Simulation and experimental study of the side LED-pumped Nd:YAG laser

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In this paper, the free-running of a LED-pumped Nd:YAG laser optically side pumped by low divergence angle and medium scale power infrared-LEDs at 810 nm is presented. In this design, a special arrangement for LED-pumping is introduced. For this purpose, 30 arrays are used with 18 LEDs in each array. The LEDs have an optical system, a divergence angle of 20 degrees and a wavelength of 810 nm with a spectral width of 30 nm. LEDs with an optical system, due to a lower divergence angle than those without an optical system, can still enter a significant amount of light into the active medium of the laser at distances far from the surface of the rod. As a result, they lead more output power to compare with the same state in LED without an optical system. Simulations for arrangements of 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided pump schemes for two LED modes with and without an optical system for two laser rods with diameters of 3 and 7 mm and 95 mm length using ZEMAX and LASCAD software programs are made. Based on the simulated outputs, the most appropriate arrangement was selected for the maximum absorbed pump power with the 30-sided pump, and finally, the output energy was 10mj for the pumped energy of 81mj.

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Keywords: Laser rod, LED-pumped Nd:YAG laser, Ray tracing, Thermal effects, ZEMAX and LASCAD software programs

1. Introduction

The LED-pumped solid-state laser is an interesting field for the laser engineering. LED pumping has some benefits such as low cost, long life, simple electronics driver, no necessity to high voltage and low thermal temperature control requirement in comparison to flash lamp and diode laser pumping. About 50 years ago, the light emitting diode (LED) was suggested as a pump source for solid-state lasers [1-7]. During early developments, the LEDs had weak electrical to optical conversion efficiency and the output power was very low. Preliminary LEDs need to be cooled to an extremely low temperature of 77K to improve the conversion efficiency and life time [2]. To overcome low absorption efficiency, Nd:YAG fiber gain medium [6] and gold reflector are applied [4]. Because of low power LED at that time, flash lamp and diode laser pumping competed with LED pumping. Nowadays, the optical power of a single LED die is able to scale up to a Watt level with a smaller chip size. In more recent experimental work, researchers have examined LED pumping of many different gain media including polymers [8], Erbium and Neodymium doped fiber amplifiers [9,10], Nd:YVO_4 [11] and Nd:Cr:YAG [12]. In 2019, Cho et al. reported the second harmonic of the Q-switched Nd:YAG laser at 532 nm with an energy of 4.5 mJ and a pulse width of 25 ns, pumped by LEDs (in 810 nm), and the maximum energy of the Q-switched laser was 14.5 mJ at 1064 nm [13]. In 2018, Tarkashvand et al. pumped the Nd:YAG laser with green and brown LEDs (520 nm and 592 nm), which achieved energies of 107 μJ and 52 μJ, respectively, by total electrical pump energy 2.6 J [14]. In this work, the electrical pump energy was selected to 2.6 J. In 2019, Zhao et al. introduced the first LED-pumped Nd:YLF laser which was actively Q-switched. In free-running mode, 165 μJ was produced and in Q-switch mode, 10.6 μJ was produced with a pulse-width of 452 ns [15].

Another method of increasing the intensity of the pump light on the laser crystal is using luminescent concentrators [16], which, of course, prevent the aggregation of the light and compact system due to their large volume. The effects of LED spectral width on Nd:YLF laser output efficiency have also been investigated [17]. The paper [18] reported segmented reflector pump systems for pumps Nd:Ce:YAG laser with visible 460nm and cool-white LEDs. As far as we know, there has been no detailed analytical discussion or numerical computation about the effect of LED divergence reducing lens in improving the absorption efficiency and the amount of absorbed pump power in different pump arrangements with different laser rod diameters. In this paper, the effect of reducing the divergence of LEDs and the diameter of the laser rod in improving and increasing the absorbed power in the laser rod are investigated. For this purpose, 5 pump arrangements, 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided were simulated in ZEMAX software.

Moreover in this paper, a precise simulation is performed for thermal effects and laser output of the side LED-pumped laser rod by using the ZEMAX and LASCAD software programs. Then, the same simulated structure is set up in the laboratory. For the 81 mJ optical pump energy, laser output energy of 10 mJ was obtained in free-running mode, which agreed well with the simulation result. The laser output energy at frequencies of

1 to 40 Hz were also measured. This high repetition rate LED pump Nd:YAG laser source can be used as a laser designator or high repetition rate laser range finder, and in the near future, it can replace high expensive diode pumped solid-state laser. The small optical conversion efficiency of the LED-pumped laser usually came from the low pump density, small spectral absorption efficiency, and weak coupling efficiency. To solve these difficulties, the pump density can first be enhanced by designing a LED array with a concentrated arrangement to increase the area fill factor. Moreover, employing high luminance LED dies can also increase the pump density. Then, higher spectral absorption efficiency can be obtained by employing LED dies with proper pump wavelength and narrower full-width-half-maximum (FWHM) with respect to the gain medium. Finally, the coupling efficiency can be significantly increased by shortening the separation between the gain medium and pump LED array without a focusing lens to overcome the large pump divergence or applying the LED die equipped with a focusing lens. Once these approaches were simultaneously accomplished, it is highly feasible to obtain a LED-pumped laser with sufficient optical conversion efficiency for practical applications.

