Resonant optical soliton perturbation with full nonlinearity and time-dependent coefficients by trial equation method

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Optical soliton solutions are retrieved for the perturbed resonant nonlinear Schrödinger's equation with time-dependent coefficients, where perturbation terms appear with full nonlinearity. The trial equation method is the integration methodology adopted in this paper. There are five forms of non-Kerr law nonlinearity considered. Bright, dark and singular solitons are reported, that come with constraint conditions for the existence of solitons.

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1. Introduction

Resonant solitons appear during bifurcation of the scattering matrix. These solitons are studied extensively with various forms of nonlinear media in presence of several perturbation terms. These perturbations are with full nonlinearity and are of Hamiltonian type. There are various forms of mathematical tools, such as trial equation method, that are applicable to study resonant solitons, and other nonlinear evolution equations, as it has been seen in the past [1-42]. These studies with resonant solitons are conducted with constant coefficients as well as time-dependent coefficients. Several forms of nonlinear media have been addressed in this context. Some of the most commonly addressed nonlinear forms are Kerr law, power law, parabolic law and dual-power law. This paper will address resonant solitons with several other forms of non-Kerr law that are less commonly studied in mathematical photonics. These are quadratic-cubic law, anti-cubic law, cubic-quintic-septic law, triple-power law and log-law. The governing equation is the resonant nonlinear Schrödinger's equation (RNLSE) that will be addressed in this paper with time-dependent coefficients. The integration methodology adopted here is the trial equation scheme. Bright, dark and singular soliton solutions will be retrieved for this model and these solitons will exist under certain restrictions that are also presented as constraint conditions.

1.1. Governing model

The RNLSE with time-dependent coefficients is of the form [1-14, 17-19, 21-28, 30]:

$$iq_t + \alpha(t)q_{xx} + \beta(t)F(|q|^2)q + \gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q = 0 \tag{1}$$

here, in (1), the first term is the linear evolution, while α(t) is the coefficient of group velocity dispersion (GVD) and β(t) is the coefficient of nonlinearity. Finally, γ(t) is quantum or Bohm potential that appears in the context of chiral solitons in quantum Hall effect. Also, the functional F meets the following technical criteria: F is a real-valued algebraic function and it is necessary to have the smoothness of the complex function $F(|q|^2)q : C \to C$. Considering the complex plane C as a two-dimensional linear space R^2, the function $F(|q|^2)q$ is k times continuously differentiable, so that

$$F(|q|^2)q \in \bigcup_{m,n=1}^{\infty} C^k\big((-n,n)\times(-m,m);R^2\big)$$

In presence of perturbation terms with time-dependent coefficients, RNLSE is modified to [10]

$$iq_t + \alpha(t)q_{xx} + \beta(t)F(|q|^2)q + \gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q = i\left[\delta(t)q_x + \lambda(t)\left(|q|^{2m}q\right)_x + \mu(t)\left(|q|^{2m}\right)_x q\right] \quad (2)$$

where δ(t) is the inter-modal dispersion, λ(t) represents the coefficient of self-steepening for short pulses and μ(t) is the higher-order dispersion coefficient. The parameter *m* is the full nonlinearity parameter.

2. Short review of the integration algorithm

A short review of trial equation method begins as [15, 16, 20, 29]:

Step 1: Suppose a nonlinear PDE with time-dependent coefficients

$$P(u, u_t, u_t, u_{tt}, u_{xt}, u_{xx}, ...) = 0 \quad (3)$$

can be converted to an ordinary differential equation (ODE)

$$Q(U, U', U'', U'''...) = 0 \quad (4)$$

using a travelling wave hypothesis $u(x,t) = U(\xi)$, $\xi = x - vt$, where $U = U(\xi)$ is an unknown function, *Q* is a polynomial in the variable *U* and its derivatives. If all terms contain derivatives, then Eq. (4) is integrated where integration constants are considered zeros.

Step 2: Take the trial equation

$$(U')^2 = F(U) = \sum_{l=0}^{N} a_l U^l \quad (5)$$

where a_l, $(l = 0, 1, 2, ..., N)$ are constants to be determined. Substituting Eq. (5) and other derivative terms such as *U″* or *U‴* and so on into Eq. (4) yields a polynomial *G*(*U*) of *U*. According to the balance principle we can determine the value of *N*. Setting the coefficients of *G*(*U*) to zero, we get a system of algebraic equations. Solving this system, we can determine *N* and values of $a_0, a_1, ..., a_N$.

