R in a multiple – layer structure (see Fig. 6), we use dielectrics with non – zero conductivities in a lot of practical applications. It is known, that it is of high practical importance to design an N – layer system, which at a given frequency must have reflection coefficient less than a given value. An important application of such a system is in designing and development of electromagnetic absorbers, e.g. anechoic chambers, systems preventing of false image from a ship' s mast or TV ghost, equipment improving ETC communication environment, clobber preventing interference between ETC lanes, partially loaded screened rooms etc. The absorbers must consint of lossy materials, so that the EM energy can be dissipated within them. A typical procedure for designing absorbers is to handle parameterized permittivities and conductivities applying an appropriate for this purpose model ([8] – [11]).

On the other hand, one can find important application of multiple – layer dielectric structures in geoscience and remote sensing systems, studying the response behaviour of a layered structure to a pulse of specific characteristics.

3. Conclusions

The electromagnetic response of an N- layer medium for the incidence of an oblique TM plane wave has been presented. The reflection coefficient has been derived for any number of layers and the expected periodicity and zeros from the geometrical and electrical parameters was found. In the case of the design of an N – layer medium a procedure with the suitable parameterization can give the number of layers for thickness and $|R|$ less than given values. It is believed that this technique could be useful for several Geophysical and EMC applications.

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