One of the main ideas of this paper is to use the focal optics system to reduce the divergence of LEDs to achieve relatively high output energies (in the milli-joules range) despite using LEDs with medium power density. In this paper, the feasibility, design, and construction of a cavity pump with LED from 30 directions around the laser rod with perfectly optimal coupling in the laser crystal are presented. Increasing the number of LED arrays by equipping them with a focal lens and increasing the cross-sectional area of the laser rod will increase the pump power and absorption power in the active medium and the output power. As far as we know, the maximum possible power of LEDs in terms of construction is 5 times that of the LEDs we used in this article. Since we did not have high-power LEDs available, we attempted to achieve an optimal design in terms of the pairing of pump light on the laser rod by using a focal lens and increasing the number of dies. In this paper, first a precise simulation of five pump arrangements: 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided is performed with ZEMAX software for LEDs with and without lens and rod diameters 3 and 7 mm, 95 mm length are employed to evaluate the pump operation after the thermal effects and laser output of the side LED-pumped laser simulated with LASCAD software. Then, we set up the best simulated structure in the laboratory. For 30-sided pump scheme with 81 mJ pump energy, we obtained laser output energy of 10 mJ in free-running mode, which agreed well with the simulation results. The laser output energy at frequencies of 1 to 40 Hz was also measured.

2. Simulation

2.1. Simulation of ray tracing by ZEMAX software

5 pump arrangements were accurately simulated in order to achieve high absorption efficiency as well as the highest possible output energy with not very powerful LEDs (with maximum peak power of about 0.6W). The arrangement of 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided pumps with 18 LEDs per array was simulated in ZEMAX with the aim of calculating the absorption efficiency and optimum efficiency of pump power density. The graph of the peak power changes of each die according to the current is shown in Fig. 1. The energy of each array of LEDs is 2.7 mJ at a maximum current of 2.3 A.

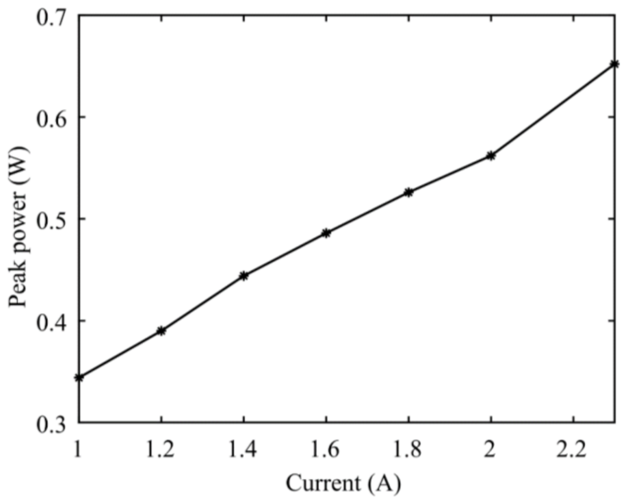


Fig. 1. Peak power versus current for single LED die (SFH4780S OSRAM CO)

In Fig. 2, there is a diagram of the arrangement of 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided pumps by replacing laser rods with diameters of 3 and 7 mm, 95 mm length; and LEDs with divergence angles of 20 degrees (with lens) and 120 degrees (without lens) are displayed. In different arrangements, the distance between the LED and the center of the laser rod varies from 0.32 to 18.9 mm depending on the diameter of the rod and the schematic LED arrangement.

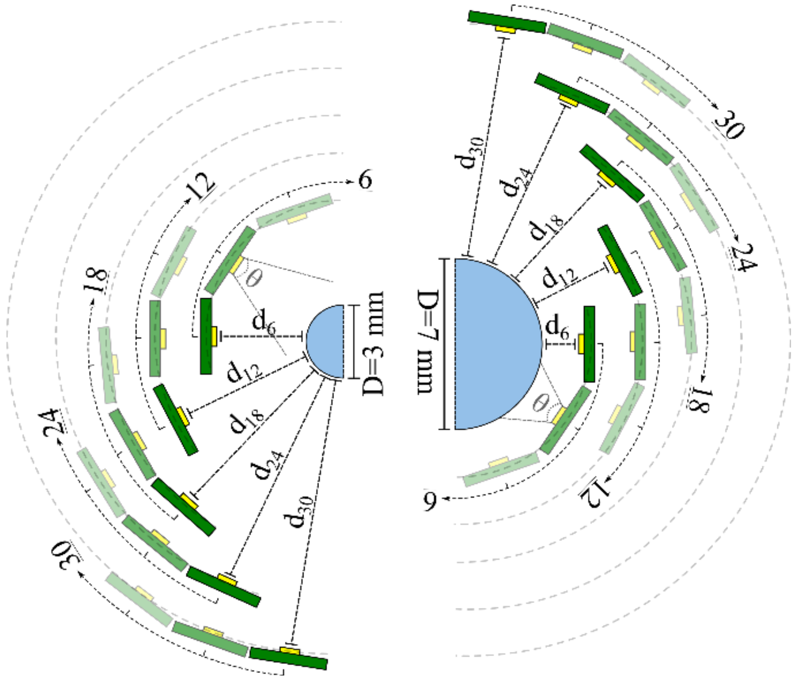


Fig. 2. Pump schematic LED arrangement of 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided pump, for laser rods with diameters of 3 and 7 mm (color online)

By considering the number of LEDs, their distance from each other, their distance to the laser rod surface, their pump distribution, their divergence angles and optical power, the pump radiation transfer from the LED arrays to the laser rod is modeled by the non-sequential mode of ZEMAX software (see Fig. 3).