Step 3: Rewrite Eq. (5) by the integral form

$$\pm(\xi - \xi_0) = \int \frac{dU}{\sqrt{F(U)}} \quad (6)$$

According to the complete discrimination system of the polynomial, we classify the roots of *F*(*U*), and solve the integral Eq. (6). Thus we obtain the exact solutions to Eq. (3).

3. Soliton solutions

In order to solve Eq. (2) by the trial equation method, we use the following wave transformation

$$q(x,t) = U(\xi)e^{i(-\kappa x + \omega(t)t)}, \; \xi = x + 2\kappa\int_0^t \alpha(t)dt \quad (7)$$

Substituting Eq. (7) into Eq. (2) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$\delta(t) + \left((2m+1)\lambda(t) + 2m\mu(t)\right)U^{2m} = 0 \quad (8)$$

while the real part gives

$$\left(\alpha(t) + \gamma(t)\right)U'' - \left(t\omega'(t) + \omega(t) + \kappa^2\alpha(t) + \kappa\delta(t)\right)U - \kappa\lambda(t)\,U^{2m+1} + \beta(t)F(U^2)U = 0 \quad (9)$$

Eq. (9) can be integrated to determine the soliton profile.

3.1. Quadratic-cubic law

Here,

$$F(s) = \beta_1(t)\sqrt{s} + \beta_2(t)s$$

where $\beta_1(t)$ and $\beta_2(t)$ are time-dependent coefficients. Therefore, RNLSE is given by

$$iq_t + \alpha(t)q_{xx} + \left(\beta_1(t)|q| + \beta_2(t)|q|^2\right)q + \gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q = i\left[\delta(t)q_x + \lambda(t)\left(|q|^{2m}q\right)_x + \mu(t)\left(|q|^{2m}\right)_x q\right] \quad (10)$$

and Eq. (9) simplifies to

$$\left(\alpha(t) + \gamma(t)\right)U'' - \left(t\omega'(t) + \omega(t) + \kappa^2\alpha(t) + \kappa\delta(t)\right)U - \kappa\lambda(t)\,U^{2m+1} + \beta_1(t)U^2 + \beta_2(t)U^3 = 0 \quad (11)$$

Balancing U″ with U^{2m+1} in Eq. (11), then we get $N = 2m + 2$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$U^{2m+1}\ Coeff.:\ \left(\alpha(t) + \gamma(t)\right)(m+1)\,a_{2m+2} - \kappa\lambda(t) = 0$$

$$U^3\ Coeff.:\ 2\left(\alpha(t) + \gamma(t)\right)a_4 + \beta_2(t) = 0$$

$$U^2\ Coeff.:\ \frac{3\left(\alpha(t) + \gamma(t)\right)a_3}{2} + \beta_1(t) = 0$$

U^1 *Coeff.*:

$$\left(\alpha(t) + \gamma(t)\right)a_2 - (t\omega'(t) + \omega(t) + \kappa^2\alpha(t) + \kappa\delta(t)) = 0$$

$$U^0 \ \ Coeff.: \ \ (\alpha(t)+\gamma(t))a_1 = 0$$

Solving the above system leads to

$$a_1 = 0, \quad a_3 = -\frac{2\beta_1(t)}{3(\alpha(t)+\gamma(t))},$$

$$a_4 = -\frac{\beta_2(t)}{2(\alpha(t)+\gamma(t))}, \ a_{2m+2} = \frac{\kappa\lambda(t)}{(\alpha(t)+\gamma(t))(m+1)}$$

$$\omega(t) = -\frac{1}{t}\int_0^t \{\kappa^2\alpha(t) + \kappa\delta(t) - (\alpha(t)+\gamma(t))a_2\}dt \quad (12)$$

Substituting these results into Eqs. (5) and (6), we get

$$\pm(\xi-\xi_0) = \int \begin{bmatrix} a_0 + a_2U^2 - \dfrac{2\beta_1(t)}{3(\alpha(t)+\gamma(t))}U^3 - \\ \dfrac{\beta_2(t)}{2(\alpha(t)+\gamma(t))}U^4 + \\ \dfrac{\kappa\lambda(t)}{(\alpha(t)+\gamma(t))(m+1)}U^{2m+2} \end{bmatrix}^{-\frac{1}{2}} dU \quad (13)$$

