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Reflection from and transmission through the double-negative slab

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In this study, reflection from and transmission through the double-negative slab is presented to find the complex reflection and transmission coefficients in the closed form, for both TE and TM waves. After determining the fields in all regions, the reflection and the transmission coefficients can be obtained by imposing the boundary conditions at the interfaces. Finally, numerical results are presented for perpendicular and parallel polarization cases to show the effects of the incidence angle and the frequency on the complex reflection and transmission coefficients.

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1. Introduction

The double-negative (DNG) materials with a real negative electric permittivity and magnetic permeability have been gaining a considerable attention due to the possibility of manufacturing them at the microwave, millimeter-wave, and optical frequencies. Such materials were first envisioned in the 1968 by the Russian physicist Veselago [1]. The DNG substances were defined and the general properties of wave propagation in the DNG medium are presented in his study. Recently, Pendry *et al.* showed how the negative-permittivity materials could be realized from arrays of wires [2] and negative-permeability materials from arrays of split rings [3]. In 2000, Smith *et al.* constructed a composite material which has negative permittivity and permeability using the combinations of copper rings and wires, and did several microwave experiments to confirm that the properties of this material is unlike from any existing material [4]. Experimental observation of the negative effective index of refraction on the DNG materials at microwave frequencies is performed by Shelby *et al.* in 2001 [5]. Conceptual and speculative ideas for potential applications of the DNG materials were suggested, and physical remarks and intuitive comments were provided in [6]. Kong studied the electromagnetic wave interaction with stratified DNG isotropic media [7]. In his study, general formulations for the wave interaction with stratified media were given, and the field solutions of guided waves in stratified media were obtained. The reflected and transmitted powers due to the interaction of electromagnetic waves with a double-negative slab were analyzed by Sabah, Ögücü and Uckun in 2006 [8], [9]. The transfer matrix method [10] was used in the analysis to find the formulations in closed form. Also, the effects of the structure parameters, incidence angle and the frequency on the reflected and the transmitted powers were presented numerically.

In this work, the reflection and transmission analysis of the plane electromagnetic wave in the presence of the double-negative slab (DNS) between the two semi-infinite dielectric media is presented. Specifically, the complex reflection and transmission coefficients are computed and presented both in analytically and numerically. The incident electric field is assumed to be a plane electromagnetic wave. The reflected and transmitted electric and magnetic fields are determined using the Maxwell's equations. Then, imposing the boundary conditions at the interfaces, the complex reflection and transmission coefficients can be found easily. Finally, the numerical results are illustrated to show the effects of the incidence angle and the frequency on the reflection and transmission coefficients.

2. Theory

Consider a plane electromagnetic wave of perpendicular polarization (TE) to be incident on the DNS with the incidence angle θ_i as shown in Fig. 1. To obtain the general formulation for the complex reflection and transmission coefficients, it is necessary to examine the interfaces shown in Fig. 1 in detail.

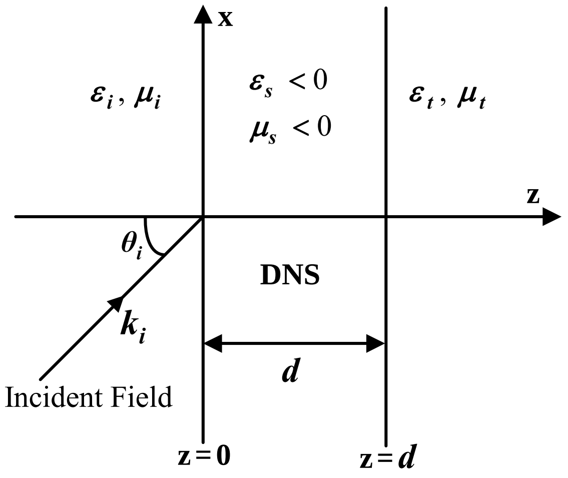


Fig. 1. The structure of the DNS between two dielectric media.

In the analysis, $\exp(j\omega t)$ time dependence is assumed and it is suppressed throughout this work. The total TE polarized electric fields in all regions can be expressed as follows:

$$\boldsymbol{E} = \begin{cases} \boldsymbol{a}_y E_i \cdot [\exp(-jk_{ix}x)] \cdot [\exp(-jk_{iz}z) + R \cdot \exp(jk_{iz}z)], & z \le 0 \\ \boldsymbol{a}_y [\exp(-jk_{sx}x)] \cdot [A \cdot \exp(-jk_{sz}z) + B \cdot \exp(jk_{sz}z)], & 0 \le z \le d \\ \boldsymbol{a}_y T E_i \exp[-j(k_{tx}x + k_{tz}(z-d))], & z \ge d \end{cases} \tag{1}$$

where k_i, k_s, and k_t are the wave-numbers, E_i is the amplitude of the incident electric field, A and B are the amplitudes of the electric fields in the DNS, d is the slab thickness, R is the complex reflection coefficient and T is the complex transmission coefficients. The magnetic fields in all regions can be determined by using Maxwell's equations.