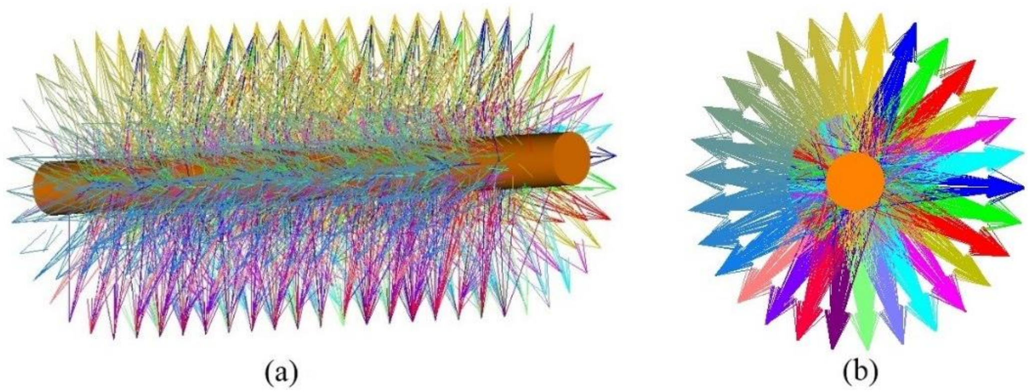


Fig. 3. (a) Modeling a side LED-pumped laser rod by 30 LED arrays in ZEMAX software, (b) Cross section of pumping rod (color online)

The Nd:YAG rod was added as a new material in a custom glass catalog. Both the refractive and absorption properties are entered. A detector volume, consisting of 150× 150×151 = 3397500 voxels, is used to display the tracking and absorption of rays. The detector volume records the absorbed flux in each volume. 3,000,000 rays have been traced, and the absorbed flux in the laser rod is measured.

The absorbed flux and its data in cross-section of the laser rod for five state pump schemes 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided for both selected rod diameters are shown in Fig. 4. The uniformity of the absorbed pump in Fig. 4 shows that we have achieved the purposes of using the symmetric pump design mentioned in this article. The absorption data were exported as a text file for subsequent analysis by LASCAD software.

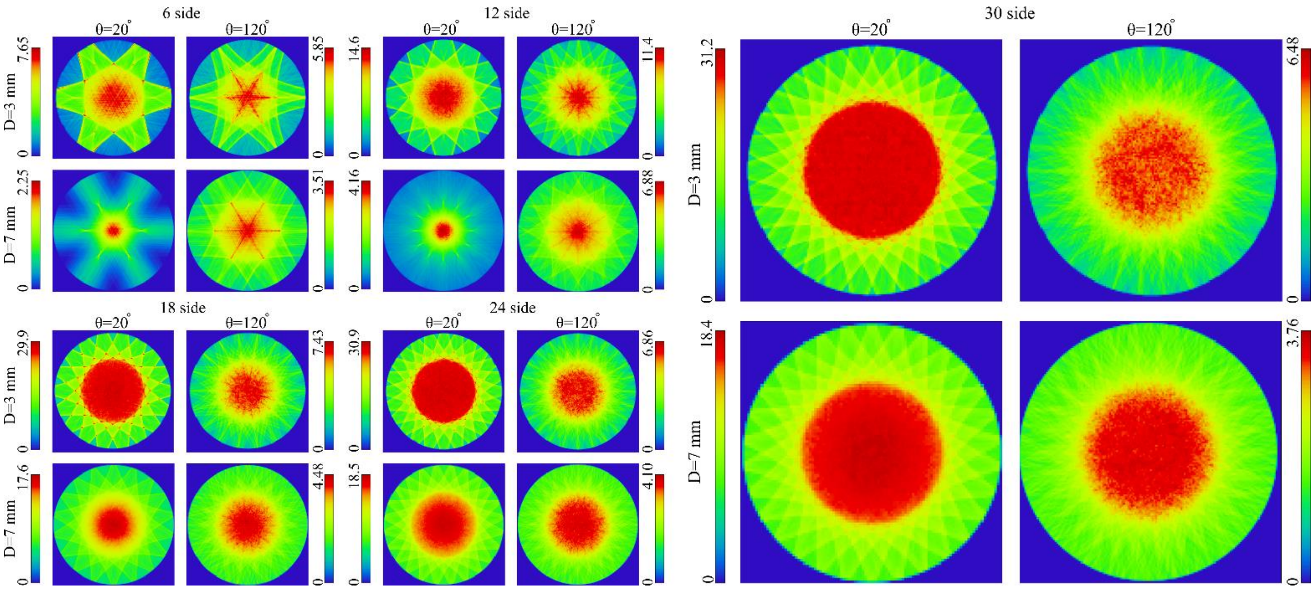
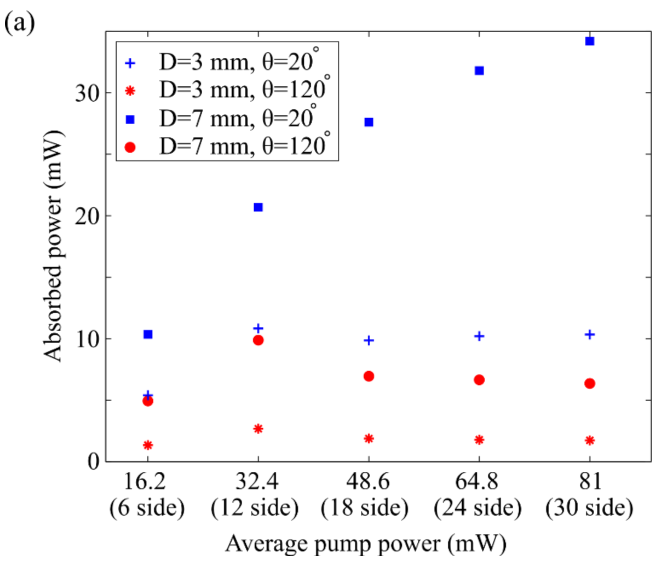