In order to carry out the integration of Eq. (13) it is necessary to choose *m* = 1. Thus, with *m* = 1, new Eq. (13) is following:

$$\pm(\xi-\xi_0) = \int \begin{bmatrix} a_0 + a_2U^2 - \dfrac{2\beta_1(t)}{3(\alpha(t)+\gamma(t))}U^3 - \\ \dfrac{\kappa\lambda(t)-\beta_2(t)}{2(\alpha(t)+\gamma(t))}U^4 \end{bmatrix}^{-\frac{1}{2}} dU \quad (14)$$

<u>Case 1</u>

Eq. (14) can be integrated with respect to U if we set

$$a_0 = 0, \quad a_2 = \frac{2\beta_1^2(t)}{9(\alpha(t)+\gamma(t))(\kappa\lambda(t)-\beta_2(t))},$$

Thus, we obtain exact solutions of Eq. (10):

$$q(x,t) = \frac{\beta_1(t)}{3(\kappa\lambda(t)-\beta_2(t))} \cdot \left[1 \pm \tanh\left(\sqrt{\frac{\beta_1^2(t)}{18(\alpha(t)+\gamma(t))(\kappa\lambda(t)-\beta_2(t))}} \cdot \left(x + 2\kappa\int_0^t \alpha(t)dt + \xi_0\right)\right)\right] e^{i(-\kappa x+\omega(t)t)} \quad (15)$$

$$q(x,t) = \frac{\beta_1(t)}{3(\kappa\lambda(t)-\beta_2(t))} \cdot \left[1 \pm \coth\left(\sqrt{\frac{\beta_1^2(t)}{18(\alpha(t)+\gamma(t))(\kappa\lambda(t)-\beta_2(t))}} \cdot \left(x + 2\kappa\int_0^t \alpha(t)dt + \xi_0\right)\right)\right] e^{i(-\kappa x+\omega(t)t)} \quad (16)$$

where $\omega(t)$ is given by Eq. (12).

Eq. (15) and Eq. (16) represent dark and singular soliton solution respectively. These solitons are valid for

$$(\alpha(t)+\gamma(t))(\kappa\lambda(t)-\beta_2(t)) > 0$$

<u>Case 2</u>

Eq. (14) can be integrated with respect to U if we set

$$a_0 = 0, \quad \beta_2(t) = \kappa\lambda(t)$$

Thus, we obtain exact solutions of Eq. (10):

$$q(x,t) = \pm\left\{\frac{3a_2(\alpha(t)+\gamma(t))}{2\beta_1(t)}\right\} \cdot \left[\mathrm{sech}^2\left(\frac{\sqrt{a_2}}{2}\left(x + 2\kappa\int_0^t \alpha(t)dt + \xi_0\right)\right)\right] e^{i(-\kappa x+\omega(t)t)} \quad (17)$$

$$q(x,t) = \pm\left\{\frac{3a_2(\alpha(t)+\gamma(t))}{2\beta_1(t)}\right\} \cdot \left[\mathrm{csch}^2\left(\frac{\sqrt{a_2}}{2}\left(x + 2\kappa\int_0^t \alpha(t)dt + \xi_0\right)\right)\right] e^{i(-\kappa x+\omega(t)t)} \quad (18)$$

where $\omega(t)$ is given by Eq. (12).

Eq. (17) and Eq. (18) represent bright and singular soliton solutions respectively. These solitons are valid for

$$q(x,t) = \pm\left\{-\frac{3a_2(\alpha(t)+\gamma(t))}{2\beta_1(t)}\right\} \cdot \left[\sec^2\left(\frac{\sqrt{-a_2}}{2}\left(x + 2\kappa\int_0^t \alpha(t)dt + \xi_0\right)\right)\right] e^{i(-\kappa x+\omega(t)t)} \quad (19)$$

$$q(x,t)=\pm\left\{-\frac{3a_2(\alpha(t)+\gamma(t))}{2\beta_1(t)}\right\}\cdot\left[\csc^2\left(\frac{\sqrt{-a_2}}{2}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]e^{i(-\kappa x+\omega(t)t)} \tag{20}$$

where $\omega(t)$ is given by Eq. (12).