By imposing the boundary conditions at the interfaces, the reflection and transmission coefficients can be obtained easily. Thus, the formulations of the complex reflection and transmission coefficients can be expressed as:

$$R = \frac{R_{01} + R_{12} \cdot \exp(j2k_{sz}d)}{1 + R_{01}R_{12} \cdot \exp(j2k_{sz}d)} \tag{2}$$

$$T = \frac{4 \cdot \exp(jk_{sz}d)}{(1 + r_{01})(1 + r_{12})[1 + R_{01}R_{12} \cdot \exp(j2k_{sz}d)]} \tag{3}$$

in which R_{01} and R_{12} are the Fresnel's reflection coefficients:

$$R_{01} = \frac{1 - r_{01}}{1 + r_{01}} \tag{4a}$$

$$R_{12} = \frac{1 - r_{12}}{1 + r_{12}} \tag{4b}$$

where for TE waves

$$r_{01} = \frac{\mu_i k_{sz}}{\mu_s k_{iz}}, \quad r_{12} = \frac{\mu_s k_{tz}}{\mu_t k_{sz}} \tag{5a}$$

and for TM waves

$$r_{01} = \frac{\varepsilon_i k_{sz}}{\varepsilon_s k_{iz}}, \quad r_{12} = \frac{\varepsilon_s k_{tz}}{\varepsilon_t k_{sz}} \tag{5b}$$

For both polarizations the conservation of the power is obtained as follows [7], [8]:

$$\left| R^{TE,TM} \right|^2 + \left| \frac{k_{tz}\mu_i}{k_{iz}\mu_t} \right| \cdot \left| T^{TE,TM} \right|^2 = 1 \tag{6}$$

Note that the incident power is normalized to unity in Equation (6).

3. Numerical results

In this section, the effects of the incidence angle and the frequency on the complex reflected and the transmitted powers are presented numerically for both TE and TM waves. To check the correctness of the results used in these computations, firstly the conservation of power given in Equation (6) is satisfied for all examples. The power conversion results are given in References [7] and [8] in detail. Secondly, a transmission line equivalent circuit model is derived for the structure given in Figure 1 [11] to find the complex reflection and transmission coefficients. It is seen that both methods give the same numerical values for these coefficients. Thus, the results are verified by means of two concepts, the conservation of power and the transmission line equivalent circuit model.

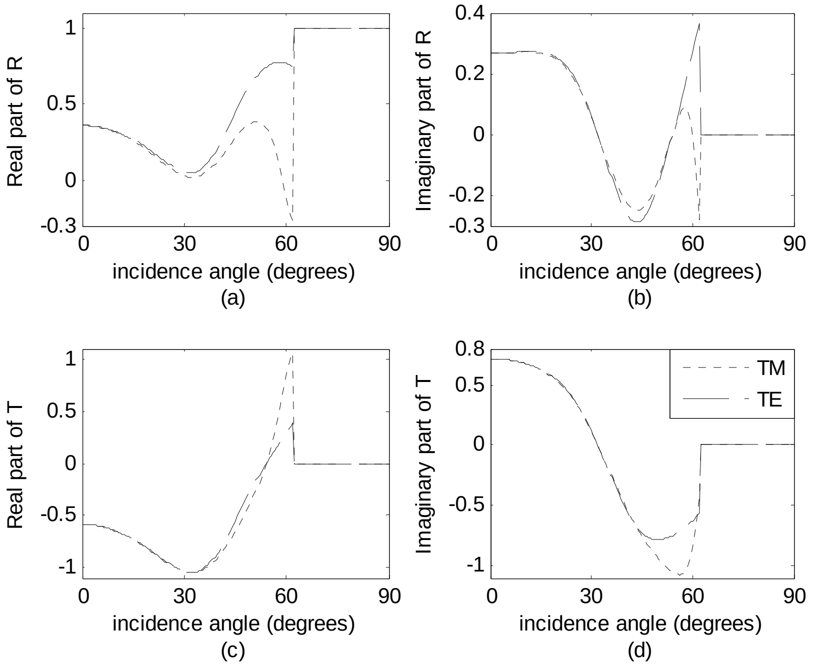
The complex reflection and transmission coefficients are calculated as a function of the incidence angle and the frequency. In the calculations, the operation frequency is assumed to be f_o = 10.9 GHz. The permeabilities of the first and the last media in Fig. 1 are selected to be equal to the permeability of the free space, $\mu_i = \mu_t = \mu_o$. The permittivity and the permeability of the DNS are calculated using Lorentz medium model equations [5] given in Equation (7) and Equation (8) with lossless case, $\Gamma_m = \Gamma_e = \Gamma = 0$.

$$\mu(\omega) = \mu_o \left(1 - \frac{f_{mp}^2 - f_{mo}^2}{f^2 - f_{mo}^2 + j\Gamma_m \omega} \right) \tag{7}$$

$$\varepsilon(\omega) = \varepsilon_o \left(1 - \frac{f_{ep}^2 - f_{eo}^2}{f^2 - f_{eo}^2 + j\Gamma_e \omega} \right) \tag{8}$$

where f_{mo} is the magnetic resonance frequency, f_{mp} is the magnetic plasma frequency, f_{eo} is the electronic resonance frequency, and f_{ep} is the electronic plasma frequency.