Fig. 4: Absorbed flux and its data in cross-section of laser rod arrangement of 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided pump schemes, for laser rods with diameters of 3 and 7 mm, as well as LED arrays with and without lenses, the units of colorbars are ($\frac{\mu w}{mm^3}$) (color online)

Simulations of 6-sided, 12-sided, 24-sided, 18-sided, and 30-sided pumps were made, considering that the average pump power for each LED array is 2.7 mW. The ray tracing results are shown in Fig. 5, where the numerical values of the absorbed power and absorption efficiency are given.



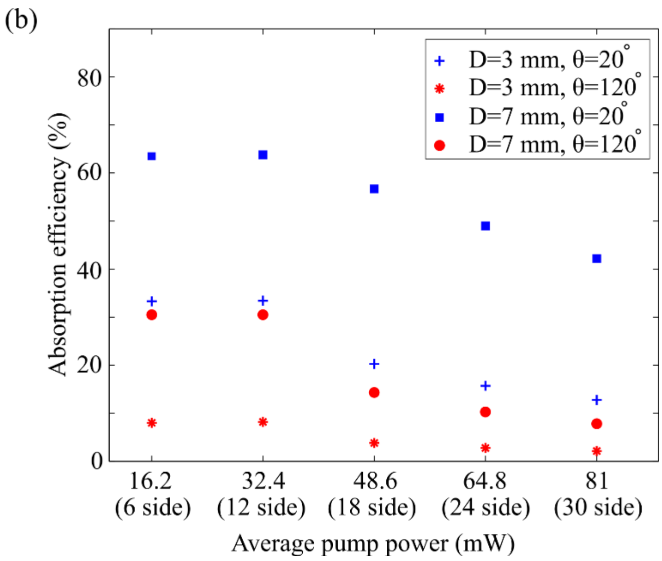


Fig. 5. Absorbed power and absorption efficiency in 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided pump schemes, for laser rods with diameters of 3 and 7 mm, as well as LED arrays with and without lenses (color online)

Figs. 5(a) and 5(b) show the absorbed power and absorption efficiency for all arrangements of 6-sided, 12-sided, 18-sided, 24-sided, and 30-sided with laser rods with diameters of 3 and 7 mm and divergence angles of 20 and 120 degrees. According to diagram 5(a), it can be seen that the absorbed power in the case of using LED with lens is much more than that in the case of LED without lens for both diameters of the rod used. When using an LED with a lens in the rod with a diameter of 7 mm, despite the decrease in inclination, the absorbed power continues to increase at longer distances. While for LEDs without lenses at distances greater than 12 sided, despite the increase in pumped power due to the increase in the number of arrays, the power absorbed by the rod is decreasing. In diagram 5(b), the effective absorption efficiency can be seen, as the number of pump arrays increases, or in other words, the distance of the arrays from the rod surface increases, the effective absorption efficiency decreases. Of course, the absorption efficiency for LED with lens is higher than that for LED without a lens. Generally, although the absorption efficiency of 30-sided pump for LED with a lens is a little lower than that for the arrangements of 6-sided, 12-sided, 18-sided, and 24-sided pumps, its absorbed power is more than other arrangements.

2.2. Analysis of thermal effects by LASCAD software

Finite Element Analysis (FEA) is used to compute the temperature distribution, deformation, and stress or fracture mechanics in the laser rod. The FEA code of LASCAD software was specifically applied to calculate the thermal effects. Since the distribution of the absorbed pump power density has been computed by ZEMAX software, data generated with this software can be used as input for LASCAD software, which interpolates the 3D data set with respect to the grid used with the FEA code. Now, by specifying the dimensions of the laser rod, cooling conditions, material parameters, and meshing structure, LASCAD software can be run. After completing calculations and simulations by LASCAD software, the graphs of thermal effects such as the temperature and stress distributions, the pump absorbed distribution in directions x and y and the deformations are available. 3D temperature distributions for the laser rod are shown in Fig. 6. According to this figure, the minimum and maximum temperatures of the rod are 298 and 305.9 K, respectively. 2D temperature distributions in the directions of width and thickness of the rod (x and y axes) are shown in Fig.7 According to Fig. 7, the temperature distribution in directions x and y axes is symmetric. LASCAD software also calculates the focal length of the thermal lens. For the simulation presented in this paper, the focal lengths were obtained 531 m and 650 m in y-z and x-z planes, respectively. All thermal simulations were performed at a maximum accessible average pump power of 3 W (30-sided pump scheme) for a 7 mm rod diameter, 95mm length at a frequency of 40 Hz.