Eq. (19) and Eq. (20) represent singular periodic solutions. These solutions are valid for

$$a_2<0.$$

3.2. Anti-cubic law

In this case,

$$F(s)=\frac{\beta_1(t)}{s^2}+\beta_2(t)s+\beta_3(t)s^2$$

where $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ are time-dependent coefficients. Therefore, RNLSE is given by

$$iq_t+\alpha(t)q_{xx}+\left(\beta_1(t)|q|^{-4}+\beta_2(t)|q|^2+\beta_3(t)|q|^4\right)q+\gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q=i\left[\delta(t)q_x+\lambda(t)\left(|q|^{2m}q\right)_x+\mu(t)\left(|q|^{2m}\right)_x q\right] \tag{21}$$

and Eq. (9) simplifies to

$$\left(\alpha(t)+\gamma(t)\right)U''-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)U-\kappa\lambda(t)\,U^{2m+1}+\beta_1(t)U^{-3}+\beta_2(t)U^3+\beta_3(t)U^5=0 \tag{22}$$

By using transformation $U=V^{1/2}$, Eq. (22) becomes

$$\left(\alpha(t)+\gamma(t)\right)\left(-(V')^2+2VV''\right)-4V^2\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)-4\kappa\lambda(t)\,V^{m+2}+4(\beta_1(t)+\beta_2(t)V^3+\beta_3(t)V^4)=0 \tag{23}$$

Balancing VV'' or $(V')^2$ with V^{m+2} in Eq. (23), then we get $N=m+2$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$V^{2m+1}\ Coeff.:\ \left(\alpha(t)+\gamma(t)\right)(m+1)\,a_{m+2}-4\kappa\lambda(t)=0$$

$$V^4\ Coeff.:\ 3\left(\alpha(t)+\gamma(t)\right)a_4+4\beta_3(t)=0$$

$$V^3\ Coeff.:\ 2\left(\alpha(t)+\gamma(t)\right)a_3+4\beta_2(t)=0$$

$$V^2\ Coeff.:\ \left(\alpha(t)+\gamma(t)\right)a_2-4(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t))=0$$

$$V^0\ Coeff.:\ -\left(\alpha(t)+\gamma(t)\right)a_0+4\beta_1(t)=0$$

Solving the above system leads to

$$a_0=\frac{4\beta_1(t)}{\alpha(t)+\gamma(t)},\quad a_3=-\frac{2\beta_2(t)}{\alpha(t)+\gamma(t)}$$

$$a_4=-\frac{4\beta_3(t)}{3\left(\alpha(t)+\gamma(t)\right)},\quad a_{2m+2}=\frac{4\kappa\lambda(t)}{\left(\alpha(t)+\gamma(t)\right)(m+1)}$$

$$\omega(t)=-\frac{1}{t}\int_0^t\{\kappa^2\alpha(t)+\kappa\delta(t)-\left(\alpha(t)+\gamma(t)\right)\frac{a_2}{4}\}dt \tag{24}$$

Substituting these results into Eqs. (5) and (6), we get

$$\pm(\xi-\xi_0)=\int\left[\frac{4\beta_1(t)}{\alpha(t)+\gamma(t)}+a_1V+a_2V^2-\frac{2\beta_2(t)}{\alpha(t)+\gamma(t)}V^3-\frac{4\beta_3(t)}{3(\alpha(t)+\gamma(t))}V^4+\frac{4\kappa\lambda(t)}{\left(\alpha(t)+\gamma(t)\right)(m+1)}V^{m+2}\right]^{-\frac{1}{2}}dV \tag{25}$$

To carry out the integration of Eq. (25) requires that $m=1$. Thus, with $m=1$, new Eq. (25) is following:

$$\pm(\xi-\xi_0)=\int\left[\frac{4\beta_1(t)}{\alpha(t)+\gamma(t)}+a_1V+a_2V^2-\frac{2(\kappa\lambda(t)-\beta_2(t))}{\alpha(t)+\gamma(t)}V^3-\frac{4\beta_3(t)}{3(\alpha(t)+\gamma(t))}V^4\right]^{-\frac{1}{2}}dV \tag{26}$$

Eq. (26) can be integrated with respect to V if we set

$$a_1=0,\quad \beta_2(t)=\kappa\lambda(t)$$

$$\beta_1(t)=-\frac{3a_2^2(\alpha(t)+\gamma(t))}{64\beta_3(t)}$$

Thus, we obtain exact solutions of Eq. (21):

$$q(x,t)=\pm\left\{\left(\sqrt{\frac{3a_2(\alpha(t)+\gamma(t))}{8\beta_3(t)}}\right)\cdot\left[\tan\left(\sqrt{\frac{a_2}{2}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega(t)t)} \quad (27)$$

$$q(x,t)=\pm\left\{\left(\sqrt{\frac{3a_2(\alpha(t)+\gamma(t))}{8\beta_3(t)}}\right)\cdot\left[\cot\left(\sqrt{\frac{a_2}{2}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega(t)t)} \quad (28)$$

where $\omega(t)$ is given by Eq. (24).