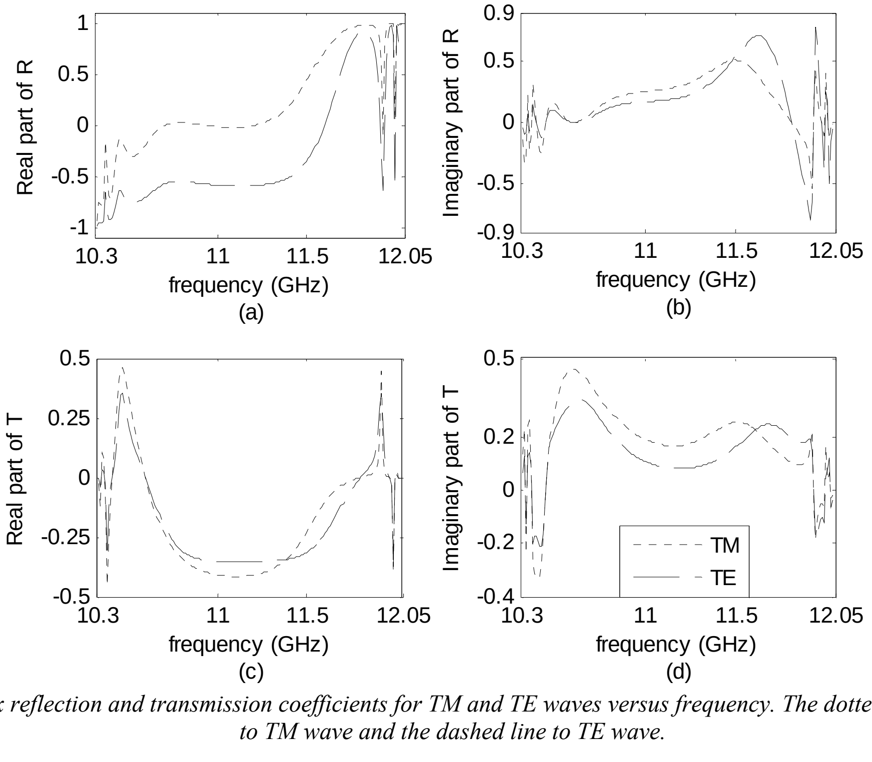
Example 1: In this example, it is considered the first and the last media are glass and mica whose relative permittivities are 8.0 and 7.0, respectively. The permittivity and permeability of the DNS are calculated as $\varepsilon_s = -3.5401\varepsilon_o$ and $\mu_s = -1.7641\mu_o$. In the calculations, the following parameters are used [3], [5]: f_{mp} = 8.50 GHz, f_{mo} = 12.05 GHz, f_{ep} = 12.80 GHz, and f_{eo} = 10.30 GHz. The slab thickness is assumed to be a quarter-wavelength long at the operation frequency f_o. Fig. 2 presents the complex reflection (R) and the transmission coefficients (T) for TE and TM waves as a function of the incidence angle. From Fig. 2, it is seen that, the real parts of R and T for both waves become unity and the imaginary parts of these coefficients become zero at the incidence angle grater then or equal to 62.07^o which is the critical angle for this configuration. It means total internal reflection occurs at this angle.



(a) (b) (c) (d)

Fig. 2. Complex reflection and transmission coefficients for TM and TE waves as a function of the incidence angle. The dotted line corresponds to TM wave and the dashed line to TE wave.

Example 2: In the second example, frequency response of the complex reflection and the transmission coefficients for TE and TM waves are investigated. The relative permittivities of the first and the last media are selected to be 1.0 (free-space) and 6.0 (mica), respectively. The permittivity and permeability of DNS are not constant in this case. They are calculated using Equation (7) and Equation (8). The angle of incidence is 60^o. The slab thickness is the same with the previous example. The complex reflection and transmission coefficients versus frequency are presented in Fig. 3. It can be said that, although *R* and *T* for TE and TM waves show similar behavior at some frequencies, the characteristics of the *R* and *T* for both waves are very sensitive to the varying frequency due to the frequency-dependent permittivity and permeability.



(a) (b) (c) (d)

Fig. 3. Complex reflection and transmission coefficients for TM and TE waves versus frequency. The dotted line corresponds to TM wave and the dashed line to TE wave.

4. Conclusions

In this work, the complex reflection and transmission coefficients due to the interaction of the plane electromagnetic waves with a DNS between the two semi-infinite dielectric media are presented in detail. The electromagnetic fields are written in all regions, and then by imposing the boundary conditions for the electric and magnetic fields, the complex reflection and transmission coefficients for TE and TM waves are found analytically. The effects of the incidence angle and the frequency on these coefficients are studied in the numerical results. It is shown that the incidence angle and the frequency affect the behavior of the complex reflection and transmission coefficients. Also, the characteristics of the complex reflection and transmission coefficients for TE and TM waves are very sensitive to the change of frequency because of the frequency-dependent permittivity and permeability.

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