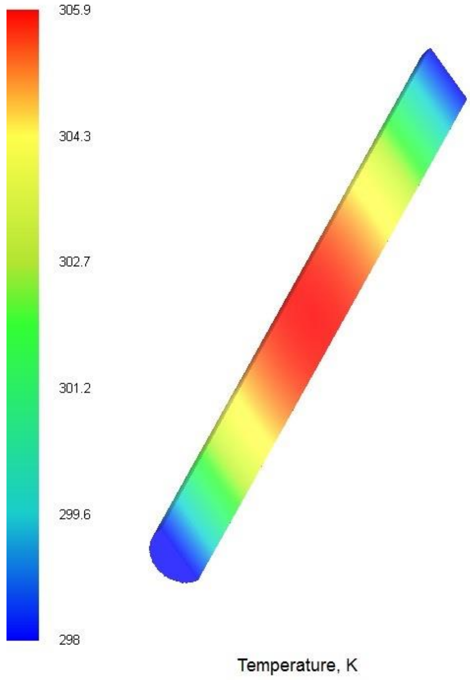


Fig. 6. 3D temperature distribution for the laser rod with a longitudinal cut from the middle of it (color online)

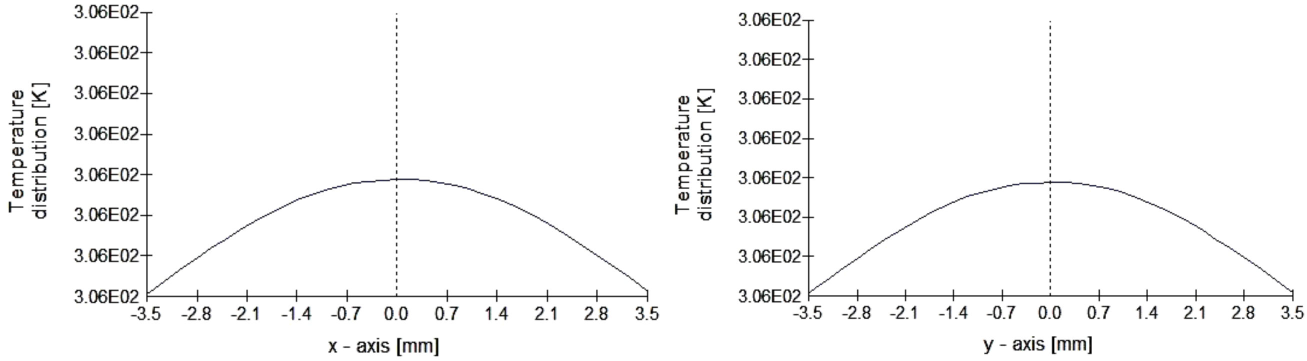


Fig. 7. 2D temperature distribution in the middle of the rod (z=47.5 mm) in directions of x and y axes

The simulated graph of the output power in terms of the output mirror reflectivity is also extracted from the LASCAD software to ensure that the choice of output mirror reflectivity is optimal (Fig. 8). According to the diagram in Fig. 8, it is clear that the optimum reflectivity for the output mirror is 94% to 97%. In the laboratory, it was found out that a reflectivity of 94% yields the best results. Another parameter that was extracted from LASCAD software was the laser output energy.

Fig. 9 shows the laser output energy at 40 Hz for different pump energies. According to Fig. 9, for the maximum pump energy (81 mJ), which is equivalent to the absorbed energy of 40 mJ, the laser output energy is simulated at about 10 mJ, and the optical-to-optical efficiency for free-running is 12.3%.

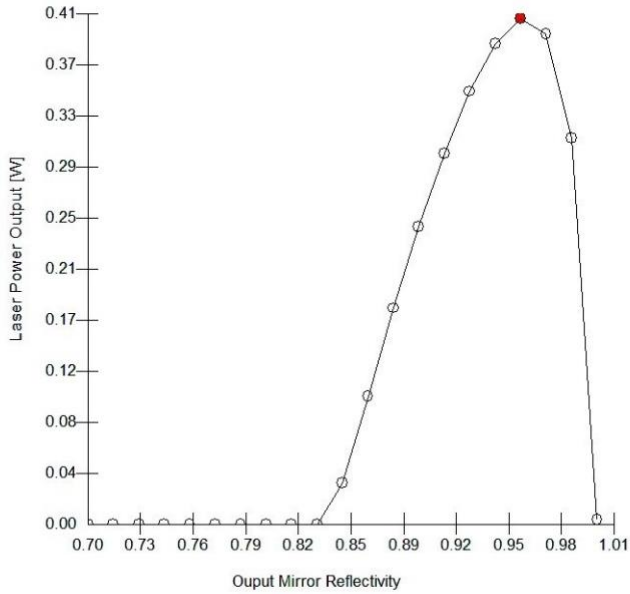


Fig. 8. Laser output power in terms of output mirror reflectivity (color online)