Eq. (27) and Eq. (28) represent singular periodic solutions. These solutions are valid for

$$a_2>0$$

$$q(x,t)=\pm\left\{\left(\sqrt{-\frac{3a_2(\alpha(t)+\gamma(t))}{8\beta_3(t)}}\right)\cdot\left[\tanh\left(\sqrt{-\frac{a_2}{2}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega(t)t)} \quad (29)$$

$$q(x,t)=\pm\left\{\left(\sqrt{\frac{3a_2(\alpha(t)+\gamma(t))}{8\beta_3(t)}}\right)\cdot\left[\cot\left(\sqrt{-\frac{a_2}{2}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega(t)t)} \quad (30)$$

where $\omega(t)$ is given by Eq. (24).

Eq. (29) and Eq. (30) represent dark and singular soliton solution respectively. These solitons are valid for

$$a_2<0.$$

3.3. Cubic-quintic-septic law

In this case,

$$F(s)=\beta_1(t)s+\beta_2(t)s^2+\beta_3(t)s^3$$

where $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ are time-dependent coefficients. Therefore, RNLSE is given by

$$iq_t+\alpha(t)q_{xx}+\left(\beta_1(t)|q|^2+\beta_2(t)|q|^4+\beta_3(t)|q|^6)\right)q+\gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q=i\left[\delta(t)q_x+\lambda(t)\left(|q|^{2m}q\right)_x+\mu(t)\left(|q|^{2m}\right)_x q\right] \quad (31)$$

and Eq. (9) simplifies to

$$\begin{aligned}&(\alpha(t)+\gamma(t))U''-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)U\\&-\kappa\lambda(t)\,U^{2m+1}+\beta_1(t)U^3+\beta_2(t)U^5+\beta_3(t)U^7=0\end{aligned} \quad (32)$$

By using transformation $U=V^{1/2}$, Eq. (32) becomes

$$\begin{aligned}&(\alpha(t)+\gamma(t))\left(-(V')^2+2VV''\right)-\\&4V^2\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)\\&-4\kappa\lambda(t)\,V^{m+2}+4(\beta_1(t)V^3+\beta_2(t)V^4+\beta_3(t)V^5)=0\end{aligned} \quad (33)$$

Balancing VV'' or $(V')^2$ with V^{m+2} in Eq. (33), then we get $N = m + 2$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$V^{2m+1}\ Coeff.:\ (\alpha(t)+\gamma(t))(m+1)\,a_{m+2}-4\kappa\lambda(t)=0$$

$$V^5\ Coeff.:\ (\alpha(t)+\gamma(t))a_5+\beta_3(t)=0$$

$$V^4\ Coeff.:\ 3(\alpha(t)+\gamma(t))a_4+4\beta_2(t)=0$$

$$V^3\ Coeff.:\ (\alpha(t)+\gamma(t))a_3+2\beta_1(t)=0$$

$$\begin{aligned}&V^2\ Coeff.:\ (\alpha(t)+\gamma(t))a_2-\\&4(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t))=0\end{aligned}$$

$$V^0\ Coeff.:\ (\alpha(t)+\gamma(t))a_0=0$$

Solving the above system leads to

$$a_0=0,\ a_3=-\frac{2\beta_1(t)}{\alpha(t)+\gamma(t)},\ a_4=\frac{4\beta_2(t)}{3(\alpha(t)+\gamma(t))}$$

$$a_5=-\frac{\beta_3(t)}{\alpha(t)+\gamma(t)}\ ,\ a_{2m+2}=\frac{4\kappa\lambda(t)}{(\alpha(t)+\gamma(t))(m+1)}$$

$$\omega(t)=-\frac{1}{t}\int_0^t\{\kappa^2\alpha(t)+\kappa\delta(t)-(\alpha(t)+\gamma(t))\frac{a_2}{4}\}dt \quad (34)$$

Substituting these results into Eqs. (5) and (6), we get

$$\pm(\xi-\xi_0)=\int\left[\begin{array}{l}+a_1V+a_2V^2-\dfrac{2\beta_1(t)}{\alpha(t)+\gamma(t)}V^3-\\ \dfrac{4\beta_2(t)}{3(\alpha(t)+\gamma(t))}V^4-\dfrac{\beta_3(t)}{\alpha(t)+\gamma(t)}V^5+\\ \dfrac{4\kappa\lambda(t)}{(\alpha(t)+\gamma(t))(m+1)}V^{m+2}\end{array}\right]^{-\frac{1}{2}}dV \tag{35}$$