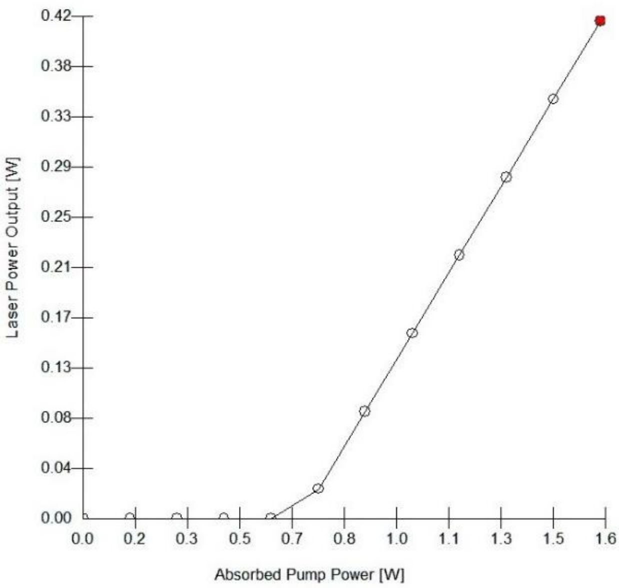


Fig. 9. Laser output energy at 40 Hz for different pump energies (color online)

3. Experimental setup

The experimental setup of a LED-pumped Nd:YAG laser is schematically shown in Fig. 10. The resonator length should be as short as possible because a compact laser is to be designed. The unsymmetrical resonator, plano-concave resonator, is selected.

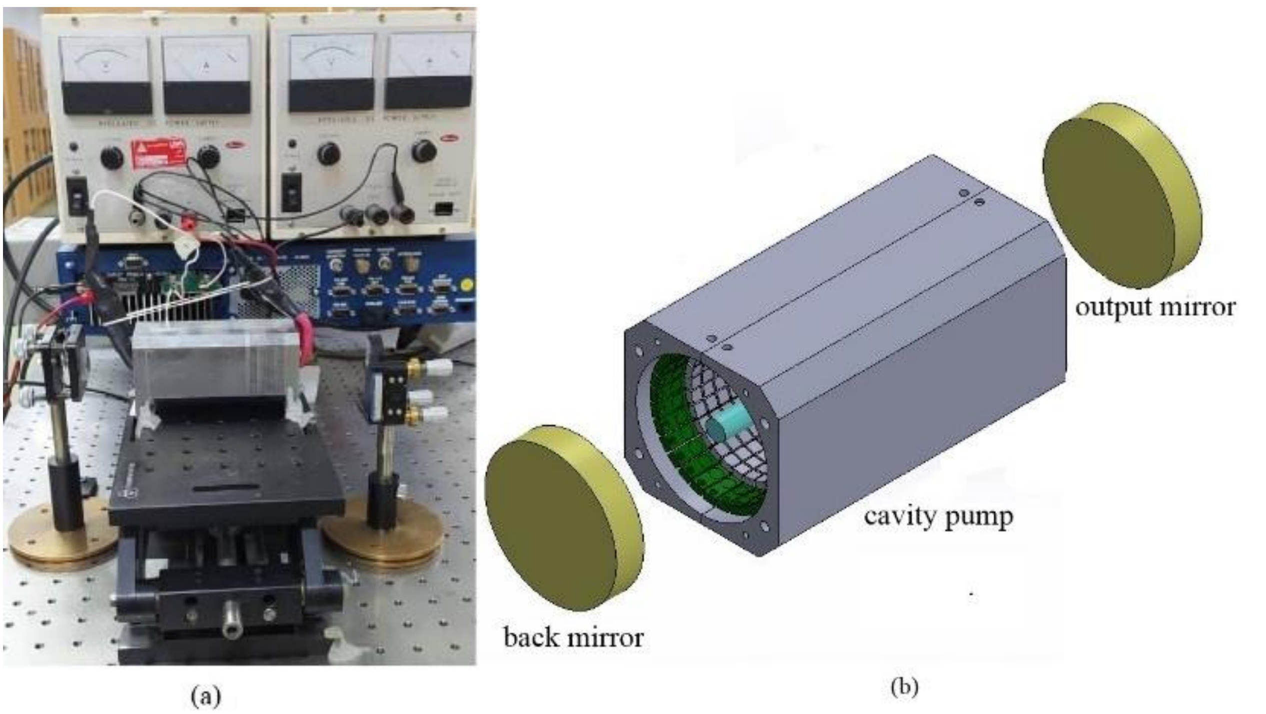


Fig. 10. Experimental setup for a side LED-pumped Nd:YAG laser, (a) the picture of setup (b) schematic drawing of the setup (color online)

The total length of the resonator was selected to be 225 mm in order to have a compact laser. The back mirror is a concave mirror and its concave surface has high-reflectivity coated at 1064 nm with a curvature radius of +2000 mm, and the output mirror is a flat output coupler mirror with a reflectivity of 94% at 1064 nm. The reason for choosing the output mirror and the amount of its reflection is to achieve a high mode volume for TEM_{00} mode.