To carry out the integration of Eq. (35) requires that $m = 3$. Thus, with $m = 3$, new Eq. (35) is following:

$$\pm(\xi-\xi_0)=\int\left[\begin{array}{l}+a_1V+a_2V^2-\dfrac{2\beta_1(t)}{\alpha(t)+\gamma(t)}V^3-\\ \dfrac{4\beta_2(t)}{3(\alpha(t)+\gamma(t))}V^4-\dfrac{\kappa\lambda(t)-\beta_3(t)}{\alpha(t)+\gamma(t)}V^5\end{array}\right]^{-\frac{1}{2}}dV \tag{36}$$

Eq. (36) can be integrated with respect to V if we set

$$a_1=0,\quad \beta_3(t)=\kappa\lambda(t)$$

$$a_2=-\frac{3\beta_1^2(t)}{4\beta_2(t)(\alpha(t)+\gamma(t))}$$

Thus, we obtain exact solutions of Eq. (31):

$$q(x,t)=\sqrt{-\frac{3\beta_1(t)}{8\beta_3(t)}}\cdot\left\{1\pm\tanh\left(\sqrt{-\frac{3\beta_1^2(t)}{16\beta_2(t)(\alpha(t)+\gamma(t))}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right\}^{\frac{1}{2}}\cdot e^{i(-\kappa x+\omega(t)t)} \tag{37}$$

$$q(x,t)=\sqrt{-\frac{3\beta_1(t)}{8\beta_3(t)}}\cdot\left\{1\pm\coth\left(\sqrt{-\frac{3\beta_1^2(t)}{16\beta_2(t)(\alpha(t)+\gamma(t))}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right\}^{\frac{1}{2}}\cdot e^{i(-\kappa x+\omega(t)t)} \tag{38}$$

where $\omega(t)$ is given by Eq. (34).

Eq. (37) and Eq. (38) represent dark and singular soliton solution respectively. These solitons are valid for

$$\beta_2(t)(\alpha(t)+\gamma(t))<0$$

3.4. Triple-power law

In this case,

$$F(s)=\beta_1(t)s^n+\beta_2(t)s^{2n}+\beta_3(t)s^{3n}$$

where $\beta_1(t)$, $\beta_2(t)$ and $\beta_3(t)$ are time-dependent coefficients. Therefore, RNLSE is given by

$$\begin{array}{l}iq_t+\alpha(t)q_{xx}+\left(\beta_1(t)|q|^{2n}+\beta_2(t)|q|^{4n}+\beta_3(t)|q|^{6n}\right)q+\\ \gamma(t)\left(\dfrac{|q|_{xx}}{|q|}\right)q=i\left[\delta(t)q_x+\lambda(t)\left(|q|^{2m}q\right)_x+\mu(t)\left(|q|^{2m}\right)_x q\right]\end{array} \tag{39}$$

and Eq. (9) simplifies to

$$\begin{array}{l}(\alpha(t)+\gamma(t))U''-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)U\\ -\kappa\lambda(t)U^{2m+1}+\beta_1(t)U^{2n+1}+\beta_2(t)U^{4n+1}+\beta_3(t)U^{6n+1}=0\end{array} \tag{40}$$

By using transformation $U=V^{1/2}$, Eq. (40) becomes

$$\begin{array}{l}(\alpha(t)+\gamma(t))\left((1-2n)(V')^2+2nVV''\right)-\\ 4n^2V^2\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)\\ -4n^2\kappa\lambda(t)V^{\frac{m}{n}+2}+4n^2(\beta_1(t)V^3+\beta_2(t)V^4+\beta_3(t)V^5)=0\end{array} \tag{41}$$

Balancing VV'' or $(V')^2$ with $V^{\frac{m}{n}+2}$ in Eq. (41), then we get $N=\frac{m}{n}+2$. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$V^{\frac{m}{n}+2}\ Coeff.:\ (\alpha(t)+\gamma(t))(m+1)a_{\frac{m}{n}+2}-4n^2\kappa\lambda(t)=0$$

$$V^5\ Coeff.:\ (\alpha(t)+\gamma(t))(3n+1)a_5+4n^2\beta_3(t)=0$$

$$V^4\ Coeff.:\ (\alpha(t)+\gamma(t))(2n+1)a_4+4n^2\beta_2(t)=0$$

$$V^3\ Coeff.:\ (\alpha(t)+\gamma(t))(n+1)a_3+4n^2\beta_1(t)=0$$

$$\begin{array}{l}V^2\ Coeff.:\ (\alpha(t)+\gamma(t))a_2-\\ 4n^2(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t))=0\end{array}$$

$$V^1\ Coeff.:\ (\alpha(t)+\gamma(t))(1-n)a_1=0$$

$$V^0\ Coeff.:\ (\alpha(t)+\gamma(t))(1-2n)a_0=0$$

Solving the above system leads to

$$a_0=0,\ \ a_1=0,\ \ a_3=-\frac{4n^2\beta_1(t)}{(\alpha(t)+\gamma(t))(n+1)},$$

$$a_4=\frac{4n^2\beta_2(t)}{(\alpha(t)+\gamma(t))(2n+1)},\ \ a_5=-\frac{4n^2\beta_3(t)}{(\alpha(t)+\gamma(t))(3n+1)},$$

$$a_{\frac{m}{n}+2}=\frac{4n^2\kappa\lambda(t)}{(\alpha(t)+\gamma(t))(m+1)}$$

$$\omega(t)=-\frac{1}{t}\int_0^t\{\kappa^2\alpha(t)+\kappa\delta(t)-(\alpha(t)+\gamma(t))\frac{a_2}{4n^2}\}dt \tag{42}$$

Substituting these results into Eqs. (5) and (6), we get

$$\pm(\xi-\xi_0)=\int\left[\begin{array}{l}a_2V^2-\frac{4n^2\beta_1(t)}{(\alpha(t)+\gamma(t))(n+1)}V^3-\\ \frac{4n^2\beta_2(t)}{(2n+1)(\alpha(t)+\gamma(t))}V^4-\frac{4n^2\beta_3(t)}{(\alpha(t)+\gamma(t))(3n+1)}V^5\\ +\frac{4n^2\kappa\lambda(t)}{(\alpha(t)+\gamma(t))(m+1)}V^{\frac{m}{n}+2}\end{array}\right]^{-\frac{1}{2}}dV \tag{43}$$

To carry out the integration of Eq. (43) requires that $m = 3n$. Thus, with $m = 3n$, new Eq. (43) is following:

$$\pm(\xi-\xi_0)=\int\left[\begin{array}{l}a_2V^2-\frac{4n^2\beta_1(t)}{(\alpha(t)+\gamma(t))(n+1)}V^3-\\ \frac{4n^2\beta_2(t)}{(2n+1)(\alpha(t)+\gamma(t))}V^4+\frac{4n^2(\kappa\lambda(t)-\beta_3(t))}{(\alpha(t)+\gamma(t))(3n+1)}V^5\end{array}\right]^{-\frac{1}{2}}dV \tag{44}$$

Eq. (44) can be integrated with respect to V if we set

$$\beta_3(t)=\kappa\lambda(t)\qquad a_2=-\frac{(2n+1)n^2\beta_1^2(t)}{(n+1)^2\beta_2(t)(\alpha(t)+\gamma(t))}$$

Thus, we obtain exact solutions of Eq. (39):

$$q(x,t)=\left(-\frac{(2n+1)\beta_1(t)}{4(n+1)\beta_3(t)}\right)^{\frac{1}{2n}}\cdot\left\{\left[1\pm\tanh\left(\sqrt{-\frac{n^2(2n+1)\beta_1^2(t)}{4(n+1)^2\beta_2(t)(\alpha(t)+\gamma(t))}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]\right\}^{\frac{1}{2n}}\cdot e^{i(-\kappa x+\omega(t)t)} \tag{45}$$

$$q(x,t)=\left(-\frac{(2n+1)\beta_1(t)}{4(n+1)\beta_3(t)}\right)^{\frac{1}{2n}}\cdot\left\{\left[1\pm\coth\left(\sqrt{-\frac{n^2(2n+1)\beta_1^2(t)}{4(n+1)^2\beta_2(t)(\alpha(t)+\gamma(t))}}\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)\right)\right]\right\}^{\frac{1}{2n}}\cdot e^{i(-\kappa x+\omega(t)t)} \tag{46}$$

where $\omega(t)$ is given by Eq. (42).