In this design, our final goal is to increase the pumping efficiency and suitability for easy assembly. Moreover, compactness and reliability are important. A schematic drawing of the pump head, consisting of 30 arrays arranged in the thirty-fold symmetry direction around the laser rod, is illustrated in Fig. 11(a). This pump design is selected mostly because of the uniformity of the absorbed pump in the laser rod and the uniform distribution of thermal effects. Such a pump design and mechanical arrangement around the rod itself is an innovation for such lasers. The Nd:YAG rod used in the experiment had a length of 95 mm and a diameter of 7 mm with 1.1% Nd^{3+} dopant concentration. Both end faces of the rod are flat-flat, and antireflection coated at 1064 nm reduces the intra-cavity losses. The rod was pumped by 30 arrays of LEDs (SFH4780S OSRAM CO.), and the energy of each LED array was 2.7 mJ at a current of 2.3 A. Therefore, the maximum total pumping energy was 81 mJ. Each LED array was 92 mm in length and 4 mm in width, and contained 18 dies, Fig. 11(b). The wavelength of the LEDs was 810 nm. The peak-point wavelength was 819 nm and the full width at half-maximum (FWHM) of the spectral bandwidth was 30 nm. The pulse width of the LED pump was selected to be 230 microseconds due to the fluorescence lifetime of the Nd:YAG crystal. The presence of a converging lens on the LEDs leads to the optimal coupling of the pump light in the laser rod. Each die requires a voltage of 3.8V and a current of 2A to start. The distance from the outer surface of each die to the lateral surface of the laser rod is 15.4 mm that makes an optimal coupling of LED light to the rod surface. Each die uses a focal lens to reduce divergence, which reduces the divergence half angle by ±10 degrees.

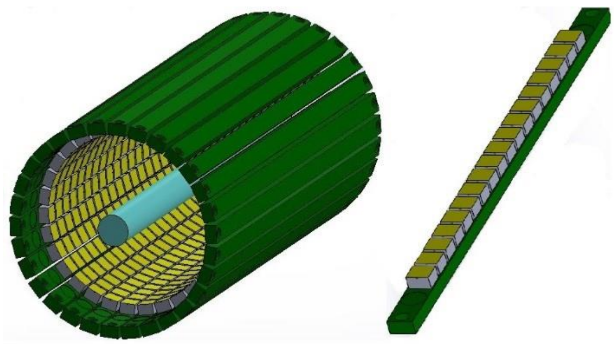


Fig. 11. (a) Schematic drawing of the LED-pumped laser head, (b) An array of 18 dies (color online)

4. Experimental results

The laser output energy in free-running mode at a frequency of 40 Hz with a pulse width of 230 μs for different pump energies is shown in Table1. The optical-to-optical efficiency for free-running is 12.3%.

Table 1. The laser output energy in free running mode at a frequency 40 Hz with a pulse width of 230 μs for different pump energy

Total current (A)	E_{pump} (mJ)	E_{output} (mJ)
70	81	10
67	78	9.7
65	75	9.2
60	69	8.7
54	65	6.8
48	60	5.5
42	55	4.5
36	48	2.5
30	42	1.2
28	37	0.8
25	27	0.6

The maximum, minimum and average output energies of the laser, in free running mode at a frequency of 1 to 40 Hz and total current of 70 A (pump energy of 81 mJ) are shown in Table 2. These measurements were recorded after 500 laser pulses.

Table 2. Experimental results for laser output energy in free running mode at a frequency of 1 to 40 Hz

f (HZ)	E_{max} (mJ)	E_{min} (mJ)	E_{avg} (mJ)	P_{avg} (mW)	PTP stability	RMS stability
1	10.9	10.1	10.4	20.8	7.69%	1.677%
5	10.6	9.07	9.97	49.8	15.34%	2.640%
10	10.5	9.11	9.68	96.8	14.35%	3.778%
20	10.1	9.01	9.27	185	11.76%	5.206 %
30	9.93	8.07	8.84	265	21.04%	5.406%
40	10.7	8.58	9.46	378	23.04%	5.711%

The output beam distribution of the laser from designed resonator is monitored by a beam profiler (wincam D_UCD12 Dataray Co.) and the output intensity distribution at a frequency of 40 Hz is shown in Fig. 12. In this figure, the 2D and 3D output beam profiles and Gaussian fit are also shown.

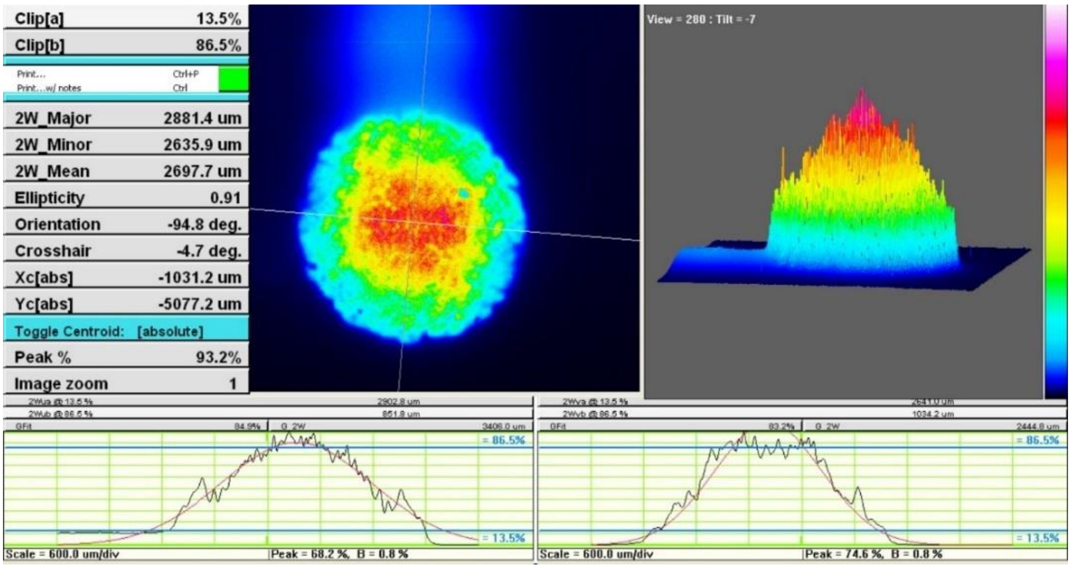


Fig. 12. 2D and 3D spatial profile for the laser beam measured by a CCD camera in near field at distance 20 cm from the output mirror (color online)

5. Discussions

The results of the investigations show that reducing the divergence angle of the LEDs gives the ability to arrange the LEDs at a further distance from the laser rod. With this method, more LED arrays can be installed around the laser rod (increasing the pump power), moreover, increasing the diameter of the laser rod increases the absorbed pump power and then the output energy of the laser. The important and noteworthy point in comparing this paper with that by Cho et al. [13] is the peak power of LEDs. The peak power of our LED was 0.6 W and that of theirs was 2.6 W. The peak power of their LED was about 4 times stronger, but their LED was without a lens, which causes a decrease in absorption efficiency. For this purpose, in this paper, the effects of the lens reducing the divergence and the diameter of the laser rod in increasing the absorption efficiency and absorbed pump energy have been investigated. The absorption efficiency of reference [13] is about 20% and that of our article is 42%. If we use lensed LEDs with a peak power of 2.6 W in the pump arrangement of our article, it will lead to output energy of 81 mJ, which leads to a 157% increase in laser output energy compared to the paper by Cho et al. [13] with an energy of 31.5 mJ. In the arrangement of 30-sided, reducing the divergence angle from 120 to 20 degrees by using a laser rod with a diameter of 7 mm increases the absorption efficiency of 7 percent to 42 percent and the absorbed power from 6.36 mW to 34.2 mW, and the arrangement results in an output energy of 10 mJ, an optical-to-optical efficiency of 12.5% and a slope efficiency of 18.5% in free running mode at 40 Hz despite the use of less powerful LEDs (0.6 W peak power per die). The agreement of the simulation results of the LASCAD software with the laboratory results confirms the results of the absorption efficiency calculated from the ray tracing method in ZEMAX with those of LASCAD software. Laser output energy was measured for frequencies from 1 to 40 Hz, and PTP stability changed from 7.69% to 23.04% with increasing repetition rate from 1Hz to 40Hz. The experimental and simulated results were compared. According to the data in Table 1, (experimental data at 40 Hz frequency) and Fig. 13 (simulated results with LASCAD), we drew a new graph and combined both results, is shown in Fig. 13. In this figure, it can be clearly seen that there is an excellent agreement between the experimental and simulated results. After 500 shots of laser operation, relatively good stability is seen in the laser output for different frequencies. According to these results slope efficiency (σ_{slope}), is calculated 18.5%.

$$\sigma_{slope} = \frac{E_{out}}{E_{pump} - E_{th}} = \frac{10mJ}{81mJ - 27mJ} = 0.1852$$

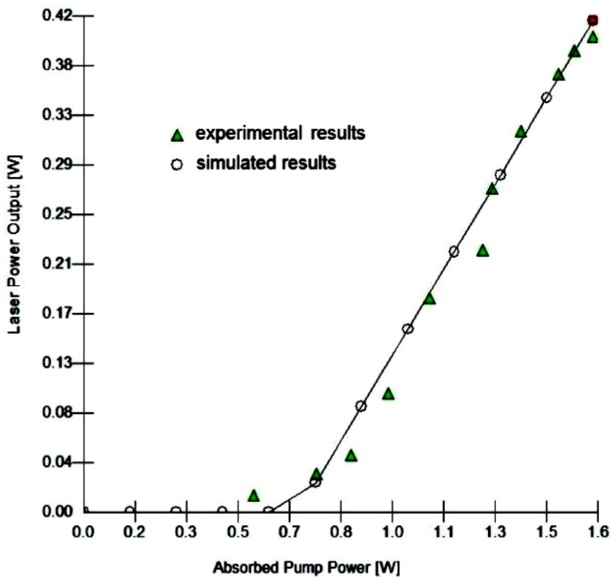


Fig. 13. Comparative diagram of experimental and simulated results for laser output power at 40 Hz frequency and maximum pump energy of 81 mJ (color online)

6. Conclusion

In the case of using LEDs without an optical system for pumping, the 12-sided mode results in higher absorption efficiency and absorbed power than other modes do. As the distance from the surface of the laser rod increases, although the number of LED arrays can be increased, the power absorbed by the Nd:YAG rod remains relatively unchanged. In other words, due to the high divergence angle of LEDs without an optical system, at longer distances, despite the increase in the number of arrays, less power from each array reaches the laser rod. As a result, the absorbed power is approximately equal to the states with fewer arrays (closer distances). However, if LEDs with an optical system are used, this happens at much farther distances so that in the case of 30-sided pump, despite the decrease in absorption efficiency, the absorbed power is still increasing. The 30-sided pump mode for LEDs with an optical system results in about 5.3 times more absorbed power than the same mode in LEDs without an optical system. The maximum absorbed power for the LED with an optical system occurs on 30 pump sides, which is about 3.1 times as much as the absorbed power for LEDs without an optical system, which occurs on 12-sided pump scheme. To promote and develop research activities in the future, the combination of high-power LEDs equipped with divergence-reducing lenses can create a significant improvement in scaling up the power of LED pumped lasers. Furthermore, the field of producing intense pulses is created by the Q-switch method.

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