Eq. (45) and Eq. (46) represent dark and singular soliton solution respectively. These solitons are valid for

$$\beta_2(t)(\alpha(t)+\gamma(t))<0$$

3.5. Log-law

In this case,

$$F(s)=\ln s \tag{47}$$

so that Eq. (2) is given by

$$\begin{gathered}iq_t+\alpha(t)q_{xx}+\beta(t)\ln(|q|^2)q+\gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q=\\ i\left[\delta(t)q_x+\lambda(t)\left(|q|^{2m}q\right)_x+\mu(t)\left(|q|^{2m}\right)_xq\right]\end{gathered} \tag{48}$$

and Eq. (9) simplifies to

$$\begin{gathered}(\alpha(t)+\gamma(t))U''-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)U\\ -\kappa\lambda(t)\,U^{2m+1}+\beta(t)U\ln U=0\end{gathered} \tag{49}$$

To obtain an analytic solution, we use the transformation $U=e^V$ in Eq. (49) to find

$$\begin{gathered}(\alpha(t)+\gamma(t))\left((V')^2+V''\right)-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)\\ -\kappa\lambda(t)\,e^{2mV}+2\beta(t)V=0\end{gathered} \tag{50}$$

In order to carry out the balancing procedure in Eq. (50), it is helpful to set $\lambda(t)=0$. This indicates that the perturbed RNLSE with log-law nonlinearity can be integrated only when the self- steepening term is not present. In this case, the perturbed RNLSE collapses to

$$\begin{gathered}iq_t+\alpha(t)q_{xx}+\beta(t)\ln|q|q+\gamma(t)\left(\frac{|q|_{xx}}{|q|}\right)q=\\ i\left[\delta(t)q_x+\mu(t)\left(|q|^{2m}\right)_xq\right]\end{gathered} \tag{51}$$

and Eq. (50) reduces to

$$\left(\alpha(t)+\gamma(t)\right)\left((V')^2+V''\right)-\left(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t)\right)$$
$$+2\beta(t)V=0 \tag{52}$$

Balancing V'' with V in Eq. (52), then we get N = 2. Using the solution procedure of the trial equation method, we obtain the system of algebraic equations as follows:

$$V^2\ Coeff.:\ \left(\alpha(t)+\gamma(t)\right)a_2=0$$

$$V^1\ Coeff.:\ \left(\alpha(t)+\gamma(t)\right)(a_2+a_1)+2\beta(t)=0$$

$$V^2\ Coeff.:\ \left(\alpha(t)+\gamma(t)\right)\left(\frac{a_1}{2}+a_0\right)-$$
$$(t\omega'(t)+\omega(t)+\kappa^2\alpha(t)+\kappa\delta(t))=0$$

Solving the above system leads to

$$a_1=-\frac{2\beta(t)}{\alpha(t)+\gamma(t)},\ \ a_2=0,$$
$$\omega(t)=-\frac{1}{t}\int_0^t\{\kappa^2\alpha(t)+\kappa\delta(t)+\beta(t)-\left(\alpha(t)+\gamma(t)\right)a_0\}dt \tag{53}$$

Substituting these results into Eqs. (5) and (6), we get

$$\pm(\xi-\xi_0)=\int\left[a_0-\frac{2\beta(t)}{\alpha(t)+\gamma(t)}V\right]^{-\frac{1}{2}}dV \tag{54}$$

Integrating Eq. (54), we obtain the exact Gausson solutions of Eq. (51) as

$$q(x,t)=Ae^{-B^2\left(x+2\kappa\int_0^t\alpha(t)dt+\xi_0\right)^2}\cdot e^{i(-\kappa x+\omega(t)t)} \tag{55}$$

where $\omega(t)$ is given by Eq. (53) and the amplitude A and the inverse width B are

$$A=\exp\left(\frac{a_0\left(\alpha(t)+\gamma(t)\right)}{2\beta(t)}\right) \tag{56}$$

$$B=\sqrt{\frac{\beta(t)}{2\left(\alpha(t)+\gamma(t)\right)}} \tag{57}$$

Naturally, the width of the Gausson proposes the constraint

$$\beta(t)\left(\alpha(t)+\gamma(t)\right)>0$$

4. Conclusions

This paper lists the soliton solutions to RNLSE that is studied with time-dependent coefficients and perturbation terms with full nonlinearity. The extended trial equation method extracts these soliton solutions to the model. Besides the constraint conditions that are needed for the existence of the solitons it is seen that an additional technical condition is needed, namely the Riemann integrability of the coefficient of GVD. Bright, dark and singular soliton solutions are obtained in this paper. The results are thus encouraging to study resonant solitons further in future. Later, this model will be addressed by the aid of different integration techniques such as extended trial equation method, modified simple equation method, semi-inverse variational principle, Lie symmetry and several others. The results of those research are surely going to be available down the road.

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